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Ἄρθρα - Articles

Why the Comparison and Ordering of Techniques is Impossible

by
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Summary

*On the occasion of a recent paper by Bidard and Klimovsky, we summarize the findings of our past work, according to which a) the usual comparison and ordering of techniques –not only with the w-r criterion but also with the cost minimization criterion and, lastly, Bidard's criterion– is not a comparison and ordering of the techniques themselves, but a comparison and ordering of certain –different for each of these three criteria– systems, which use these techniques, b) that the ordering of these systems is not unambiguous, c) that the comparison and ordering of given techniques as such is impossible and d) that the only possible unambiguous comparison and ordering of techniques is their comparison and ordering in the form of corn systems** in which real wages have the same composition, which (systems) use these techniques, or in the form of Charasoffian standard systems, which use these techniques.*

In their paper "Switch in Methods and Wage Maximization", Christian Bidard and Edith Klimovsky (2001) investigate whether the criterion of the w-r relationship constitutes a sound criterion for the unambiguous ordering of techniques with respect to their profitability. As is known, according to this criterion, from given neighbouring techniques (or systems) we choose for a given rate of profit r that with the highest nominal wage rate w.[1]

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** By 'corn system' we mean a Sraffian standard system, in which real wages have the same composition as the net product and consequently as the surplus product, i.e. a system, in which real wages, surplus product (and therefore net product) and means of production have the same composition.

1. In reality, the criterion of the w-r relationship consists in the –for given w– choice of that

Is this criterion for comparing and ordering techniques and choosing the most profitable of these correct? Bidard and Klimovsky (2001) maintain that this criterion, which Sraffa also uses, is correct, when used in the case of single production techniques, and incorrect, when used in the case of joint production techniques.

Concerning the case of single production techniques they write:

"One must notice that the standard (we shall henceforth call the standard or numeraire the normalization commodity – G.S.) plays a role in the measure of wage. However, the maximum profit rates R_{12} and R_{13} (these are the highest rates of profit of two square neighbouring single production techniques, of which the former uses processes 1 and 2 and the latter uses processes 1 and 3 – G.S.) and the switch-point(s) do not depend on it. Therefore, the relative position of the curves (i.e. of the w-r curves – G.S.) are independent of the standard. The conclusions of the analysis are robust, as far as single production is only concerned" (Bidard and Klimovsky 2001, p. 2).

This claim is erroneous. We have long since and repeatedly shown that, in cases of decomposable single production techniques, the highest rates of profit and the switch-point(s) and therefore also the ordering of techniques vary with the normalization commodity (see Stamatis 1983, 1984, 1988, 1989, 1992, 1993, 1994, 1994a, 1998, 1998a, 2000).[2]

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technique which maximizes r. However, because the envelope of the w-r curves is a one-to-one relationship, i.e. because to each w corresponds one and only one r and to each r corresponds one and only one w, the w-r criterion may be taken as a criterion, according to which –for given r– we choose the technique with the highest w.

2. For the doubting reader, we present the following example of two neighbouring decomposable single production techniques. Assuming the techniques $[A^{(\alpha)}, \ell^{(\alpha)}]$ and $[A^{(b)}, \ell^{(b)}]$ with

$$A^{(\alpha)} = \begin{bmatrix} 0.5 & 0.25 \\ 0 & 0.75 \end{bmatrix}, \ \ell^{(\alpha)} = (0.5, 0.5)$$

and

$$A^{(b)} = \begin{bmatrix} 0.5 & 0.25 \\ 0 & 0.749 \end{bmatrix}, \ \ell^{(b)} = (0.5, 0.252)$$

If we normalize the prices of both techniques α and b by means of

$$p_1^{\alpha} = p_1^{b} = 1,$$

then we get for both techniques α and b the same w-r-relation, i.e. the w-r relation

$$w = 1 - r,$$

Thus, in the case of *decomposable* single production techniques, the

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with

$$w_{max}^{\alpha} = w_{max}^{b} = 1$$

and

$$r_{max}^{\alpha} = r_{max}^{b} = 1.$$

According to this normalization, both techniques are for each r, $0 \leq r \leq r_{max}^{\alpha} = r_{max}^{b} = 1$ equivalent.

However, if we normalize prices with

$$0.25p_1^{\alpha} + 0.25p_2^{\alpha} = 0.25p_1^{b} + 0.25p_2^{b} = 1,$$

then for the w-r relation of technique α we get

$$w^{\alpha} = 1 - 3r$$

with

$$w_{max}^{\alpha} = 1$$

and

$$r_{max}^{\alpha} = 1/3$$

and for the w-r relation of technique b we get

$$w^{b} = \frac{[1-0.5(1+r)][1-0.749(1+r)]}{0.188-0.062625(1+r)}$$

with

$$w_{max}^{b} = 1.001$$

and

$$r_{max}^{b} = 0.3351.$$

Here also we have two switch-points, one at point $r = r_1 = 0.00205$ and one at point $r = r_2 =$ $= 0.326145$.

The ordering of two neighbouring techniques having as the criterion the w-r criterion thus varies for each r different from r_1 and r_2 with varying normalization. In the first normalization, the two techniques are of equivalent for each r. In the second normalization however, they are of equivalent only for $r = r_1$ and $r = r_2$, while for every other r one technique prevails over the other. Lastly, with varying normalization, the maximum rates of profit of the two techniques also vary. In the first normalization, the maximum rates of profit of the two techniques were equal to each other and equal to 1, while in the second normalization the maximum rate of profit of technique α became equal to 1/3 and the maximum rate of profit of technique b became equal to 0.3351.

The above apparently paradoxical phenomena are by to means paradoxical, but rather have a very simple explanation. With the w-r criterion it is not given techniques which are being ordering of given techniques – having as the criterion the w-r relation criterion – varies with the normalization commodity.[3]

As we explain in our aforementioned papers, this happens because in ordering techniques having as criterion the w-r relation criterion, it is not these techniques themselves which are being ordered but in reality the –for each given normalization commodity– normalization subsystems corresponding to these techniques, i.e. those subsystems, each of which uses one of the given techniques or part of one of these techniques and produces as its net product the normalization commodity given each time. So, when the normalization commodity varies and consequently the corresponding normalization subsystems also vary, then naturally the ordering of these latter also varies. This does not hold only (a) in the case of indecomposable single production, (b) in the case of decomposable single production, when the maximum rate of profit of the non-basic part of the technique is greater than the maximum rate of profit of the basic part of the technique, and (c) in the case of 'normally behaving' joint production (see Stamatis 1995).[4] This is not to say that in these three cases the ordering of techniques does not vary when the normalization commodity varies. It does indeed vary in these cases. However, although the ordering of techniques varies with the normalization commodity, the most profitable technique does not vary with the latter, but remains the same for each given r, irrespective of which bundle of commodities functions as normalization commodity.

According to the above, the assertion of Bidard and Klimovsky, that the w-r criterion is erroneous when used for the ordering of joint production techniques, is correct. (However, as we shall see later on, the explanation which they give for their correct assertion is erroneous).

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compared and ordered, but rather the normalization subsystems for each given normalization which correspond to these techniques. With varying normalization however, these normalization subsystems may vary and consequently their ordering may also vary.

3. As shown by Theodore Mariolis (1994), even in the case of *indecomposable* techniques of single production, their ordering varies with the normalization commodity. In this case however, the most profitable technique does not vary with the normalization commodity, but rather remains –irrespective of the normalization commodity– the same.

4. By 'maximum rate of profit of the basic part' of a decomposable technique we mean here the rate of profit of that part of the technique, which results for w = 0. And by 'maximum rate of profit of the non-basic part' of a decomposable technique we mean here the rate of profit of that part of the technique, which results for w = 0 and for zero prices of all basic commodities.

The analytical framework in which Bidard and Klimovsky expand on their assertion that the w-r criterion –when applied in the case of joint production– is erroneous, is restrictive. For they restrict themselves to only that case of joint production in which negative or zero or undetermined prices of commodities do not appear and the maximum rates of profit of the techniques and the switch-points, which they themselves consider to be genuine and not fake switch-points, do not vary with the normalization commodity (see Bidard and Klimovsky 2001, p. 2).[5]

However, the restrictive analytical framework introduced by Bidard and Klimovsky does not, even in the slightest, affect the correctness of their assertion that the w-r criterion, when applied in joint production techniques, leads to erroneous results. For it evidently remains correct, even when the w-r criterion leads –when comparing and ordering only certain types and not all possible types of joint production techniques– to erroneous results.

