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The Ricardian Determination of the General Rate of Profit prior to and independent of Prices or The Sraffian Standard System as a misconception of the Ricardian Corn Production Sector

by
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Whoever has dealt albeit in an elementary way with the problem of determining prices of production and the general rate of profit for a given wage rate knows that its solution often appears to the mathematically ignorant as a vicious circle. For –while here the prices of production and the general rate of profit can be determined and indeed 'simultaneously'– someone who is mathematically ignorant tends, because of the fact that prices of production are dependent on the general rate of profit and, conversely, the general rate of profit is dependent on prices of production, to conclude that it is impossible for one to determine prices of production without already knowing the general rate of profit and impossible to determine the general rate of profit without already knowing the prices of production, and for precisely this reason to subsequently ascertain that the determination of the general rate of profit and prices of production for a given wage rate is impossible.

The above conclusion must have been drawn also by Ricardo from the interdependence of the general rate of profit and prices of production. However, this did not inevitably lead him to the ascertainment that the determination of prices of production and the general rate of profit for an exogenously given wage rate –or more precisely, for an exogenously given *real* wage rate, since in determining the prices of production and the general rate of profit, Ricardo considered the real and not the nominal wage rate to be given– is impossible. He must have considered this determination to be possible, despite the vicious circle which he thought it entailed, i.e. ultimately he must have believed that this vicious circle was only apparent and consequently that it

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simply constituted a technical-mathematical, a technical-calculating difficulty, because otherwise his ingenious idea for resolving the problem cannot be explained.

This idea consists in one first determining the general rate of profit prior to and independent of production prices and then –for a now known general rate of profit– the prices of production. That is, he believed ultimately that although the general rate of profit is dependent on prices of production, it is possible for it to be determined prior to and independent of them, and then for a now known rate of profit to determine the prices of production and thus resolve the problem –for a given real wage rate– of determining the general rate of profit and prices of production, the solution to which initially seemed like breaking a vicious circle and therefore impossible. And he did indeed solve the problem in this way! Let us see exactly how he did so.

Ricardo must have thought as follows: because the general rate of profit is by definition an equal rate of profit in all sectors of production, then to know it, it suffices to know the rate of profit of any sector. He must then have wondered under what conditions is it possible for a given real wage rate and without prior knowledge of prices of production for one to know the rate of profit of a sector. Evidently, only when this latter can be defined as the ratio of two homogenous physical magnitudes, namely the surplus product and physical capital, i.e. the aggregate of the means of production and real wages, of the said sector. And when can the rate of profit of a sector be defined in this way? Clearly, when that sector produces a commodity, which it uses both as a means of production and as a wage commodity, without simultaneously using further means of production and further wage commodities other than the aforesaid commodity. However, because the real wage rate is by assumption the same in all sectors, all the sectors use the same –single or joint– wage commodity. Therefore, the commodity produced by the said sector is used not only by that sector itself but also by the other sectors as the only wage commodity.

So, Ricardo constructs just such a sector, which he graphically calls a corn production sector. Under the above conditions, corn is the only reproductive commodity of the entire production system. This means, firstly, that corn is a reproductive commodity and secondly, that there is no other reproductive commodity other than corn. Corn is a reproductive commodity because it is a wage commodity and as such, it directly enters into the (re)production of labor power and thus, indirectly, also into the production of all commodities. Any other commodity could also be a reproductive commodity either if it was, like corn, a wage commodity or if it directly or indirectly entered into the production of a wage commodity. The former can be ruled out, because there is no other wage commodity apart from corn. The latter can also be ruled out because, firstly, as we noted, corn is the only wage commodity and secondly, no other commodity apart from corn itself enters directly –and therefore nor indirectly– into the production of corn. Consequently, there is no other reproductive commodity apart from corn. All the other commodities are non-reproductive.

It is clear that for a given *real* wage rate, one can determine the rate of profit of the corn production sector prior to and independent of the prices of production of the other commodities as the ratio of two homogenous physical magnitudes, as the ratio of the surplus product to the aggregate of the means of production and real wages of the corn production sector. Let's see how.

The gross product of the sector in question is a certain quantity of corn. If we deduct from this quantity the means of production, which also consist of corn, then we get the net product of the sector as a quantity of corn. If from the net product we deduct total real wages, which also consist of corn, we get the surplus product of the sector as a quantity of corn.

The capital of the corn production sector consists –from a physical viewpoint– just like the capital of every other sector, of the means of production and real wages. However, in the corn production sector, both the means of production and real wages consist of corn. Consequently in this sector, the capital too consists, from a physical point of view, only of corn.

Thus, because in the corn production sector both the surplus product and physical capital consist of corn and consequently are homogenous magnitudes, we can form the ratio of the former to the latter, which clearly is none other than the rate of profit of the corn production sector. So, the rate of profit of this sector is given prior to and independent of the prices of production. For, not only do we not need prices in order to calculate the rate, but even if we calculate it with the help of prices, i.e. the only price which we need in the case at issue, namely the price of corn, then it does not vary, whatever the latter may be. As long as it is not equal to zero. If we calculate it with the help of prices, i.e. as the ratio of the price of the surplus product to the price of physical capital of the corn production sector, then clearly it does not vary, whatever the price of corn may be. For all we are doing is multiplying the numerator and the denominator of the initial fraction by the price of a unit of corn, i.e. by the same (positive) number.

However, because, as we have already pointed out, the general rate of profit is by definition an equal rate of profit in all production sectors, the rate of profit of the corn production sector, which we calculated prior to and independent of prices, is equal to the general rate of profit of the entire production system. We thus calculated –according to Ricardo– the general rate of profit prior to and independent of prices.

With the general rate of profit given, in the above manner, Ricardo then goes on to calculate the prices of production. The apparent vicious circle has been resolved. And it was resolved in such a simple manner which befits the genius of someone like Ricardo.

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Let us now see the above more vividly with the help of mathematics. The given system of production is a system of production [A, l, X], which uses the production technique [A, l] and produces the strictly positive gross product X, $X > 0$. The matrix of technical coefficients is symbolized by A and the vector of labor inputs per unit of produced commodity by l. We have

$$A \geq 0$$

and

$$l > 0.$$

Because the technique is productive, the maximum eigenvalue λ_m^A of A is smaller than unit. And because $A \geq 0$, λ_m^A is positive. Therefore

$$0 < \lambda_m^A < 1$$

Let sector 1 be the corn production sector. Then

$$\alpha_1 = (\alpha_{11}, 0, 0, \ldots, 0)^T,$$

where α_1 the first column of A.

Let d,

$$0 \leq d \leq Y / lX,$$

be the real wage rate, where Y,

$$Y = (I - A)X,$$

the net product of the system and consequently, Y/lX the average physical productivity of labor. The real wage rate consists only of commodity 1, i.e. corn. Consequently

$$d = (d_1, 0, 0, ..., 0)^T,$$

where d_1 the quantity of corn which workers receive for one hour of labor.

The matrix dl of inputs in real wages is according to the above

$$dl = \begin{bmatrix} d_1 l_1 & d_1 l_2 & \cdots & d_1 l_n \\ 0 & 0 & \cdots & 0 \\ & & \cdots & \\ . & & \cdots & . \\ & & \cdots & . \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

and the matrix $\bar{A}$ of technical coefficients supplemented by real wages is

$$\bar{A} = A + dl = \begin{bmatrix} (\alpha_{11} + d_1 l_1) & (\alpha_{12} + d_1 l_2) & \cdots & (\alpha_{1n} + d_1 l_n) \\ 0 & \alpha_{22} & \cdots & \alpha_{2n} \\ . & & \cdots & \\ . & & \cdots & \\ . & & \cdots & \\ 0 & \alpha_{n2} & \cdots & \alpha_{nn} \end{bmatrix}$$

As a consequence of $0 < \lambda_m^A < 1$ and $0 \le d \le Y/lX$

$$0 < \lambda^{A} < 1$$

i.e. for the given real wage rate the given technique is surplus productive, that is to say, it produces for each exogenously given surplus product U,

$$U = Y - dlX = (I - \bar{A})X$$

a positive or strictly positive X and here, where X is exogenously given and strictly positive, a positive or strictly positive surplus product U, $U \ge 0$ or $U > 0$.

Just like A, $\bar{A}$ too is reducible. Consequently, on the condition that the real wages are paid in advance at the beginning of the period, we have for prices of production

$$P_1 = (1+r)P_1(\alpha_{11} + d_1 l_1) \tag{1}$$

and

$$\hat{P} = (1+r)\hat{P}A_{22} + (1+r)P_1[(\alpha_{12} + d_1 l_2), \ldots, (\alpha_{1n} + d_1 l_n)], \tag{2}$$

where P_1 the price of commodity 1, and $\hat{P}$,

$$\hat{P} = (P_2, P_3, \ldots, P_n),$$

the prices of commodities 2, 3, ..., n, r the general rate of profit and A_{22}.

$$A_{22} = \begin{bmatrix} \alpha_{22} & \alpha_{23} & \cdots & \alpha_{2n} \\ \alpha_{32} & \alpha_{33} & \cdots & \alpha_{3n} \\ . & . & \cdots & . \\ . & . & \cdots & \\ . & . & \cdots & . \\ \alpha_{n2} & \alpha_{n3} & \cdots & \alpha_{nn} \end{bmatrix}$$

For $P_1 > 0$ we get from (1)

$$r = \frac{1-(\alpha_{11} + d_1 l_1)}{\alpha_{11} + d_1 l_1} \tag{3}$$

As a consequence of $(0<)\lambda_m^{\bar{A}} < 1$, we have $(0<)\alpha_{11} + d_1 l_1 < 1$ and consequently r is positive.

The r given by (3) is the rate of profit of sector 1 (= corn production sector) and at the same time the general rate of profit.

We wish to stress that the general rate of profit, which we calculated here according to Ricardo prior to and independent of prices, was calculated *before we normalized* prices. It is important to remember this.

If we normalize prices with

$$P_1 = 1, \tag{4}$$

then by replacing (3) and (4) in (2) we get the absolute prices of the other commodities, i.e. of commodities 2, 3, ..., n.

Thus, for a given real wage rate we determined –without having previously normalized prices– firstly the general rate of profit prior to and independent of prices and subsequently, after normalizing prices, for a now given general rate of profit the (absolute) prices of the commodities.

Of course, this solution of the problem then gives only positive prices for all commodities, when and only when $(\alpha_{11} + d_1 l_1) > \lambda_m^{A_{22}}$, where $\lambda_m^{A_{22}}$ the maximum eigenvalue of A_{22}. Otherwise, i.e. when $(\alpha_{11} + d_1 l_1) < \lambda_m^{A_{22}}$, Ricardo's solution gives, as emerges from (2) and $P_1 > 0$, for r, which is defined by (3), negative prices for certain of the non-reproductive commodities, i.e. the commodities 2, 3, ..., n.

In addition, Ricardo's solution is compatible with each normalization of prices, only when $(\alpha_{11} + d_1 l_1) > \lambda_m^{A_{22}}$. When, on the contrary, $(\alpha_{11} + d_1 l_1) < \lambda_m^{A_{22}}$, it is compatible only with normalization (4), in which the only reproductive commodity (commodity 1) functions as the normalization commodity. Because if, when $(\alpha_{11} + d_1 l_1) < \lambda_m^{A_{22}}$, one normalizes prices with the normalization commodity being any bundle of commodities which contains even just one non-reproductive commodity, one gets from (1) and (2)

$$r = \frac{1 - \lambda_m^{A_{22}}}{\lambda_m^{A_{22}}},$$

$$P_1 = 0$$

and

$$\hat{P} > 0$$

while Ricardo's solution gives

$$r = \frac{1 - (\alpha_{11} + d_1 l_1)}{\alpha_{11} + d_1 l_1}$$

$$P_1 > 0$$

and a $\hat{P}$, which contains also negative components. In this same case, i.e. in the case in which $(\alpha_{11} + d_1 l_1) < \lambda_m^{A_{22}}$ and prices have been normalized with the normalization commodity being a bundle of commodities containing even just one non-reproductive commodity, the rate of profit of the corn production sector is equal to

$$\frac{[1-(\alpha_{11}+d_1 l_1)]P_1}{(\alpha_{11}+d_1 l_1)P_1}$$

and consequently, because of $P_1 = 0$, undetermined. For precisely this reason, it can be set equal to the general rate of profit $(1-\lambda_m^{A22})/\lambda_m^{A22}$ of the non-reproductive sectors and thus create the impression that there is a general –for all sectors equal– rate of profit.

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Ricardo's solution appears –because of the exceptionally restrictive prerequisites, that there is a sector, which uses solely and exclusively the commodity which it produces not only as a means of production but also as a wage commodity and that the commodity of this sector is the only wage commodity used also by the other sectors– to be of limited validity. This must have been what Georg Charasoff thought when he gave a generally valid solution to the problem of determining – for a given real wage rate –the general rate of profit prior to and independent of prices. However, as we shall see, Charasoff's solution shows ultimately that Ricardo's solution is, when it is possible, generally valid, i.e. that Ricardo's idea can be applied –exactly the same– also when there is not, as indeed there is not in reality, a corn production sector.

We shall describe Charasoff's solution. Assuming that there is no Ricardian corn production sector, as we described above, but that of the n sectors of the production system, the m first, 1 < m < n, are reproductive, while the remaining n-m are non-reproductive sectors. The real wage rate d consists of course, because all the wage commodities are reproductive, of some or all of the reproductive commodities and no non-reproductive commodity.

The matrix $\bar{A}$ consequently has the form

$$\bar{A}=\begin{bmatrix}\bar{A}_{11} & \bar{A}_{12}\\ 0 & A_{22}\end{bmatrix},$$

with

$$0<\lambda_m^{\bar{A}}<1.$$

The interpretation of $\bar{A}_{11}$, $\bar{A}_{12}$ and A_{22} is clear. In addition, $\bar{A}_{11}$ and A_{22} are irreducible.

Charasoff constructed –in 1910, half a century before Sraffa!– a composite sector consisting of all the reproductive sectors, and only those, in proportions such that its gross product, physical capital and surplus product have the same composition, i.e. they are, from a physical viewpoint, homogenous magnitudes. That is, he constructed a composite 'corn' production sector! The gross product of the sector $\bar{q}$ is defined as follows

$$\alpha\bar{q}=\bar{A}_{11}\bar{q}. \tag{5}$$

Because $\bar{q}>0$, it follows from (5) that

$$\alpha=\lambda_m^{\bar{A}_{11}}$$

and consequently

$$\lambda_m^{\bar{A}_{11}}=\bar{A}_{11}\bar{q}, \tag{6}$$

i.e. that $\bar{q}$ is the –with the exception of a fully determined scalar– strictly positive right eigenvector of $\bar{A}_{11}$, which corresponds to its maximum eigenvalue. Let us normalize $\bar{q}$ in any manner whatsoever, so that it is fully determined. Then the surplus product $\bar{u}$ which corresponds to the gross product $\bar{q}$ is

$$\bar{u}=(I_{11}-\bar{A}_{11})\bar{q}=\bar{q}-\bar{A}_{11}\bar{q}=\bar{q}-\lambda_m^{\bar{A}_{11}}\bar{q}=(1-\lambda_m^{\bar{A}_{11}})\bar{q}. \tag{7}$$