Let us now take a look at the analytical framework of Bidard and Klimovsky, which is in effect delineated by an arithmetical example given by them. They start out with three production processes, namely processes 1, 2

5. At this point, Bidard and Klimovsky accept that –according to Manara– "the maximum profit rate cannot be defined (rather: cannot be determined – G.S.) in general as the first positive root of the $\det(B-(1+r)A)=0$ (because the corresponding polynomial may have complex roots only)". And they continue: "...we assume that the maximum profit of every square system can be identified etc.". However, they do not tell us how they determine this maximum profit rate which, according to them, can, in every case, be identified, i.e. determined. Evidently they mean that the maximum rate of profit is indeed determined in general as the first positive root of $\det(B-(1+r)A)=0$, but that they confine themselves to those cases of square systems of joint production, in which the maximum rate of profit of every square system –defined as the first positive root of $\det(B-(1+r)A)=0$– can be identified, i.e. determined.

The maximum rate of profit of a square system of joint production is not in the general case given before the normalization of prices, but results after the normalization of prices. Therefore, this is not the first positive root of $\det(B-(1+r)A)=0$, but rather is in the general case the first positive root of $\det(B^*-(1+r)A^*)=0$, where the matrices B^* and A^* are the matrix of outputs and the matrix of inputs in means of production of the normalization subsystem at each time. For this reason, the matrices B^* and A^* and consequently the maximum rate may vary with the normalization commodity. Naturally, there are also cases of square systems of joint production in which, for certain or all possible normalizations $B^*=B$ and $A^*=A$, in which case the maximum rate of profit is the first positive root of $\det(B-(1+$ $+r)A)=0$. These cases however do not constitute the rule. One such case is the systems of the arithmetical example of Bidard and Klimovsky, which we shall look at further on.

and 3, each of which produces two commodities, namely commodities X and Y. All three processes use the same quantity of labor and specifically one unit of labor. These three processes constitute three square neighbouring joint production techniques, i.e. techniques (12), (13) and (23), of which the first consists of processes 1 and 2, the second of processes 1 and 3 and the third of processes 2 and 3.

They present the following arithmetical example:

$$\begin{aligned} &\text{Process 1: } 20X \oplus 20Y \oplus 1L \rightarrow 21X \oplus 27Y \\ &\text{Process 2: } 20X \oplus 20Y \oplus 1L \rightarrow 23X \oplus 25Y \\ &\text{Process 3: } 30X \oplus 30Y \oplus 1L \rightarrow 36X \oplus 34Y, \end{aligned}$$

where L symbolizes labor. These give three square neighbouring techniques, techniques (12), (13) and (23).

If we normalize prices of production using

$$p_X = a, \quad a > 0,$$

where p_X the price of one unit of commodity X, i.e. using as normalization commodity one unit of commodity X and setting its price equal to a positive constant, then the w-r curves of the three techniques are represented in the following Figure 1 of Bidard and Klimovsky (2001).

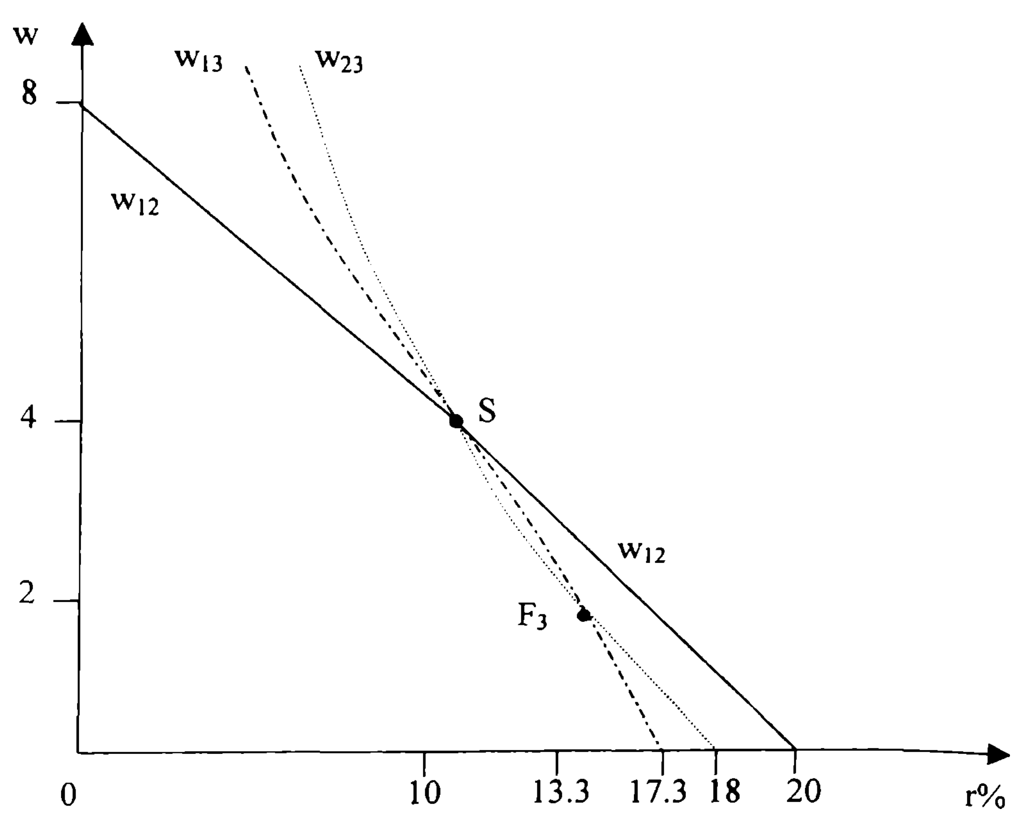

Figure 1

If we normalize prices of production using

$$p_Y = a, \quad a > 0,$$

where p_Y the price of one unit of commodity Y, i.e. using as normalization commodity one unit of commodity Y and setting its price equal to a positive constant, then the w-r curves of the three techniques are represented in the following Figure 2 of Bidard and Klimovsky (2001).

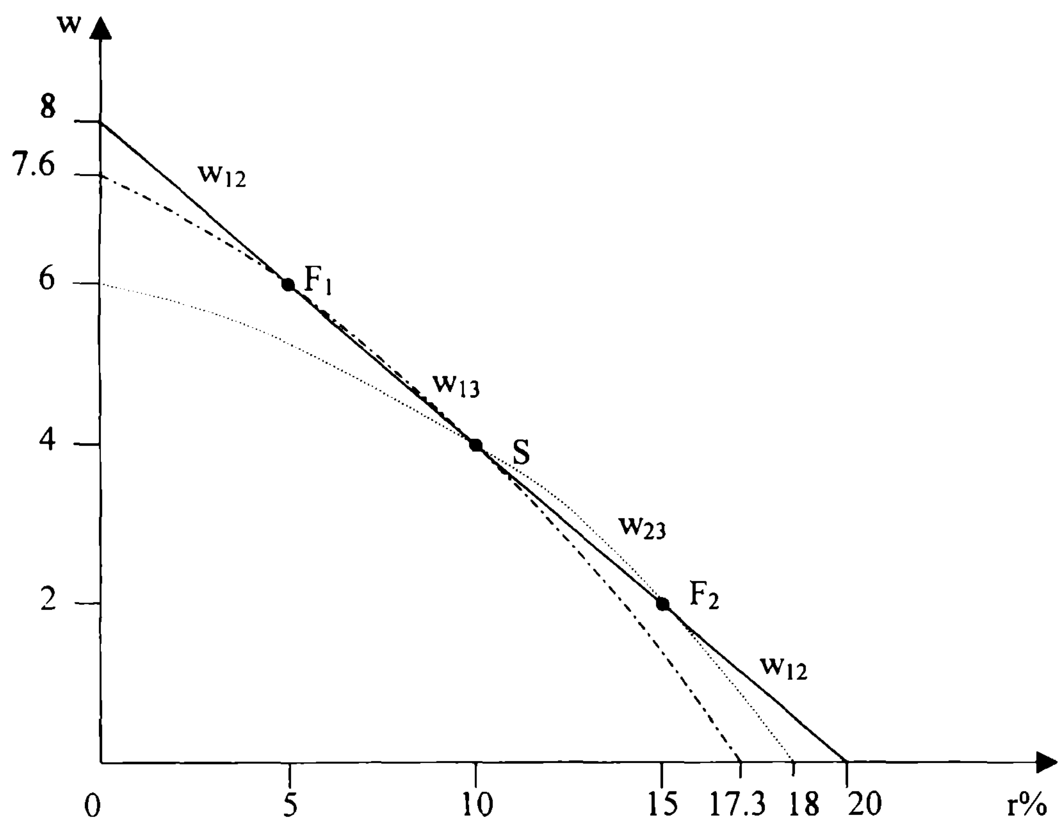

Figure 2

If we accept the definition of switch-point given by Bidard and Klimovsky (2001), i.e. the definition that, in the case where we have three techniques, genuine switch-points are only those points of the upper envelope of the w-r curves of the three techniques, at each of which all three w-r curves are intersected (and that every other point on the upper envelope of the three techniques, at which less than three w-r curves are intersected is a fake switch-point), then a comparison of Figures 1 and 2 shows that with varying normalization commodity, neither the maximum rates of profit of the three techniques vary nor the one and only and –according to Bidard and Klimovsky– genuine switch-point. The maximum rates of profit of the three techniques (12), (13) and (23) are for both normalizations of prices equal to 0.2, 0.173 and

0.18 respectively. And the –according to Bidard and Klimovsky– only genuine switch-point appears in both normalizations of prices for rate of profit $r = 0.1$.