The means of production and the real wages $\bar{A}_{11}\bar{q}$, i.e. the physical capital, which is directly necessary for the production of $\bar{q}$, is according to (6) equal to $\lambda_m^{\bar{A}_{11}}\bar{q}$. Consequently, the ratio of the surplus product to the physical capital of the composite 'corn' production sector, and therefore also the rate of profit of this sector, is equal to

$$\frac{(1-\lambda_m^{\bar{A}_{11}})\bar{q}}{\lambda_m^{\bar{A}_{11}}\bar{q}}.$$

For known reasons, the general rate of profit r of the overall production system is also

$$r=\frac{(1-\lambda_m^{\bar{A}_{11}})\bar{q}}{\lambda_m^{\bar{A}_{11}}\bar{q}}.$$

and because $\bar{q}>0$,

$$r=\frac{1-\lambda_m^{\bar{A}_{11}})}{\lambda_m^{\bar{A}_{11}}}. \tag{8}$$

We wish to again stress that here, the general rate of profit was determined prior to and independent of prices *without the latter having previously been normalized.*

Charasoff constructs this composite corn sector as consisting of two reproductive sectors, the products of which are both used as wage commodities. The overall production system of Charasoff also includes one other non-reproductive sector.

For prices P_I and P_{II} of the reproductive and non-reproductive commodities, the following holds respectively

$$P_I[I_{11}-(1+r)\bar{A}_{11}]=0 \tag{9}$$

and

$$P_{II}=P_{II}(1+r)A_{22}+P_I(1+r)\bar{A}_{12}. \tag{10}$$

If one normalizes prices, setting the price of a bundle of only reproductive commodities equal to a positive constant, e.g.

$$P_I\bar{q}=1, \tag{11}$$

then (8), (9) and (11) determine the absolute prices of the reproductive commodities. If one then replaces these latter prices and (8) in (10), one will get the absolute prices P_{II} of the non-reproductive commodities.

Charasoff thus showed that Ricardo's solution had general validity, because one does not have to construct a fictitious corn production sector, which does not necessarily contain the production system given at any time, but rather one can *on the basis of the given production system* construct a composite 'corn' production sector, which makes it possible to determine the general rate of profit –without first normalizing prices– prior to and independent of prices.

Charasoff's solution then of course only gives positive prices for all the commodities, when $\lambda_m^{\bar{A}_{11}} > \lambda_m^{A_{22}}$. Otherwise, i.e. when $\lambda_m^{\bar{A}_{11}} < \lambda_m^{A_{22}}$, it gives, as emerges from (10) and $P_I > 0$, for the r defined by (8), also negative prices for certain non-reproductive commodities.

In addition, Charasoff's solution is compatible with each normalization of prices, only when $\lambda_m^{\bar{A}_{11}} > \lambda_m^{A_{22}}$. When, on the contrary, $\lambda_m^{\bar{A}_{11}} < \lambda_m^{A_{22}}$, then it is compatible only with normalizations of type (11), i.e. only with normalizations in which a bundle of only reproductive commodities functions as the normalization commodity. Because, if, when $\lambda_m^{\bar{A}_{11}} < \lambda_m^{A_{22}}$, one normalizes using a bundle of commodities as normalization commodity, which (bundle) contains even just one non-reproductive commodity, then one gets

$$r = \frac{1-\lambda_m^{A_{22}}}{\lambda_m^{A_{22}}}$$

$$P_I = 0$$

and

$$P_{II} > 0$$

while Charasoff's solution gives

$$r = \frac{1-\lambda_m^{\bar{A}_{11}}}{\lambda_m^{\bar{A}_{11}}},$$

$$P_I > 0$$

and a P_{II} which contains also negative components. In the same case, i.e. in the case in which $\lambda_m^{\bar{A}_{11}} < \lambda_m^{A_{22}}$ and prices have been normalized with the normalization commodity being a bundle of commodities which contains even just one non-reproductive commodity, the rate of profit of the joint corn production sector is equal to

$$\frac{P_I(1-\lambda_m^{\bar{A}_{11}})\bar{q}^*}{P_I\lambda_m^{\bar{A}_{11}}\bar{q}^*}$$

and consequently, because of $P_I = 0$, undetermined. For precisely this reason, it can be set equal to the general rate of profit of the non-reproductive sectors, a fact which creates the impression that there is a general –for all sectors equal– rate of profit[1].

1. In the case that $\lambda_m^{\bar{A}_{11}} < \lambda_m^{A_{22}}$, there is not only one composite corn sector, that of Charasoff, the gross product $\bar{q}^*$ of which is defined as the right eigenvector of $\bar{A}_{11}$ corresponding to the maximum eigenvalue $\lambda_m^{\bar{A}_{11}}$ of $\bar{A}_{11}$, and which has the same composition as the surplus product of $(1-\lambda_m^{\bar{A}_{11}})\bar{q}^*$ and the physical capital of $\lambda_m^{\bar{A}_{11}}\bar{q}^*$ consisting of its means of production and its aggregate real wages, but also a second composite corn sector, namely the one whose gross product $\bar{q}^{**}$ is defined as the right eigenvector of A corresponding to the maximum eigenvalue $\lambda_m^{\bar{A}}$ of A and has the same composition as the surplus product of $(1-\lambda_m^{\bar{A}})\bar{q}^{**}$ and the physical capital of $\lambda_m^{\bar{A}}\bar{q}^{**}$ consisting of its means of production and its real wages. Charasoff's composite corn production sector consists of all the reproductive sectors and only these and its maximum rate of profit is equal to $(1-\lambda_m^{\bar{A}_{11}})/\lambda_m^{\bar{A}_{11}}$. The aforementioned second composite corn production sector consists of all the –reproductive and non-reproductive– sectors and its maximum rate of profit is equal to $(1-\lambda_m^{\bar{A}_{11}})/\lambda_m^{\bar{A}_{11}}$. In the given case, in which $\lambda_m^{\bar{A}_{11}} < \lambda_m^{A_{22}}$ and consequently there is the aforesaid second composite corn production sector, Charasoff's method, i.e. the generalized method of Ricardo, does not allow a unique determination of the rate of profit. Because, depending on the composite corn production sector with which one begins, the rate of profit is equal to $(1-\lambda_m^{\bar{A}_{11}})/\lambda_m^{\bar{A}_{11}}$ or equal to $(1-\lambda_m^{\bar{A}})/\lambda_m^{\bar{A}}$ in which clearly $(1-\lambda_m^{\bar{A}_{11}})/\lambda_m^{\bar{A}_{11}} > (1-\lambda_m^{\bar{A}})/\lambda_m^{\bar{A}}$.

It emerges of course from that which has been set out so far that the general rate of profit of the overall production system calculated by Ricardo and Charasoff is in reality the general rate of profit of the single or composite corn production sector, i.e. of the reproductive sector, reduced to the general rate of profit of the overall production system. This reduction however is not always possible. For, as we saw, when $(\alpha_{11} + d_1 l_1) < \lambda_m^{A_{22}}$ or respectively $\lambda_m^{\bar{A}_{11}} < \lambda_m^{A_{22}}$, then this reduction is either not possible (when we normalize with the normalization commodity being a bundle of commodities containing even just one non-reproductive commodity) or it entails negative prices of non-reproductive commodities (when we normalize with the normalization commodity being a bundle of only reproductive commodities).