It is very easy (since it is simply a matter of arithmetical calculations) for one to show that the above also holds for any other normalization of prices, i.e. for each normalization, the normalization commodity of which is a bundle of commodities, which contains commodities X and Y in any positive quantities whatsoever.

Thus, if we normalize prices with normalization commodity the bundle of commodities (1, 3) by means of

$$p_X + 3p_Y = 1,$$

we get the following w-r relations

$$w_{12} = 2 - 10r,$$

$$w_{13} = \frac{-9 + 110r}{-9 + 10r}$$

and

$$w_{23} = \frac{-9 + 50r}{-5 + 10r}.$$

For $w_{12} = 0$ we get for the maximum rate of profit $r_{12(max)}$ of technique (12)

$$r_{12(max)} = 0.2.$$

For $w_{13} = 0$ we get for the maximum rate of profit $r_{13(max)}$ of technique (13)

$$r_{13(max)} = 0.1727272.$$ [6]

And for $w_{23} = 0$ we get for the maximum rate of profit $r_{23(max)}$ of technique (23)

$$r_{23(max)} = 0.18.$$

For $r = 0.1$ we get from the above three w-r relations

$$w_{12} = w_{13} = w_{23} = 1.$$

So, for $r = 0.1$ we have also in the case of normalization with normalization commodity the bundle of commodities (1, 3) a –according to Bidard and Klimovsky– genuine switch-point.

6. Bidard and Klimovsky round 0.1727272 to 0.173.

We may therefore conclude that in the arithmetical example of Bidard and Klimovsky, not only the maximum rates of profit of the three techniques but also the 'genuine' switch-point, which appears for a certain normalization at point r = 0.1, do not vary with normalization.

However, when we normalize prices with (1), only one switch-point appears on the upper envelope (at r = 0.1), which according to Bidard and Klimovsky is a genuine switch-point, because all three w-r curves are intersected there, and no fake switch-point appears. *However*, when we normalize prices with (2), the above same 'genuine' switch-point appears at the same point of the upper envelope (at r = 0.1) but, at the same time, at points r = 0.05 and r = 0.15, appear two switch-points which are –according to Bidard and Klimovsky– fake switch-points, because at the first point only the w-r curves of techniques (12) and (13) are intersected, while at the second point only the w-r curves of techniques (12) and (23) are intersected.

According to the w-r criterion, as used also by Sraffa, and consequently also according to Sraffa, these two –according to Bidard and Klimovsky– fake switch-points, which appear in the normalization with (2), are genuine switch-points. Bidard and Klimovsky assert that they are not genuine but fake switch-points. And they justify this assertion by saying that they are not genuine but fake switch-points because only two of the total three given w-r curves are each time intersected there.

This explanation for the fake switch-points is erroneous. The correct explanation is the following: Assuming that these two switch-points, which appear in the normalization with (2) at points r = 0.05 and r = 0.15, were –as is the switch-point which appears not only in the normalization with (1) but also in the normalization with (2) at point r = 0.1– genuine switch-points. If this were the case, then these two switch-points should appear at the aforementioned points not only in the normalization with (2) but also in the normalization with (1) – as is the case with the switch-point which appears at point r = 0.1. However, in the normalization with (1), these two switch-points disappear, they do not exist.

So, according to the w-r criterion, in the normalization with (1) at point r = 0.05, technique (23) prevails over techniques (13) and (12), and technique (13) prevails over technique (12). While according to this same criterion, in the normalization with (2) at this same point r = 0.05 the techniques (12) and (13) are equivalent and prevail over technique (23) (compare Figures 1 and 2 of Bidard and Klimovsky).

Furthermore, according to the w-r criterion, in the normalization with (1) at point r = 0.15, technique (12) prevails over techniques (23) and (13), and technique (23) prevails over technique (13). While according to the same criterion, in the normalization with (2) at this same point r = 0.15, techniques (12) and (23) are equivalent and prevail over technique (13).

The correct conclusion from the above is therefore the following: The w-r criterion as a criterion for the unambiguous ordering of given square neighbouring techniques is not correct, because the ordering of techniques according to this criterion is not unambiguous, since it varies with the normalization of prices. As we noted above, this has been known for quite some time.

However, in the above arithmetical example of Bidard and Klimovsky, with varying normalization commodity the –according to Bidard and Klimovsky– genuine switch-point remains invariable. But this does not hold generally. In the general case, these 'genuine' switch-points also vary (they shift or appear and disappear). In the arithmetical example of Bidard and Klimovsky, this 'genuine' switch-point, which appears for r = 0.1, remains invariable with varying normalization commodity because the techniques of this example are sui generis.

In a word, they are techniques which –for a certain arithmetical value of the rate of profit (in the arithmetical example of Bidard and Klimovsky, for the arithmetical value of r, r = 0.1)– fulfil certain conditions which we shall elaborate on below.

Thus, assuming that all the techniques for comparison and ordering are according to Sraffa's w-r criterion, at r = r*, for each normalization commodity equivalent. Consequently, they all have for r = r* and for each normalization commodity the same nominal wage rate w = w*.

This nominal wage rate w*, which –for each given normalization commodity– is equal for all the techniques, may of course –remaining equal for all the techniques– vary with the normalization commodity. (In the example of Bidard and Klimovsky, it does not vary in normalizations (1) and (2), because in the transition from one normalization to the other, not only the relative but also the absolute prices of commodities do not vary).

So, assuming that –for each given normalization commodity– we consider exogenously given not r* but the w* corresponding to the given normalization commodity, which is equal for all the techniques. Then, for each normalization commodity, the rate of profit of each technique will be equal to r*.

When does the above occur? When, that is, according to Bidard and Klimovsky, do we have a genuine switch-point?

Only when each of the techniques for comparison and ordering fulfils the following conditions: (a) For the given normalization commodity and for the corresponding –common for all the techniques– w* there exists in the normalization subsystem, which corresponds to that technique, a real wage rate (naturally compatible for the given prices with the nominal wage rate w*) *such, that* the surplus product of this normalization subsystem has the same composition as the means of production of that same normalization subsystem, (b) for the given normalization commodity the ratio of the surplus product to the means of production, i.e. the rate of profit determined independent of prices, is equal to all the normalization subsystems, and (c) condition (b) is fulfilled for each normalization commodity. Condition (a) is fulfilled by all possible techniques. Condition (b) is fulfilled by all the techniques which, for given w* and *given normalization commodity*, are equivalent and condition (c) is fulfilled only in the case of those techniques which, for the given w*, are equivalent *for each normalization commodity*. Precisely these three conditions are fulfilled in the arithmetical example of Bidard and Klimovsky.[7]

7. If we normalize prices with

$$p_X = 1, \tag{a}$$

then the net product of the normalization subsystem of each technique is equal to (1, 0). For the normalization subsystem of technique (12) we have the following data:

Activity level: (-0.3125, 0.4375),
Gross product: (3.5, 2.5)
Means of production: (2.5, 2.5), and
Aggregate labor: 0.125 units

For the normalization subsystem of technique (13) we have:

Activity level: (-0.10526, 0.1844211),
Gross product: (4.421053, 3.421053)
Means of production: (3.421053, 3.421053), and
Aggregate labor: 0.078951 units

And for the normalization subsystem of technique (23) we have:

Activity level: (-0.22222, 0.277778),
Gross product: (4.888889, 3.888889)
Means of production: (3.888889, 3.888889), and
Aggregate labor: 0.055558 units

For each of the three normalization subsystems there are real wages, for which the surplus

Bidard and Klimovsky tell us that for r = 0.1 the three techniques of their arithmetical example can be –on the basis of the w-r criterion– unambiguously

product of the normalization subsystem has the same composition as the means of production. These real wages are in the normalization subsystem of technique (12) equal to (0.75, -0.25), in the normalization subsystem of technique (13) equal to (0.657895, -0.34211) and in the normalization subsystem of technique (23) equal to (0.611111, -0.38889).

The corresponding real wage rates are

$$(0.75,\ -0.25)\frac{1}{0.125},$$

$$(0.657895,\ -0.34211)\frac{1}{0.078951} \quad \text{and}$$

$$(0.611111,\ -0.38889)\frac{1}{0.055558}.$$

For normalization with (a) and for r = 0.1 we get for the price p_Y of commodity Y in each of the three normalization subsystems:

$$p_Y = 1.$$

Consequently, in all three normalization subsystems, for r = 0.1 the following holds

$$p_X = p_Y = 1.$$

As one may easily ascertain, the nominal wage rates, which correspond for these prices of commodities to the above real wage rates of the three normalization subsystems, are all equal to 4.