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In order for one to determine for a given real wage rate the general rate of profit and prices of production, it is of course today no longer necessary to resort to the artifice of Ricardo and Charasoff. Dmitriev had already shown at the end of the 19th century how one could solve the problem by simultaneously determining the rate of profit and prices, without resorting to Ricardo's artifice: Namely, by solving the system of equations consisting of (9), (10) and any normalization equation of type (11).

Dmitriev's solution gives, in the case in which $\lambda_m^{\bar{A}_{11}} > \lambda_m^{A_{22}}$ *for each normalization of prices* $r = \frac{1-\lambda_m^{\bar{A}_{11}}}{\lambda_m^{\bar{A}_{11}}}$ also positive prices of commodities. In the case however, in which $\lambda_m^{\bar{A}_{11}} < \lambda_m^{A_{22}}$ and prices have been normalized with the normalization commodity being a bundle of only reproductive commodities, it gives $r = \frac{1-\lambda_m^{\bar{A}_{11}}}{\lambda_m^{\bar{A}_{11}}}$ and negative prices for certain non-reproductive commodities. Lastly, in the case in which $\lambda_m^{\bar{A}_{11}} < \lambda_m^{A_{22}}$ and prices have been normalized with the normalization commodity being a bundle that contains even just one non-reproductive commodity, it gives

$$r = \frac{1-\lambda_m^{A_{22}}}{\lambda_m^{A_{22}}},\ P_I = 0 \text{ και } P_{II} > 0.$$

Dmitriev solved the problem of determining the general rate of profit and prices also for the case in which the *nominal* wage rate is given. This solution of Dmitriev gives, in the case in which $\lambda_m^{A_{11}} > \lambda_m^{A_{22}}$ ($\lambda_m^{A_{11}}$ = maximum eigenvalue of the matrix A_{11} of technical coefficients of the now *basic* subsystem) for each normalization of prices, positive prices of commodities. In the case in which $\lambda_m^{A_{11}} < \lambda_m^{A_{22}}$ and prices have been normalized with the normalization commodity being a bundle of only basic commodities, it gives for each nominal wage rate, to which there corresponds a r, $0 \leq r < (1-\lambda_m^{A_{22}})/\lambda_m^{A_{22}}$, positive prices of commodities, for the nominal wage rate, to which there corresponds $r = (1-\lambda_m^{A_{22}})/\lambda_m^{A_{22}}$, it gives undetermined prices of non-basic commodities and for each nominal wage rate, to which there corresponds a r, $(1-\lambda_m^{A_{22}})/\lambda_m^{A_{22}} < r \leq (1-\lambda_m^{A_{11}})/\lambda_m^{A_{11}}$, it gives negative prices for certain non-basic commodities. Lastly, in the case in which $\lambda_m^{A_{11}} < \lambda_m^{A_{22}}$ and prices have been normalized with the normalization commodity being a bundle containing even just one non-basic commodity, it gives for each nominal wage rate, to which there corresponds r, $0 \leq r < (1-\lambda_m^{A_{22}})/\lambda_m^{A_{22}}$, positive prices of commodities, while for the nominal wage rate w = 0, to which there corresponds $r = (1-\lambda_m^{A_{22}})/\lambda_m^{A_{22}}$, it gives positive prices for the non-basic commodities and zero prices for the basic commodities.

However, we should note that in Dmitriev, in (10) the vector $P_{II}(1+r)A_{22}$ is zero, because A_{22} is zero. The same holds correspondingly for Charasoff also, where the vector $P_{II}(1+r)A_{22}$ is zero, because A_{22} is zero. This is due to the fact that in Dmitriev, the non-reproductive or, respectively, non-basic commodities do not enter into the production of the non-reproductive or, respectively, non-basic commodities and in Charasoff the non-reproductive commodities do not enter into the production of the non-reproductive commodities. The same holds in the case of von Bortkiewicz, who at the beginning of the 20th century gave a simplified version of Dmitriev's solution for a given real wage rate. Although von Bortkiewicz constructs, without realizing it, a case in which $A_{22} \geq 0$ and $\lambda_m^{\bar{A}_{11}} < \lambda_m^{A_{22}}$ or $\lambda_m^{\bar{A}_{11}} > \lambda_m^{A_{22}}$, in which, that is, non-reproductive commodities enter into the production of non-reproductive commodities in such proportions that the maximum eigenvalue of $\bar{A}_{11}$ can be either smaller or greater than the maximum eigenvalue of A_{22}.

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And although it was already known since the end of the 19th century and beginning of the 20th century that for a given real wage rate one could determine the general rate of profit and prices simultaneously, without being forced to resort to the artifice of Ricardo and Charasoff, Sraffa, at least sixty years after this accomplishment, wishes to determine the general rate of profit prior to and independent of prices in a way similar to that of Charasoff, as if this way was not a contrivance of those who –for purely technical/mathematical reasons– could not simultaneously determine the general rate of profit and prices –an artifice, the aim of which was to help them overcome their technical/mathematical difficulties in determining the general rate of profit and prices–, but was a fundamental issue of political economy. To put it clearly and unequivocally in advance: Sraffa, who studied Ricardo and published all his works, has not even understood in what –from a purely technical viewpoint– the Ricardian determination of the general rate of profit prior to and independent of prices consists and more specifically, he has not understood that this determination applies, firstly, for a given *real* and not nominal wage rate and secondly, *without previously normalizing prices*. We shall see below the consequences of this lack of understanding on the part of Sraffa.

Sraffa wishes to determine the general rate of profit prior to and independent of prices not with the ingeniously simple method of Ricardo, but –exactly 50 years after Charasoff– in the manner of Charasoff. He does not realize however that Ricardo and Charasoff tackle and solve the problem of determining the general rate of profit prior to and independent of prices for a given real wage rate without first normalizing prices. Sraffa approaches and tries to solve the problem for a given *nominal* wage rate *after first normalizing prices*. Indeed, he tries to solve the problem –and believes that he has solved it– *by means of an appropriate normalization of prices*. As we shall show, the solving of the problem in this form is, before normalization of prices, impossible and after normalization of prices platitudinous – platitudinous in the sense that it is achieved not only by means of what Sraffa considers to be an appropriate normalization of prices, but by means of any normalization of prices. We shall also show that this platitudinous solution to the problem does not constitute, as he himself and his supporters believe, a *general* solution of the problem in the form set by Ricardo, i.e. a solution of the Ricardian problem of determining the general rate of profit for a given real wage rate prior to and independent of prices without previous normalization in the case in which there is no Ricardian corn production sector (such a solution was given by Charasoff), but a solution of the problem of simultaneously determining the general rate of profit and prices for a given *nominal* wage rate *after normalization of prices*. The solution to the problem had been given, as noted above, more than 60 years earlier by Dmitriev.

Let us now see what Sraffa wants to do and what he actually does. He wants to solve Ricardo's problem in exactly the same way that it was solved by Ricardo himself, without however assuming, as Ricardo assumed, the existence of a single corn production sector, but constructing, like Charasoff, a composite corn production sector. He fails to understand however that in Ricardo, it is not the nominal but the real wage rate which is given and he assumes as exogenously given not the real but the nominal wage rate. He thus constructs a composite corn production sector not like that of Charasoff, the construction of which assumes the real wage rate to be given, but a composite corn production sector, the construction of which assumes as exogenously given the nominal wage rate and as unknown –also with respect to its composition– the real wage rate. He calls this sector the standard system. The Sraffian standard system is a production subsystem, which uses the same technique as the given production system and produces all the basic commodities produced by the latter and only these in proportions such that its gross product and means of production have the same composition. Consequently, since the net product is equal to the difference between the gross product and the means of production, the net product of the Sraffian standard system has the same composition as its gross product and its means of production. Sraffa calls the composite product of the standard system the standard commodity.