The above holds for r = 0.1 also in the case of normalization with

$$p_Y = 1 \qquad \text{(b)}$$

as well as with every other normalization, in which a bundle of commodities functions as normalization commodity, which bundle consists of any positive quantities of commodities X and Y. We give here the data pertaining to the three normalization subsystems, which result for normalization with (b), and we leave to the reader the verification of our above assertion. The net product of the normalization subsystem of each technique is equal to (0, 1). For r = 0.1 the price of commodity X is in all three normalization subsystems equal to 1. Consequently

$$p_Y = p_X = 1.$$

For the normalization subsystem of technique (12) we have the following data:

Activity level: (0.1875, -0.0625),
Gross product: (2.5, 3.5)
Means of production: (2.5, 2.5), and
Aggregate labor: 0.125 units

For the normalization subsystem of technique (13) we have:

Activity level: (0.157895, -0.02632),

ordered, i.e. that their ordering does not vary with the normalization commodity. They also tell us that for each r, $r \neq 0.1$, these same techniques are not –on the basis of this same criterion– unambiguously classifiable. However, they do not explain how the ordering of these techniques for $r = 0.1$ with the w-r criterion can be correct, i.e. unambiguous, even though –according to them themselves– this criterion is erroneous. Also, they do not tell us whether the ordering of these same techniques for each r, $r \neq 0.1$, with the w-r criterion, which is –again according to them themselves– erroneous, i.e. not unambiguous, becomes correct, i.e. unambiguous, when we use another –different from the w-r criterion– correct criterion. In a word, they do not counter Sraffa's w-r criterion with a correct criterion of their own for the ordering of techniques. They simply allude to it faintly.

The criterion to which they allude (see Bidard and Klimovsky 2001, pp. 3-4), without explicitly stating it, is a criterion which was expounded by Bidard in one of his papers (see Bidard 1990).

We shall therefore present this criterion of Bidard, which we had analyzed in one of our papers in the past (see Stamatis 1996), and subsequently we shall do that which Bidard and Klimovsky do not explicitly do, i.e. –within the framework of the arithmetical example of Bidard and Klimovsky– we shall match Bidard's aforesaid criterion against Sraffa's erroneous w-r criterion, naturally presuming that at least Bidard still considers it to be correct.

In his aforementioned paper (Bidard 1990, pp. 841-842) Bidard calls each bundle of commodities c_i,

$$c_i = b_i - (1 + r) a_i \qquad (3)$$

r-net product of process i (in the arithmetical example of Bidard and Klimovsky $i = 1, 2, 3$).

The r-net product of each process i evidently varies with r and is not necessarily strictly positive but –depending on r– may be strictly positive or

Gross product: (2.368421, 3.368421),
Means of production: (2.368421, 2.368421), and
Aggregate labor: 0.131575 units

And for the normalization subsystem of technique (23) we have:

Activity level: (0.333333, -0.16667),
Gross product: (1.666667, 2.666667),
Means of production: (1.666667, 1.666667), and
Aggregate labor: 0.166663 units

semi-positive or contain, apart from positive or apart from positive and zero components, also negative components or, lastly, be strictly negative.

The above definition of the r-net product c_i of each process i evidently presupposes that the vector of physical inputs (= means of production) a_i of each process i and the vector of the surplus product ra_i of this same process i are collinear, i.e. that the means of production and the surplus product of each process have the same composition. Therefore, the rate of profit r of each process emerges as the ratio of two homogenous physical magnitudes, as the ratio of the surplus product to the means of production, and consequently it is independent of the prices of commodities.

Also, because of (3), the r-net product c_i of each process i is identical to real wages and, because of $L_i = 1$, identical to the real wage rate of this process i (see also Bidard and Klimovsky 2001, p. 3).

Bidard (1990, p. 841) then defines the r-net product of each technique (km), $k \in i$, $m \in i$ and $k \neq m$, where in the arithmetical example of Bidard and Klimovsky $i = 1, 2, 3$, as the strictly positive vectors, which result as a linear combination of the two r-net products of processes k and m of the technique (km).

Let d_{km} be the vector of the r-net product of the technique (km). Then

$$d_{km} = \alpha c_k + \beta c_m \, (> 0), \tag{4}$$

where

$$c_k = b_k - (1 + r) a_k,$$
$$c_m = b_m = (1 + r) a_m,$$
$$0 \leq \alpha \leq 1,$$ [8]
$$0 \leq \beta \leq 1$$

and

$$\alpha + \beta = 1.$$

Consequently

$$\begin{aligned} d_{km} &= \alpha b_k - \alpha(1 + r) a_k + \beta b_m - \beta(1 + r) a_m \\ &= (\alpha b_k + \beta b_m) - [\alpha(1 + r) a_k + \beta(1 + r) a_m] \\ &= (\alpha b_k + \beta b_m) - (1 + r)(\alpha a_k + \beta a_m). \end{aligned} \tag{5}$$

8. Bidard however presupposes $0 < \alpha < 1$ and $0 < \beta < 1$.

It is quite clear that the means of production $\alpha b_k + \beta b_m$ of the technique (km) and the surplus product $r(\alpha a_k + \beta a_m)$ of this same technique (km) have the same composition. Therefore, the rate of profit of this technique emerges as the ratio of two homogenous physical magnitudes, as the ratio of its surplus product to its means of production, and is therefore independent of prices.

It is also clear –since it emerges directly from (5)– that the r-net product d_{km} of the technique (km) is identical to real wages and –because of $L_k = L_m = 1$ and $\alpha + \beta = 1$ and consequently also of $\alpha L_k + \beta L_m = 1$– identical to its real wage rate.

Let the technique (kn), $n \in i$, which is evidently neighbouring to the technique (km) and for which there are r-net products d_{kn},

$$d_{kn} > 0.$$

Then, according to Bidard (1990), the two neighbouring techniques (km) and (kn) can be unambiguously ordered for given r, where there is at least one pair of collinears d_{km} and d_{kn}, i.e. when there is at least one pair of real wage rates of the two techniques (km) and (kn), which has the same composition. Bidard calls this common composition of these two real wage rates d_{km} and d_{kn} "common direction d".

The conditions for the unambiguous ordering of techniques, which are set forth by Bidard, are therefore the conditions (a), (b) and (c) which we set out above. So, according to Bidard (1990), when –for given r– the given techniques fulfil the above conditions, then we choose the one with the greatest d, i.e. the one with the greatest real wage rate. And, when –for given r– only certain of the given techniques fulfil the above conditions, then only these latter techniques are comparable and can be ordered and of these, the one with the greatest d, i.e. with the greatest real wage rate, is the most profitable.[9]

So, the criterion of Bidard (1990) is a real wage maximization criterion. Also, it is a criterion, which compares and orders given techniques, firstly, without requiring prior normalization of prices and, secondly, in complete independence of prices – even though it appears that Bidard (1990) himself does not realize this latter point.

Below, we shall apply Bidard's criterion to the specific arithmetical

9. In this way, Bidard is saying of course –albeit probably without realizing it– that given techniques are not for given r always and unconditionally comparable and can be ordered with respect to their profitability.

example of Bidard and Klimovsky and compare the results we get not only with the results we get from applying Sraffa's w-r criterion but also with the results obtained by Bidard and Klimovsky (2001).

First, however, we consider it expedient to clarify the following: Because, firstly, each of the three techniques (12), (13) and (23) which produces the commodities X and Y consists of two production processes and, secondly, each of the three production processes 1, 2 and 3 produces both commodities X and Y, the techniques (12), (13) and (23) are separable. This means that commodities X and Y may be produced not only by each of these three techniques, but also by only certain processes of each of these three techniques (here: by only one production process of each of these three techniques), i.e. that commodities X and Y may also be produced by the non-square techniques (10), (02) and (03), that is, by processes 1, 2 and 3. Therefore, the square and non-square techniques to be compared are six, namely techniques (12), (13), (23), (10), (02) and (03). As we shall see, this fact is of some importance for the possibility of defining the genuine switch-point within the framework of Bidard's criterion.

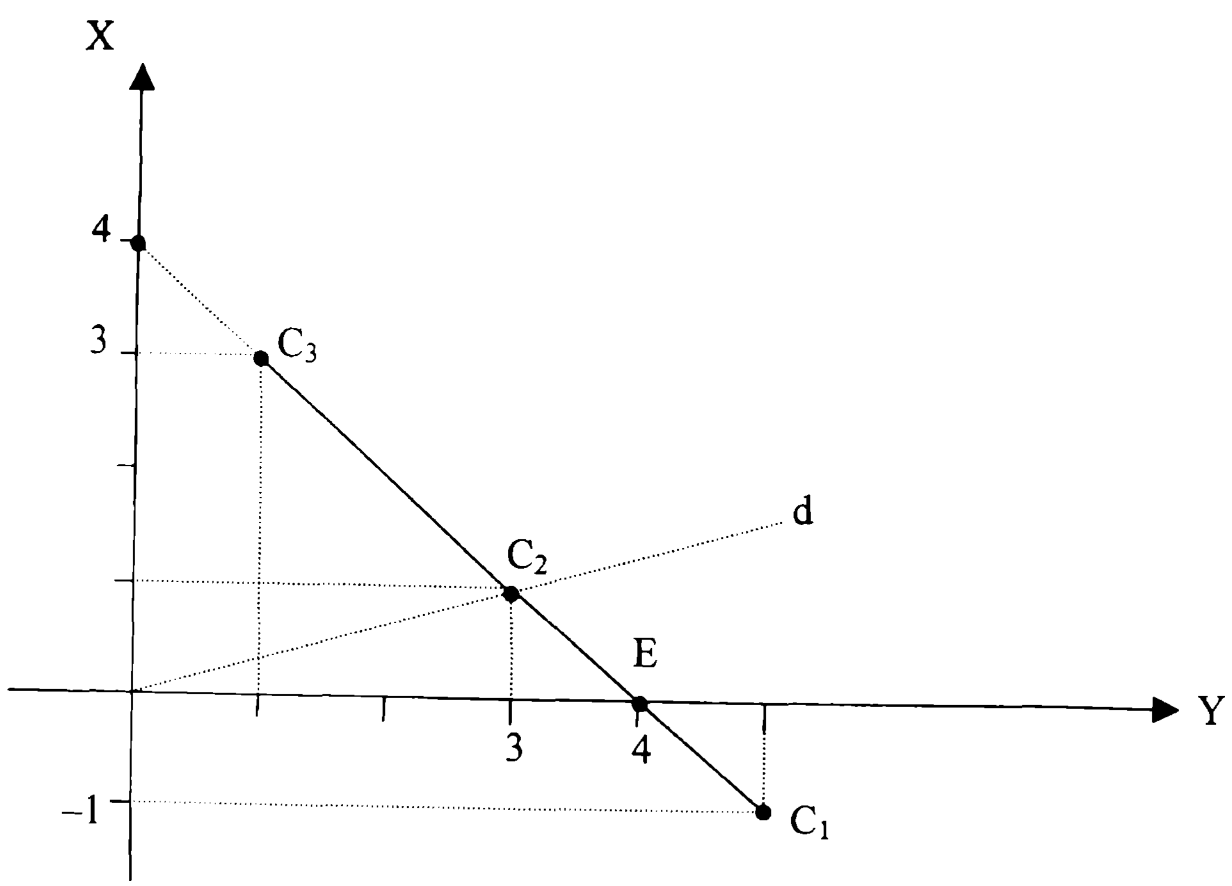

Figure I

So, let r = 0.1. Then in the arithmetical example of Bidard and Klimovsky

$$c_1 = (-1, 5),$$

$$c_2 = (1, 3),$$

and

$$c_3 = (3, 1).$$

In Figure I above, we depict c_1 with C_1, c_2 with C_2 and c_3 with C_3.

The points C_3, C_2, E and C_1 evidently lie on the same straight line, namely on the straight line

$$X = 4 - Y.$$

The point C_3 is the r-net product of the non-square technique (03), i.e. of process 3. Each point on the segment C_3E –except points C_3 and E!– is r-net product of technique (13). Each point on the segment C_3C_2 *–except* points C_3 and C_2!– is r-net product of square technique (23). Point C_2 is r-net product of the non-square technique (02), i.e. of process 2. And each point on segment C_2E *–except* points C_2 and E!– is r-net product of square technique (12).

According to Sraffa's w-r criterion, for r = 0.1 the techniques (12), (13) and (23) are equivalent and their ordering does not vary with the normalization commodity (compare Figures 1 and 2 of Bidard and Klimovsky 2001), i.e. they can be unambiguously ordered.

These three techniques can be, for r = 0.1, unambiguously ordered and are, for r = 0.1, equivalent also according to Bidard and Klimovsky (2001). However, Bidard and Klimovsky do not tell us –given that they consider Sraffa's w-r criterion to be erroneous– according to which other –different to Sraffa's w-r criterion– criterion they consider that these three techniques can be for r = 0.1 unambiguously ordered and are for r = 0.1 equivalent.

Let us now see what happens if we order the three square techniques (12), (13) and (23) using Bidard's criterion. According to this criterion, for r = 0.1, the above techniques cannot be unambiguously ordered, because they do not fulfil conditions (a), (b) and (c), i.e. Bidard's conditions for unambiguous ordering, since there is no common direction d for them.

Direction d, which passes through point C_3 does not constitute the r-net product of any of the three techniques. Each direction d, which passes between points C_3 and C_2, constitutes common direction d only for techniques (13) and (23) but not for technique (12). Thus, according to Bidard's criterion, for each direction d, which passes between points C_3 and C_2, techniques (13) and (12) are, for r = 0.1, equivalent, while for technique (12) nothing can be said, for it cannot be compared to the other two techniques.

Direction d, which passes through point C_2, does not constitute r-net product of any of the three techniques. Consequently, for direction d, which passes through point C_2, the three techniques cannot be compared and ordered according to Bidard's criterion.

Each direction d, which passes between points C_2 and E, constitutes r-net product of techniques (12) and (13) but not r-net product of technique (23). Therefore, according to Bidard's criterion for each direction d, which passes between points C_2 and E, techniques (12) and (13) are equivalent, while for technique (23) nothing can be said because it cannot be compared with the other two techniques.

Conclusion: According to Bidard's criterion, at r = 0.1 there is no switch-point of any kind, neither a genuine nor a fake switch-point, because for r = 0.1 there is no common direction d for all three techniques.

For all the more reason, this conclusion holds if in the comparison we include all three non-square techniques (10), (02) and (03). In this case, we would merely have to include in the analysis also direction d, which passes through point C_3, as well as direction d which passes through point C_2. In this case, according to Bidard's criterion for direction d, which passes through point C_3, the six techniques cannot be compared to one another, because for only one of these, namely for technique (03), is there r-net product, the composition of which coincides with direction d, which passes through point C_3.

Also according to Bidard's criterion for direction d, which passes through point C_2, the six techniques cannot be compared to one another, because for only one of these, namely technique (02), is there r-net product, the composition of which coincides with direction d which passes through point C_2.

It emerges from the above analysis that Bidard and Klimovsky have no criterion whatsoever, on the basis of which they could say –as they nevertheless do say– that according to the –in their view erroneous– w-r criterion of Sraffa, the switch-point appearing at point r = 0.1 for each normalization of prices is a genuine switch-point.

Ultimately, this switch-point is indeed a switch-point, but a switch-point not of the three techniques but of the –for each normalization– corresponding normalization subsystems.

Now let r = 0.05. Then

$$c_1 = (0, 6),$$

$$c_2 = (2, 4)$$

and

$$c_3 = (4.5, 2.5).$$

Thus, here we get the following Figure II.