So Sraffa wants to determine, with the help of this standard system, the general rate of profit of the overall production system prior to and independent of prices, by determining the rate of profit of the standard system prior to and independent of prices and then equating it with the general rate of profit of the overall production system.

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Let us now see if this is possible in the simple case, in which the given production system is like Ricardo's initial production system and it is not, as in Ricardo, the real wage rate which is given, but, as in Sraffa, the nominal wage rate.

So, let the given system of production be the initial system of production [A, 1, X] with $\alpha_1 = (\alpha_{11}, 0, 0, ..., 0)^T$. Consequently, commodity 1 is the only basic commodity. All the other commodities are non-basic. Sraffa defines the general rate of profit r not in the way that Ricardo does, i.e. as the ratio of profit to aggregate means of production expressed in price terms, but rather as the ratio of profit to means of production expressed in price terms. This difference however is without significance for the issue of concern to us here. He also considers exogenously given not the real but the nominal wage rate w.

Thus, for prices we get

$$P_1 = P_1(1 + r)\alpha_{11} + l_1 w \tag{12}$$

and

$$\bar{P} = \bar{P}(1 + r)A_{22} + P_1(1 + r)(\alpha_{11}, \alpha_{12}, ..., \alpha_{1n}) + \bar{l}w \tag{13}$$

where

$$l = (l_1, \bar{l}),$$

$$\bar{l} = (l_2, l_3, ..., l_n)$$

and

$$\bar{P} = (P_2, P_3, ..., P_n).$$

Our system is clearly the initial Ricardian system, in which however, instead of the real wage rate, it is the nominal wage rate that is exogenously given.

Let us now see whether one can here determine –without first normalizing prices– the general rate of profit prior to and independent of prices, by determining it as the rate of profit of the Sraffian standard system. Here, the Sraffian standard system evidently coincides with the production sector of commodity 1. From (12) we get for the rate of profit of sector 1, i.e. of the Sraffian standard system,

$$r = \frac{P_1 - P_1\alpha_{11} - l_1 w}{P_1\alpha_{11}} = \frac{P_1(1-\alpha_{11}) - l_1 w}{P_1\alpha_{11}} = \frac{1-\alpha_{11}}{\alpha_{11}} - \frac{l_1 w}{P_1\alpha_{11}} \tag{14}$$

It is clear that without first normalizing prices, (14) does not determine r prior to and independent of prices. It determines r prior to and independent of prices, only if prices have first been normalized and moreover with a normalization of the type

$$P_1 = c, c > 0 \qquad (15)$$

i.e. with a normalization in which the only basic commodity functions as the normalization commodity, namely commodity 1. Then and only then we get from (14)

$$r = \frac{c - c\alpha_{11} - l_1 w}{c\alpha_{11}}, \qquad (14\alpha)$$

by virtue of which r is, for exogenously given w, determined prior to and independent of prices.

So, for given w, r is not –either without previous normalization of prices or for each normalization of prices– prior to and independent of prices given.

Also, as shown by (14a), the size of r evidently depends, for exogenously given w, on the normalization. Because for exogenously given w, each normalization of type (15) arbitrarily introduces, if we appropriately define the composition of the unknown real wage rate, a given –with respect to its size– real wage rate of this appropriately defined composition.

Let us see how this happens. If w is exogenously given and we normalize with (15) and additionally assume that the real wage rate consists of a normalization commodity, i.e. of commodity 1, then clearly (15) and the given w determine the size of the real wage rate consisting of the normalization commodity as being equal to

$$\frac{w}{P_1} = \frac{w}{c}.$$

So, for given w, the real wage rate of the aforementioned composition varies with normalization. But r varies with the real wage rate. For this reason, r varies with varying normalization also.

The –for given w– variation in the rate of profit r of the Sraffian standard system with normalization shows two things:

Firstly, that, for given w, r cannot be determined without previous normalization of prices prior to and independent of prices. And secondly, that r –since it is determined for given w in the manner just described by the normalization, since this determines for given w the size of the real wage rate consisting of the normalization commodity– is determined only then without previous normalization prior to and independent of prices, when given is, in addition to the nominal wage rate or, instead of the nominal wage rate, the real wage rate, where this latter consists, just as the normalization commodity of all normalizations which for given w allow the determination of r prior to and independent of prices, of the standard commodity, i.e. of commodity 1. So, we may conclude, that r would be without previous normalization of prices prior to and independent of prices then and only then determined, if the size of the real wage rate consisting of the standard commodity (commodity 1) was given. And this is indeed the case. Because, on the condition that the real wage rate d consists only of the standard commodity, i.e. commodity 1, and consequently is equal to d, (14) gives

$$r=\frac{1-\alpha_{11}}{\alpha_{11}}-\frac{l_1 w}{P_1\alpha_{11}}=\frac{1-\alpha_{11}}{\alpha_{11}}-\frac{l_1 P_1 d_1}{P_1\alpha_{11}}=\frac{1-\alpha_{11}-l_1 d_1}{\alpha_{11}}.$$

The fact that r is then and only then determined prior to and independent of prices, when either the nominal wage rate is given and prices have been normalized with a normalization of type (15), i.e. with a normalization, in which the standard commodity functions as the normalization commodity, or a real wage rate consisting of a standard commodity is given and prices have not been normalized, becomes understandable if one takes into consideration that, for a given nominal wage rate, each normalization of type (15), in which the standard commodity functions as normalization commodity, entails on the condition that the real wage rate consists of a normalization commodity, the determination of the size of the real wage rate consisting of the normalization, i.e. standard, commodity.

Not even the maximum rate of profit R,

$$R=r_{|w=0}$$

can be determined prior to normalization. For R of course, the following emerges from (14), for $w = 0$ and $P_1 > 0$,

$$R=\frac{1-\alpha_{11}}{\alpha_{11}}. \tag{16}$$

However, (16) holds *for each normalization* and thus also prior to normalization, only when $\alpha_{11} > \lambda_m^{A_{22}}$. If however $\alpha_{11} < \lambda_m^{A_{22}}$, then, as one can show, for normalizations with bundles of commodities which contain only basic commodities (here only commodity 1) as normalization commodities, (16) holds, while for normalizations with bundles of commodities which contain even just one non-basic commodity as normalization commodities, $P_1 = 0$ holds and

$$R = \frac{1-\lambda_m^{A_{22}}}{\lambda_m^{A_{22}}}.$$

In this case, the maximum rate of profit of the standard system is, as emerges from (14) for $w = 0$ and $P_1 = 0$, indetermined.

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Let us now see also the general case of the existence of more than one basic commodity, i.e. the framework in which Sraffa approaches and tries to solve the problem. It will become clear that in principle it does not differ at all from the case which we have just examined.

So, assuming that for A the following holds

$$A = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}.$$

For the prices of the basic and non-basic commodities P_I and P_{II}, the following holds

$$P_I = P_I(1 + r)A_{11} + l_I w$$

and

$$P_{II} = P_{II}(1 + r)A_{22} + P_I(1 + r)A_{12} + l_{II} w$$

Let us now see how Sraffa solves his problem, i.e. of determining the general rate of profit of the overall production system by determining it for a given w as the rate of profit of its standard system prior to and independent of prices.