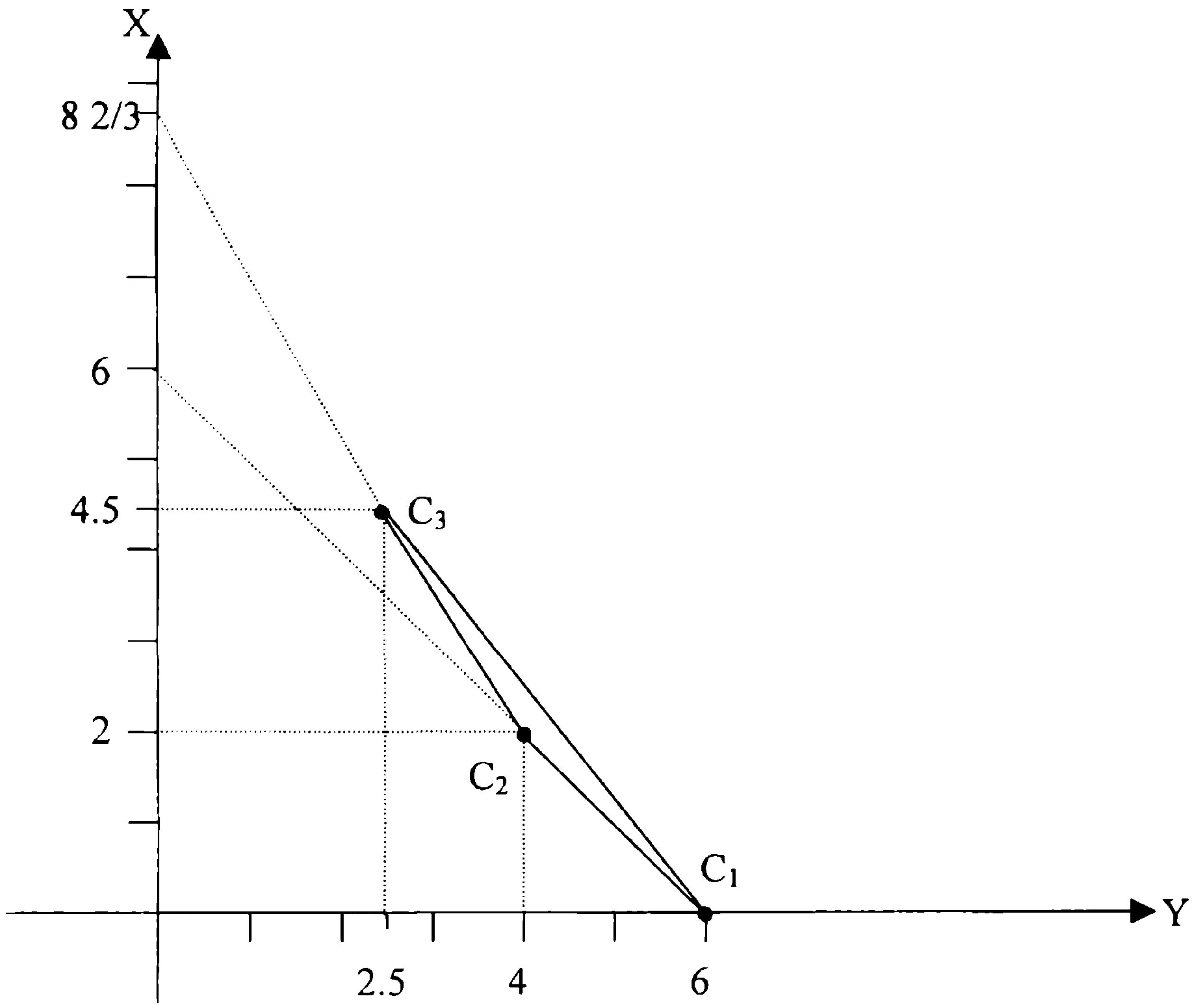

Figure II

For r = 0.05 the ordering of the three techniques with Sraffa's w-r criterion is not unambiguous, since it varies with the normalization commodity. When commodity X functions as normalization commodity, then for r = 0.05 technique (23) prevails over technique (13), and technique (13) prevails over (12) (See Fig. 1 of Bidard and Klimovsky 2001). However, when commodity Y functions as normalization commodity, then according to Sraffa's w-r criterion, techniques (12) and (13) are equivalent and prevail over technique (23) (See Fig. 2 in Bidard and equivalent Klimovsky 2001).

Bidard and Klimovsky (2001) correctly ascertain that the given three techniques cannot be, for r = 0.05, unambiguously ordered with Sraffa's w-r criterion. But they do not explain why they cannot be unambiguously ordered with this criterion. Nor do they tell us if there is another criterion, different from Sraffa's w-r criterion, with which these techniques can be unambiguously ordered. They confine themselves simply to a discussion of the switch-point, which appears, when commodity Y functions as normalization commodity, and which disappears when commodity X functions as normalization commodity. But the discussion of this switch-point and its labeling as a 'fake' switch-point is meaningless from the moment we know that this switch-point appears and disappears according to the normalization commodity used, i.e. from the moment we know that, for r = 0.05, the ordering of the given techniques with Srafffa's w-r criterion is not unambiguous, but varies with varying normalization commodity. It would be meaningful only if one explains why the ordering of the given techniques according to Sraffa's w-r criterion varies with varying normalization commodity and is therefore not unambiguous.

Let us now see what happens according to Bidard's criterion (we shall include in the comparison and ordering of techniques also the non-square techniques (10), (02) and (03)).

For direction d, which passes through point C_3, the six techniques cannot be compared to one another. For each direction d, which passes between points C_3 and C_2, only techniques (12) and (13) can be compared to each other (technique (13) prevails over technique (12)), while for the other techniques nothing can be said. For direction d, which passes through point C_2, only techniques (13) and (02) can be compared to each other (technique (13) prevails over technique (02)), while for the other techniques nothing can be said. For each direction d, which passes between points C_2 and C_1, only techniques (13) and (12) can be compared to each other (technique (13) prevails over technique (12)), while regarding the other techniques nothing can be said. And, lastly, for direction d, which passes through C_1, the six techniques cannot be compared to one another.[10]

10. In this case, direction d coincides with the r-net product of technique (10), i.e. with the r-net product of process 1.

And for r = 0.05 it is confirmed that the comparison and ordering of given techniques with Bidard's criterion is not unambiguous but varies with direction d.

This is why the assertion of Bidard and Klimovsky that "... in a two-commodity economy, fakes (i.e. fake switch-points – G.S.) occur when, for some method i (i.e. for some process i – G.S.), the surplus (i.e. the r-net product – G.S.) b_i-(1-r_0)a_i has the direction of the numeraire" (p. 3), is correct. Here, where r = 0.05, the r-net product of process 1 has the same direction, i.e. the same composition, as the normalization commodity Y. For this reason, a fake switch-point appears for r = 0.05 when commodity Y functions as the normalization commodity. As we shall see immediately below, the same holds for r = 0.15, when as normalization commodity functions the commodity Y, because the r-net product of process 2 and the normalization commodity have the same composition. In contrast, when as normalization commodity functions the commodity X for both r = 0.05 and for r = 0.15 no fake switch-point appears, because for neither r = 0.05 and r = 0.15 is there a r-net product of some process, which has the same composition as the normalization commodity (Compare Figures II and III).

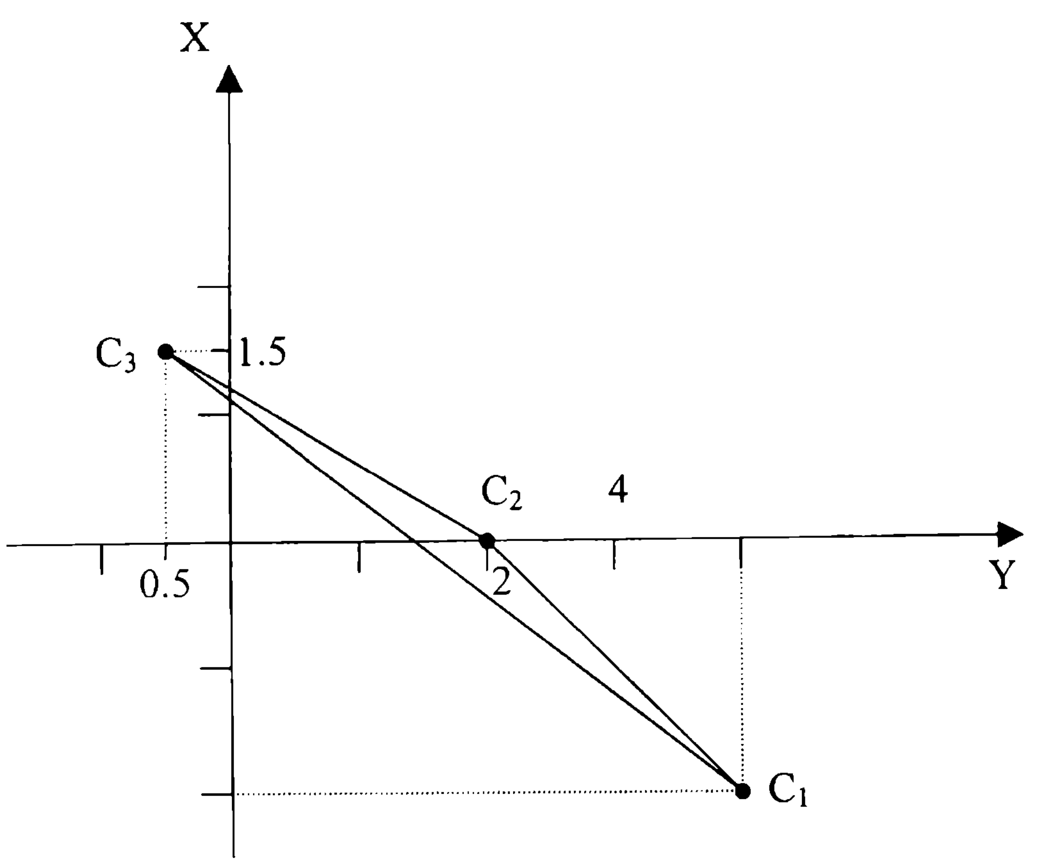

Figure III

Assuming now r = 0.15. Then

$$c_1 = (-2, 4),$$

$$c_2 = (0, 2)$$

and

$$c_3 = (1.5, -0.5).$$

We thus get Figure III.

Here we have the following situation. According to Sraffa's w-r criterion, for r = 0.15 the ordering of the three techniques is not unambiguous, because it varies with the normalization commodity. When commodity X functions as the normalization commodity, then for r = 0.15 technique (12) prevails over technique (23) and technique (23) prevails over technique (13) (compare Fig. 1 of Bidard and Klimovsky 2001). However, when commodity Y functions as normalization commodity, then for r = 0.15 the techniques (12) and (23) are of equivalent and prevail over technique (13) (compare Fig. 2 in Bidard and Klimovsky 2001).

Bidard and Klimovsky correctly ascertain that the three techniques cannot be unambiguously ordered for r = 0.15 using Sraffa's w-r criterion, because their ordering varies with the normalization commodity. However, they do not explain why, with this criterion, the given techniques cannot be unambiguously ordered. Nor do they tell us if there is another criterion –different to Sraffa's w-r criterion– with which these techniques can be unambiguously ordered. They merely confine themselves to a discussion of the switch-point, which appears when commodity Y functions as normalization commodity and which does not appear when commodity X functions as normalization commodity. But the discussion of this switch-point and its labeling as a 'fake' switch-point is meaningless from the moment we know that this switch-point appears and disappears depending on the normalization commodity used, i.e. from the moment we know that, for r = 0.15, the ordering of the given techniques with Srafffa's w-r criterion is not unambiguous, but varies with the normalization commodity.

Let us now see what happens according to Bidard's criterion (where in the comparison of techniques we shall also include techniques (10), (02) and (03)).

For direction d, which coincides with the X-axis, only techniques (13) and (23) are comparable ((23) prevails over (13)), while for the other techniques nothing can be said. For each direction d between the X-axis and the Y-axis, only the two techniques (13) and (23) are comparable ((23) prevails over ((13)), while for the other techniques nothing can be said. And for direction d, which coincides with the Y-axis (and with the r-net product of technique (02), i.e. with the r-net product of process 2), only techniques (02) and (13) are comparable (technique (02) prevails over technique (13)), while regarding the other techniques nothing can be said.

And so for r = 0.15 the ordering of the given techniques according to Bidard's criterion varies with direction d and consequently is not unambiguous. This conclusion evidently holds for each r.

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Bidard's criterion of ordering of techniques is a real wage maximization criterion. It appears as though with this criterion one ordered –for given r– the given techniques with respect to the size of their –of common composition– real wage rates and choosed as the most profitable the one with the highest real wage rate.

However this is not exactly the case. For with Bidard's criterion we do not order the techniques themselves but rather quasi Sraffian standard systems corresponding to these techniques, with respect to the size of their –of common composition– real wage rates. Thus, Bidard's criterion is a real wage rate maximization criterion for the ordering not of the given techniques, but of the corresponding quasi Sraffian standard systems.

If we ex post introduce a price normalization equation, in which functioning as normalization commodity is a bundle of commodities with the same composition as one of the compositions of the –of common composition– real wage rates of the quasi Sraffian standard systems, then Bidard's criterion is, just like Sraffa's criterion, a w-r criterion. Because then, according to this criterion, for given r, we order the techniques with respect to the size of their *nominal* wage rates and we choose as the most profitable the one with the greatest *nominal* wage rate. This ordering evidently coincides with the ordering, which results for this same r, if we order the techniques for a common direction d, which is identical to the composition of the normalization commodity used, with respect to the size of their *real* wage rates and we choose as the most profitable the one with the highest *real* wage rate. This same ordering evidently varies with the aforementioned normalization commodity, the composition of which coincides each time with a common direction d, just as the ordering according to the real wage maximization criterion of Bidard varies with varying common direction d.

So, Bidard's criterion is eventually, just like Sraffa's criterion, a w-r criterion. This is why the ordering according to Bidard's criterion varies, just like the ordering according to Sraffa's w-r criterion, with the normalization commodity.

However, we saw that –even though both these criteria are eventually w-r criteria– Bidard's criterion differs from Sraffa's w-r criterion, since both these criteria as a rule result in different types of ordering of the given techniques.

So, in what way do Sraffa's criterion and Bidard's criterion differ, given that both are w-r criteria? In the following: Neither Sraffa nor Bidard compare and order the given techniques. Rather, on the one hand, Sraffa compares and order with the w-r criterion, for given r, the normalization subsystems which correspond for the given normalization of prices and for given r to the given techniques, Bidard compares and orders *with this same w-r criterion* for given common direction d and consequently for each normalization with normalization commodity a commodity, the composition of which coincides with the given common direction d, the for given r quasi Sraffian standard systems corresponding to the techniques. This is why Sraffa's ordering varies with normalization. For with normalization and with the normalization commodity, the normalization subsystems which he compares also varies. This is also why Bidard's ordering varies with common direction d and consequently –on the condition that the normalization commodity is a commodity, the composition of which coincides with each common direction d– with normalization. For with common direction d and consequently with normalization of the above type, the quasi Sraffian standard systems –which he compares and orders for each given r– also vary.

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We economists began to 'unambiguously' compare and order linear techniques with respect to their profitability, without first having investigated whether unambiguous comparison and ordering of linear techniques with respect to their profitability –without conditions or, albeit, subject to certain conditions– is possible. The criteria for the comparison and ordering of techniques, which we have used to date, are three: the w-r criterion, the cost minimization criterion and the aforementioned criterion of Bidard.

None of these three criteria leads, as we have already seen, to the unambiguous ordering of techniques. The reason is simple: The ordering of techniques with these three criteria is not ordering of the given techniques, but ordering of certain systems of production which are different for each criterion, which (systems) use these techniques or parts of these techniques-systems, the ordering of which varies when certain of its conditions vary, such as the normalization commodity and common direction d, and for precisely this reason it is not unambiguous.

Let us now take a look at the ordering of techniques with each of the aforementioned three criteria.

As we have already noted, the ordering of techniques with Sraffa's w-r criterion is not an ordering of techniques, but an ordering of the normalization subsystems which –for the normalization commodity of the respective normalization of prices– correspond to these techniques. The ordering of these normalization subsystems varies however in the general case with the normalization commodity and consequently is not unambiguous. It does not vary with the normalization commodity and consequently is unambiguous only in special cases, to which we referred above.

And in what does the cost minimization criterion consist? With this criterion, two or more square neighbouring techniques are ordered, i.e. two or more square techniques, which differ with respect to only one production process, which produces the same commodity, with the criterion of their cheapness in the production of this commodity. With this criterion, the technique is chosen which for each given r produces the said commodity at the lowest cost.

This ordering, of course, presupposes normalization of prices. In reality therefore, with the cost minimization criterion it is not the given techniques each time which are ordered but the normalization subsystems corresponding to these techniques, which (subsystems) differ among themselves only with respect to one and only one production process, which produces the same commodity.

For this reason, this ordering varies with varying normalization commodity – for exactly the same reasons that the ordering of given techniques with the w-r criterion also varies with varying normalization commodity.

Apart from this, the cost minimization criterion cannot be applied in the case of joint production techniques, except on the condition that the process, in which two neighbouring techniques differ, produces exactly the same bundle of commodities – a prerequisite obviously given in the case of single production techniques, where each process produces only one commodity, but not in the case of joint production techniques, where each production process produces one or more commodities.

Lastly, even though both the above criteria are often used as quasi equivalent, as far as we know nobody has yet taken the trouble to investigate the relationship between them. One acts –without having investigated the issue– as though, when of two neighbouring techniques one minimizes the cost of the commodity produced by the process by which these two techniques differ, this technique was also more profitable than the other, i.e. as though the cost minimization criterion was equivalent to the w-r criterion. What is certain is that these two criteria lead to the same ordering only in the case of indecomposable single production techniques (see Stamatis 1997a).[11] For obvious reasons, the same must apply also in the case of 'normally behaving' joint production techniques. In all other cases it is more or less certain that these two criteria do not lead to the same ordering.

Whatever the situation, i.e. either ordering of techniques according to the cost minimization criterion coincides or does not coincide in each given case with ordering according to the w-r criterion, the fact remains that with the cost minimization criterion it is not the techniques given each time that are compared and ordered, but the normalization subsystems corresponding to them, with respect to the cheapness of the common bundle of commodities, which is produced by the production process by which these techniques differ.[12]

11. In this comparison of orderings, which result according to each of these two criteria, prices must of course have been normalized, both for ordering in accordance with the cost minimization criterion and for ordering according to the w-r criterion, in the same way. The only exception being the case where, in ordering according to the cost minimization criterion, prices have been normalized by $w = 1$. In this case, in ordering according to the w-r criterion, prices can be normalized in any way (different of course from normalization by means of $w = 1$, because, when $w = 1$, there are no w-r relationships of the techniques). In this case however, classifications which result from the application of the two criteria cannot be compared for r, $r \geq min R_i$, $i = 1, 2, ..., n$, R_i the maximum rate of profit of technique i and n the multitude of techniques, because evidently for each r, $r \geq min R_i$ there is no ordering of techniques according to the cost minimization criterion.