Assuming that the gross product of the standard system is q^*. Then

$$\beta q^* = A_{11} q^*.$$

Because $q^* > 0$, q^* is evidently the right eigenvector of A_{11}, which corresponds to the maximum eigenvalue $\lambda_m^{A_{11}}$ of $A_{11.}$ Consequently

$$\beta = \lambda_m^{A_{11}}$$

and

$$\lambda_m^{A_{11}} q^* = A_{11} q^*$$

For the net product $(I_{11} - A_{11})q^*$ of the standard system we get

$$(I_{11} - A_{11})q^* = q^* - A_{11}q^* = q^* - \lambda_m^{A_{11}} q^* = q^*(1 - \lambda_m^{A_{11}}).$$

Consequently this too has the same composition as the means of production $\lambda_m^{A_{11}} q^*$ and the gross product q^* of the standard system. The correspondence between the Sraffian standard system and Charasoff's joint 'corn' production sector is clear.

Let us see if one can calculate the percentage of the standard system and thus also the general rate of profit of the overall system prior to and independent of prices. For the rate of profit of the standard system, the following evidently holds

$$r = \frac{P_I q^* - P_I A_{11} q^* - l_I q^* w}{P_I A_{11} q^*} = \frac{P_I q^* - P_I \lambda_m^{A_{11}} q^* - l_I q^* w}{P_I \lambda_m^{A_{11}} q^*} \tag{20}$$

It is immediately apparent from (20) that it is impossible for given w to calculate r prior to and independent of prices. This would be possible, only when, firstly, it was not the nominal but the real wage rate that was given or both the nominal and real wage rate were given and, secondly, the real wage rate consisted only of a standard commodity. That is, only if

$$d = \gamma q^*, \;\; \gamma = \text{positive constant},$$

and

$$w = P_I d = P_I \gamma q^*.$$

Then for r we would get

$$r = \frac{P_I q^* - P_I \lambda_m^{A_{11}} q^* - l_I q^* P_I \gamma q^*}{P_I \lambda_m^{A_{11}} q^*} = \frac{P_I[1 - \lambda_m^{A_{11}} - l_I q^* \gamma] q^*}{P_I \lambda_m^{A_{11}} q^*} \tag{21}$$

This r is evidently independent of P_I, because for each $P_I > 0$ the following holds

$$r = \frac{(1-\lambda_m^{A_{11}} - l_I q^* \gamma) q^*}{\lambda_m^{A_{11}} q^*}$$

and because of $q^* > 0$,

$$r = \frac{1-\lambda_m^{A_{11}} - l_I q^* \gamma}{\lambda_m^{A_{11}}}$$

Not even the maximum rate of profit R,

$$R = r_{|w=0}$$

can be determined prior to normalization of prices. For R of course, the following emerges from (20), for $q^* > 0$, $w = 0$ and $P_I > 0$,

$$R = \frac{1-\lambda_m^{A_{11}}}{\lambda_m^{A_{11}}}. \tag{20α}$$

However, (20a) then only holds for each normalization and thus before the normalization of prices, when $\lambda_m^{A_{11}} > \lambda_m^{A_{22}}$, because only then is $P_I > 0$ for each normalization of prices. If however $\lambda_m^{A_{11}} < \lambda_m^{A_{22}}$, then, as one can show, for normalizations with bundles of commodities which contain only basic commodities as normalization commodities $P_I > 0$ and (20a) holds, while for normalizations with bundles of commodities which contain even just one non-basic commodity as normalization commodities, $P_I = 0$ and for the maximum rate of profit the following holds

$$R = \frac{1-\lambda_m^{A_{22}}}{\lambda_m^{A_{22}}}.$$

In this latter case, the maximum rate of profit of the Sraffian standard system is, as emerges from (20) because of $P_I = 0$, undetermined. And this is why it can be set equal to the maximum rate of profit

$$R = \frac{1-\lambda_m^{A_{22}}}{\lambda_m^{A_{22}}}$$

of the non-basic subsystem, thus creating the impression that there is a general –equal in all sectors– maximum general rate of profit.

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Of course, so far we have been speaking about the possibility of one determining the rate of profit of the standard system prior to and independent of prices, without having previously normalized the latter.

Let us now see what happens if we first normalize prices. If we normalize prices e.g. according to Sraffa, setting the price of the net standard commodity equal to unit, by means of

$$P_I(1-\lambda_m^{A_{11}})q^* = 1, \qquad (21\alpha)$$

then we get from (20)

$$w = \frac{1}{l_I q^*} - \frac{1}{R}\frac{1}{l_I q^*} r.$$

And if, like Sraffa, we normalize q^* with

$$l_I q^* = 1, \qquad (21\beta)$$

we get

$$w = 1 - \frac{1}{R} r, \text{ with } R = \frac{1-\lambda_m^{A_{11}}}{\lambda_m^{A_{11}}}. \qquad (22)$$

According to (22) r is, for given w, prior to and independent of prices determined. However: after first having normalized prices. And specifically: after first having normalized prices a la Sraffa, using as normalization commodity the Sraffian standard net product[2]. On the condition that the real

2. In the case in question, where $\lambda_m^{A_{11}} < \lambda_m^{A_{22}}$, the Sraffian standard system is not, as Sraffa and his exponents believe, the only system, of which the definite gross product q^*, the net product $(1-\lambda_m^{A_{11}})q^*$ and the means of production $\lambda_m^{A_{11}}q^*$ corresponding to the maximum eigenvalue

wage rate consists of a standard commodity, this normalization is tantamount to an exogenous determination of the size of this real wage rate consisting of a standard commodity. Because, on the condition that the real wage rate consists of a standard commodity and the nominal wage rate w is given, the given w and (21a) entail that the size of this real wage rate consisting of a standard commodity is equal to

$$\frac{w}{P(1-\lambda_m^{A_{11}})q^*}=w$$

i.e. equal to the size of the given nominal wage rate w and consequently also given.

Thus, the rate of profit of the standard system is –prior to and independent of prices– given either when the real wage rate consists of a standard commodity and is with respect to its size given or when instead of the above real wage rate the nominal wage rate is given and prices have been normalized with the Sraffian standard commodity as normalization commodity (i.e. à la Sraffa).

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$\lambda_m^{A_{11}}$ of the A_{11} right eigenvector of A have the same composition. As shown by Spyros Vassilakis, in this case there is also a second system, of which the definite gross product q^{**}, the net product $(1-\lambda_m^A)q^{**}$ and the means of production $\lambda_m^A q^{**}$ corresponding to the maximum eigenvalue λ_m^A of the A_{11} right eigenvector of A have the same composition. While the standard system of Sraffa produces all the basic and only the basic commodities, the standard system of Vassilakis produces all the –basic and non-basic– commodities. If we normalize 1 using $lq^{**} = 1$ and prices using $p(1-\lambda_m^A)q^{**}=1$, i.e. with the net product of Vassilakis' standard system as standard commodity, then for the w-r-relationship we get

$$w=1-\frac{1}{R}r,\ \text{with}\ R=\frac{1-\lambda_m^A}{\lambda_m^A}=\frac{1-\lambda_m^{A_{22}}}{\lambda_m^{A_{22}}}$$

This w-r-relationship is, as also (22) which results from the normalization of prices a la Sraffa, linear, but the maximum rate of profit R which emerges therefrom is different to (smaller than) that which emerges from the Sraffian w-r-relationship (22).

It is precisely at this point that the neo-Ricardian mythopoeia of Sraffa and his standard commodity begins. And what has and is not being said by the neo-Ricardians! Namely, that Sraffa introduced a measure of prices –the standard net product– which allows us to determine the general rate of profit prior to and independent of prices. That when prices are measured using this measure, they are independent of the distribution of income or –according to others, less enthusiastic– that, when prices are measured using this measure, then we are certain that the variations in prices caused by variations in the distribution of income are not due *also* to variations in the measure of prices but solely and exclusively to variations in the distribution of income, which does not occur, when prices are measured using another measure, because every other measure, in contrast with the Sraffian measure of prices, itself varies with the distribution of income. That in the standard commodity Sraffa discovered the invariable measure of prices which Ricardo tried to find – and thus succeeded, as we would say, in squaring the circle, as Marx rightly considers the discovery of such a measure. And a great deal more.

But let us stay with the argument which directly relates to the issue of concern to us here, i.e. the argument that, when we normalize prices using the Sraffian standard net product as normalization commodity, then, for a given nominal wage rate, we can determine the general rate of profit prior to and independent of prices. This accomplishment would be of interest if, in the case where we normalized prices with a normalization commodity different to the Sraffian net standard product, *we could not*, for a given nominal wage rate, determine the general rate of profit prior to and independent of prices. But this is not the case. Because each normalization of prices, not only the Sraffian, leads to a w-r-relationship which allows us, for given w, to determine r. The same is true also when prices have been normalized using labor power as normalization commodity and consequently are measured in terms of labor commanded. In this case however, because the price of labor power, i.e. the nominal wage rate w, has by virtue of the normalization equation been set equal to a positive constant, usually equal to unit, we do not get the usual w-r-relationship but a relationship between the given and constant w and r.

When the w-r-relationship, to which a certain normalization leads, is not, like the w-r-relationship, to which the Sraffian normalization leads, linear and consequently one-to-one, then to any given w there does not correspond, as in the w-r-relationship, to which the Sraffian normalization leads, only one but more than one value of r. However, we limit the economically significant interval of values of r by means of $0 \leq r \leq R_n$ where R_n is the smallest of the values of r given by the initial w-r-relationship for w = 0, so that ultimately, each w-r-relationship which results for normalizations different from the normalization of Sraffa becomes, under the above limitation, one-to-one: to each value of w corresponds only one value of r. However, it is thus proven that the determination –for given *nominal* wage rate and *certain* normalization of prices– of the general rate of profit prior to and independent of prices, which Sraffa gives, is platitudinous, because it holds not only for the Sraffian, but for *each* normalization of prices. The issue of course becomes even more interesting if one considers that the w-r-relationship and consequently also the said platitudinous method of determining –for given nominal wage rate and any normalization of prices– the general rate of profit had already been set out by Dmitriev at the end of the 19th century.

In addition, the Sraffian determination of the general rate of profit of the given system of production as the rate of profit of the standard system is neither always compatible with positive prices of commodities, nor possible for each normalization. It is compatible with positive prices of commodities and possible for each normalization of prices only when $\lambda_m^{A_{11}} > \lambda_m^{A_{22}}$. But when $\lambda_m^{A_{11}} < \lambda_m^{A_{22}}$, then it is not compatible with positive prices of commodities, because the normalization which it entails, implies for w, to which a r corresponds, $r = \frac{1-\lambda_m^{A_{22}}}{\lambda_m^{A_{22}}}$, undetermined prices of the non-basic commodities and for each w, to which a r corresponds, $\frac{1-\lambda_m^{A_{22}}}{\lambda_m^{A_{22}}} < r \leq \frac{1-\lambda_m^{A_{11}}}{\lambda_m^{A_{11}}}$, (where $\frac{1-\lambda_m^{A_{11}}}{\lambda_m^{A_{11}}}$ the r, which corresponds to w = 0) negative prices of certain non-basic commodities.

Moreover, if, in the case that $\lambda_m^{A_{11}} < \lambda_m^{A_{22}}$, we normalize not à la Sraffa or, in general, using a bundle of only basic commodities as normalization commodity, but using a bundle of commodities containing even just one non-basic commodity as normalization commodity, then for w = 0, the general rate of profit of the overall system does not turn out to be equal to the rate of profit of the Sraffian standard system, i.e. –which is the same thing– of the basic subsystem, but rather as being equal to that of the non-basic subsystem, that is, as not being equal to $\frac{1-\lambda_m^{A_{11}}}{\lambda_m^{A_{11}}}$ but rather equal to $\frac{1-\lambda_m^{A_{22}}}{\lambda_m^{A_{22}}}$. Here, prices are for each w, to which corresponds a r, $0 \le r \le (1-\lambda_m^{A_{22}})/\lambda_m^{A_{22}}$, positive with the exception of prices of basic commodities, which for w = 0 and consequently for $r = (1-\lambda_m^{A_{22}})/\lambda_m^{A_{22}}$ are zero ($P_I = 0$). In this case, because of $P_I = 0$, the maximum rate of profit of the Sraffian standard system is undetermined. And for precisely this reason it can be set equal to the maximum rate of profit of the non-basic subsystem, which is equal to $(1-\lambda_m^{A_{22}})/\lambda_m^{A_{22}}$, thus creating the impression that there is a general –equal for all sectors– maximum rate of profit.

To summarize, we can say the following: When $\lambda_m^{A_{11}} < \lambda_m^{A_{22}}$, then the Sraffian determination of the general rate of profit of the overall system as the rate of profit of the standard system is either incompatible with positive prices of commodities (if we normalize à la Sraffa) or is impossible (if we normalize using a bundle of commodities which contains even just one non-basic commodity as normalization commodity).

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Garegnani maintains that the classical and Marxian theories, which he calls 'surplus theories', differ in general from neoclassical theories in that they have a 'core', which precedes their further economic analysis, is independent of such analysis and consists in –for given gross or net product, given production technique and given real wage rate– the determination of the number of workers employed, of aggregate real wages and, above all, of the 'surplus' product. It is not our intention to discuss this view of Garegnani here. It is of interest to us only to the extent that to this 'core' of 'surplus theories' belongs also the –for given real wage rate– determination of the general rate of profit prior to and independent of prices, which he calls the 'surplus-equation method' and which he distinguishes from the –for given real wage rate– simultaneous determination of both general rate of profit and prices. For undetermined reasons, he considers this latter method, which he calls the 'price-equations method', to bear no relation whatsoever to the 'surplus theories'. Nor do we wish to discuss this view of Garegnani here. We should just like to observe that the difference between the 'surplus-equation method' and the 'price-equations method' is purely and simply technical in nature, i.e. that ultimately, the 'surplus-equation method' of Ricardo and Charasoff differs only from a technical viewpoint from the more advanced (from a technical viewpoint) 'price-equations method' of Dmitriev and that any differences between the two methods neither express nor found differences in the theory itself.

What is perplexing is the fact that Garegnani classifies –without intermediation and without any founding or explanation whatsoever– also Sraffa's method, which we described above, in the 'surplus-equation method', overlooking even the fact that in Sraffa it is the nominal and not the real wage rate which is given. This becomes understandable only if one takes into consideration the desire of Sraffa's followers and first and foremost of Garegnani himself to classify Sraffa among the proponents of the 'surplus theories', i.e. of classical political economy.

As a worthy student of Sraffa, Garegnani himself develops a 'surplus-equation method' of determining the rate of profit prior to and independent of prices – evidently in order to classify himself in the classical tradition of economic theory.

Concluding this paper, let us see in what –according to Garegnani– something so important consists, which can classify or not classify someone among the tradition of the classicists.

Garegnani wishes to determine the general rate of profit, for given real wage rate, prior to and independent of prices. Of course he has no inkling that this problem is unrelated to the basic principles of classical political economy, but is purely and simply related to the –of a technical nature– inability of Ricardo to simultaneously determine the general rate of profit and prices for an exogenously given real wage rate, as Dmitriev subsequently succeeded.

So, he considers the real wage rate to be exogenously given. And he then wishes to determine the general rate of profit of the overall system of production as the rate of profit of the subsystem which produces as its net product the real wages or, as he himself calls it, of the vertically integrated system of production of wage commodities[3], prior to and independent of

3. A vertically integrated sector of production is but a sector of production which is not defined, as usual, on the basis of its *gross* but on the basis of its *net* product as a sector which produces in addition to its *net* product also all the means of production which are *directly* and *indirectly* necessary for the production of this net product. To facilitate an understanding of the

prices. This, as we saw, is possible –without normalization of prices– only by means of Charasoff's method. Garegnani attempts it –with normalization of prices– in a different way. And he succeeds. With what result, we shall see below.

So, he normalizes prices by setting the price of the real wage rate and consequently the nominal wage rate w equal to unit. As a result of this normalization, all prices are measured in terms of labor commanded. Thus, the real wage rate buys one unit of labor power and consequently its value, i.e. the nominal wage rate, is equal to unit. As a consequence, the aggregate real wages of the given system of production buy the given aggregate labor power L and therefore their value is equal to this latter, i.e. equal to L. The net product of the vertically integrated sector of production of wage commodities is equal to the aggregate wages of the given system of production. Consequently, its value is equal to the value of these latter, i.e. equal to L.

In the vertically integrated sector of production of wage commodities, certain real wages are paid. If the labor power which is engaged in this sector is equal to L_V, then these real wages buy exactly this quantity of labor power L, and consequently their value is equal to L_V. So, the nominal wages of the vertically integrated sector of production of wage commodities is equal to L_V. Consequently, the total profit of the vertically integrated sector of production of wage commodities is equal to $L - L_V$.

Subsequently, in order to calculate the rate of profit of the vertically integrated sector of production of wage commodities, Garegnani calculates –indirectly– the value, expressed in terms of labor commanded, of the capital of the vertically integrated sector of production of wage commodities. Let us symbolize it with K. For a given production technique K is, if we express it –like Garegnani– in terms of dated labor or, to be more precise, in dated nominal wages, a function of the rate of profit r and the nominal wages L_V of the said sector. Consequently, for r we get:

$$r = \frac{L - L_V}{K} \tag{23}$$

vertically integrated sector of production, Garegnani refers the reader to Sraffa's 'subsystems of production'. Apparently he is unaware or deliberately makes no mention of the fact that it was not Sraffa who first introduced the concept of the vertically integrated sector in 1960, but Feldman in 1928.

where

$$K = f(L_V, r).$$

In a simple example, which he constructs, Garegnani does not directly calculate K, but expresses $L - L_V$ in terms of dated nominal wages and gets

$$L - L_V = r\frac{L_V}{2} + 2r\frac{L_V}{2} + r^2\frac{L_V}{2}.$$

Taking into account (23), this identity gives for K

$$K = \frac{L_V}{2} + 2\frac{L_V}{2} + r\frac{L_V}{2}.$$

(23) enables the –for given real wage rate– determination of r prior to and independent of prices. However, (23) arose after a certain normalization of prices, according to which the value of the given real wage rate –and consequently also the nominal wage rate w– was set equal to unit.

So ultimately, Garegnani does not determine the general rate of profit, like Ricardo and Charasoff *without normalization of prices* for a given real wage rate prior to and independent of prices, but *by previously normalizing prices in a certain way*.

However, he himself appears to believe that he is determining the rate of profit of the vertically integrated sector of production of wage commodities for given real wage rate prior to and independent of prices either without normalization or because of the special normalization which he introduces. We already know that the former does not hold. Because, as we saw, he normalizes prices. The latter *appears* to hold. That is, it *appears* that the fact that Garegnani is in a position to calculate this rate of profit prior to and independent of prices is due to the special type of normalization of prices which he introduces. Let us see why this impression is created.

The subsystem, the rate of profit of which Garegnani wants to calculate for a given real wage rate prior to and independent of prices, the vertically integrated sector of production of wage commodities, presents the following peculiarity: Its net product, which is nothing more than the aggregate real wages of the overall system of production, has the same composition as these latter. However, because the given real wage rate is by definition uniform, i.e. the same in all sectors, the overall real wages of the vertically integrated sector of production of wage commodities have the same composition as the real wages of the overall system and thus the same composition as the net product of the vertically integrated sector of production of wage commodities. Therefore, the vertically integrated sector of production of wage commodities is, when the real wage rate is, as here, given, a sector whose net product and real wages and consequently also the surplus product have the same composition. As we saw, Garegnani normalizes prices with the given real wage rate as normalization commodity, setting the price of the former equal to a constant (to unit). But because this normalization commodity has the same composition as the net product, the real wages and surplus product of the vertically integrated sector of production of wage commodities, Garegnani's normalization allows him to calculate the price of the net product, the price of real wages and the difference between these two prices, which is the price of the surplus product, i.e. the profit, of the vertically integrated sector of production of wage commodities, prior to and independent of prices.

In order to get the rate of profit of this sector prior to and independent of prices, Garegnani clearly must calculate also the capital of this sector prior to and independent of prices. This is possible only if he calculates it in terms of dated nominal wages, because then it is a function of the rate of profit and the value of real wages of the sector, which value, as we saw, can be calculated prior to and independent of prices. Under these conditions, Garegnani can apparently calculate also the ratio of the price of the surplus product to capital, i.e. the rate of profit, of the vertically integrated sector of production of wage commodities.

It would appear that all this is possible only for the aforementioned normalization of prices, according to which the real wage rate –i.e. a percentage of the net product of the vertically integrated sector of production of wage commodities– functions as normalization commodity. And because, if one normalizes prices with normalization commodity any bundle of commodities of a different composition to that of the real wage rate and consequently also of the net product of the vertically integrated sector of production of wage commodities, then clearly neither the profit nor the capital and consequently nor their ratio, the rate of profit of the vertically integrated sector of production of wage commodities can be calculated prior to and independent of prices.

But things are otherwise. For, as we know, if one normalizes prices in any way whatsoever different to that of Garegnani, then one gets for the given real wage rate a nominal wage rate and consequently also a rate of profit of the vertically integrated sector of production of wage commodities corresponding to the aforesaid nominal wage rate.

Therefore, that which was said of Sraffa applies also to Garegnani: Firstly, Garegnani does not determine the rate of profit prior to and independent of prices without first normalizing prices but following normalization of the latter. Secondly, the fact that Garegnani can determine the rate of profit prior to and independent of prices is not due to the special form of normalization which he uses, because this is possible for each normalization. And thirdly, he does not determine the rate of profit prior to and independent of prices for given only a real wage rate but for given also a nominal wage rate, because his normalization entails that this latter is given and equal to unit.

Of course Garegnani cannot, for the normalization which he introduces, get a usual w-r-relationship. Because his normalization presupposes a given real wage rate and at the same time a nominal wage rate corresponding to this real wage rate, which nominal wage rate is always and independent of the size of the given real wage rate equal to unit. To the given real wage rate corresponds –irrespective of the type of normalization and consequently also for Garegnani's normalization– a certain rate of profit. With the difference that according to Garegnani's normalization, when the real wage rate varies and consequently also the rate of profit, the nominal wage rate that corresponds to this varying rate of profit is always equal to unit. Therefore, the only thing which Garegnani can get is a d-r-relationship, but not a w-r-relationship of the usual form of a trade-off between w and r.

With the aim of expressing r as a function of any magnitude, which resembles the usual w-r-relationship, Garegnani expresses r as a (monotonously *increasing*) function of $(L - L_V) / L_V$, i.e. of the ratio of profits to wages (which here, because not only $L - L_V$, but also L_V, are independent of prices, is equal to the Marxian rate of surplus value). If he was cleverer, he would have expressed r as a function of $L_V / (L - L_V)$, i.e. of the ratio of wages to profits, or as a function of L_V / L, i.e. of the share of wages, for these functions, as monotonously *decreasing*, more closely resemble the w-r-relationship.

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