12. When in using the cost minimization criterion, prices have been normalized with $w = 1$, then

Let us now look at Bidard's criterion. In reality, Bidard does not introduce a new criterion for the comparison and ordering of techniques (because, as we saw above, if when using Bidard's criterion one normalizes prices ex post with a bundle of commodities as normalization commodity, the composition of which (bundle) coincides with each common direction d of Bidard, then it becomes clear that Bidard's criterion is in the final analysis a w-r criterion), but rather sets out conditions under which –according to Bidard himself– given square neighbouring techniques are unambiguously classifiable. However, as we saw previously, the conditions for the unambiguous comparison and ordering of given techniques, which are set out by Bidard, entail that with this criterion of real wage maximization or –following the aforesaid ex post normalization– with this w-r criterion, it is not the given techniques that are compared and ordered but the quasi Sraffian standard systems corresponding to these techniques, regarding which (standard systems) it has been presupposed that they have real wage rates of common composition. Thus, when Bidard sets out the above conditions, under which –according to him– given square neighbouring techniques may be unambiguously compared and ordered with respect to their profitability, it is as though he is saying –Bidard himself does not realize it– that given square neighbouring techniques cannot be unambiguously compared and ordered, except when they fulfil the aforementioned conditions, i.e. when to these correspond quasi Sraffian standard systems, the real wage rates of which have a common composition. We showed above that the ordering of these quasi Sraffian standard systems is not unambiguous but varies with the common composition of their real wage rates.

It emerges from the above that the comparison and ordering of given square neighbouring techniques, which is performed with each of the aforementioned three criteria, is not a comparison and ordering of techniques but comparison and ordering of certain –different for each criterion– systems, which use these techniques, and that the comparison and ordering of these systems is not unambiguous.

Thus, the question which arises is the following: Are there systems which can be unambiguously compared and ordered?

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each bundle of commodities d functions as normalization commodity, which (bundle) fulfils for *each* of the techniques to be compared the condition dp = w = 1. Thus, in this case it is possible for there to exist for each technique infinite normalization subsystems, which of course have the same profitability.

The answer is that there are. In fact there are two kinds of such systems: Charasoffian standard systems and corn systems, the real wages of which have the same composition.

A Charasoffian standard system is a system, of which the –defined as the aggregate of the means of production and real wages– physical capital and its surplus product have the same composition (see Charasoff 1910 and Stamatis 1999). Because in such a system physical capital is defined as the aggregate of means of production and real wages, the rate of profit of such a system is defined as the ratio of profits to the price of the aggregate of means of production and real wages.

We use the term 'corn system' for a system, the real wages, surplus product, net product and means of production of which have the same composition and the physical capital of which consists only of the means of production. So, a corn system is a Sraffian standard system, i.e. a system in which the net product and means of production have the same composition, and in which, additionally, both real wages and consequently also the surplus product have the same composition as the net product and consequently as the means of production too. Because in a corn system the physical capital consists only of the means of production, its rate of profit is defined as the ratio of profits to the price of the means of production.

We shall first explain why Charasoffian standard systems can be unambiguously compared and ordered with respect to their profitability and then why corn systems, the real wages of which have the same composition, can be unambiguously compared and ordered with respect to their profitability.

The ordering of given Charasoffian standard systems evidently emerges directly as the ordering of the rates of profits of these systems. This ordering is unambiguous, because it depends neither on normalization nor on prices, nor lastly on the distribution of income. The reasons why this ordering does not depend on any of the above three factors are the following: This ordering does not require previous normalization of prices. Prices play no role whatsoever in this ordering, because the rate of profit of each Charasoffian standard system may be taken as the ratio of two homogenous physical magnitudes, as the ratio of the surplus product to the aggregate of means of production and real wages, and consequently it is independent of prices. Lastly, distribution of income also plays no role in the ordering of Charasoffian standard systems, because in these systems the real wages of each system is –a non explicit– part of physical capital and consequently an unknown magnitude.[13]

So, according to the above, given –neighbouring or non-neighbouring, of the same or different dimensions, square or non-square– techniques are –for equal or unequal or non-comparable real wage rates– unambiguously comparable with respect to the rate of profit –defined as the ratio of profits to the price of means of production and real wages– only in the form of the Charasoffian standard systems which correspond to these techniques – if they exist and to the extent that such systems exist. Put differently: Only techniques, for which there are Charasoffian standard systems, are unambiguously classifiable with respect to their profitability – but only then in the form of the Charasoffian standard systems corresponding to them (see Stamatis 1997b and 1997-1998).

Such a comparison and ordering is eventually performed by Neumann in his splendid paper (see von Neumann 1937 and 2001). In this paper Neumann –without realizing it– tacitly states the aforementioned condition, under which given –neighbouring or non-neighbouring, square or non-square, of the same or different dimensions– techniques are unambiguously comparable with respect to their profitability.

We dare say, despite what has been written about the importance and value of von Neumann's paper, that its real value and importance consists only in the following: While in his paper he intends to tell us which commodities in what quantities are offered and purchased and at what prices these quantities of commodities are offered and purchased, in the end he tells us nothing about all this,[14] but tells us, indirectly and without he himself realizing it, in which

13. It is worth noting here that the comparison and ordering of Charasoffian standard systems with respect to their profitability does not require, precisely because real wages constitute a non explicit part of physical capital, the real wage rate to have the same composition in each system.
14. Ultimately, von Neumann determines only which commodities are offered and purchased in a state of equilibrium. He also determines not absolute but only relative quantities of these commodities. Lastly, neither does he determine the relative prices of these commodities. For while the vector of equilibrium prices which he gets is positive, it is also *random*. Furthermore, the commodities, which according to von Neumann are produced in a state of equilibrium, are not determined by demand but rather emerge as the commodities produced by the most profitable Charasoffian standard system of the Charasoffian standard systems which correspond to the given techniques (see Stamatis 1997b and 1997-1998).

form any given techniques are unambiguously comparable with respect to their profitability.

Let us now see whether corn systems, the real wages of which have the same composition, can be compared and ordered unambiguously with respect to their profitability. These systems may –for given r–[15] be compared and ordered unambiguously with respect to the size of their –of common composition –real wage rates and consequently– if we presuppose that a system with a higher real wage rate than another system is for given r more profitable than this latter – with respect to their profitability. A system like the above, which has –for given r– a real wage rate higher than another system is more profitable than this latter system.

The ordering of these systems is unambiguous because it depends neither on the normalization commodity nor on prices, but only on the ordering criterion itself, i.e. on the size of the real wage rates –of common composition– of the systems to be compared and ordered. This ordering does not depend on normalization because it does not presuppose previous normalization of prices, and it does not depend on prices because the rate of profit of each of the said systems is defined as the ratio of two homogenous physical magnitudes, i.e. as the ratio of the surplus product to the means of production, and consequently it is independent of prices.

So, according to the above, given neighbouring or non- neighbouring square or non-square but of the same dimensions techniques may be compared and ordered unambiguously with respect to their profitability only in the form of the corn systems with real wages of common composition corresponding to them (and only those to which such systems correspond). Put differently: Only techniques, for which there are systems like the above, can –for given r– be compared and ordered unambiguously with respect to their profitability.

It is on the comparison and ordering of techniques, for which there are corn systems with real wages of common composition, that –if I am not mistaken– the construction of Samuelson's surrogate production function is based.

As is known, within the context of the remarkable but now forgotten Capital Controversy of the two Cambridges, i.e. of the still today neoclassical

15. The ordering of given corn systems, the real wages of which have the same composition, can be performed –in contrast with the ordering of given Charasoffian standard systems– only for given r.

US Cambridge and the once neo-Ricardian English Cambridge, the neo-Ricardians rejoined to Samuelson and the neoclassicists that this case of, as they called it, a “one good economy”, was not the general case, but rather an exceptionally extraordinary case. They overlooked however and continue to overlook the fact that the foundation of their arguments against Samuelson and the neoclassicists, i.e. the –for given r– supposedly unambiguous comparison and ordering of techniques with the w-r criterion, on the basis of which they ascertained the celebrated reswitching phenomenon, which they turned against Samuelson and the neoclassicists, was and still is unsound, because the –for given r– truly unambiguous comparison and ordering of techniques is possible only in the case where it is also possible to formulate a neoclassical surrogate production function.

Before we proceed with our analysis, we wish to clarify the following: We have referred both to the comparison and ordering of Charasoffian standard systems and the comparison and ordering of corn systems with real wages of common composition as the *only* unambiguous comparison and ordering. This is not a contradiction. For the comparison and ordering of Charasoffian standard systems is performed without any precondition whatsoever, while the comparison and ordering of corn systems with real wages of common composition is performed on the condition that r is given. What we have therefore are two ‘unique’ unambiguous types of comparison and ordering. For they are carried out under different conditions. This is the consequence of the fact that different systems are each time being compared and ordered.

But let us see how we reach the ascertainment that only techniques for which there are Charasoffian standard systems can be, not they themselves but in the form of these Charasoffian standard systems, unambiguously compared and ordered with respect to their profitability.

Let us again begin with the w-r criterion. What do we do when we try to compare and order given techniques with this criterion? We start with the fact that each technique is characterized by two things, namely by w and by r, which as a rule differ from technique to technique. In order to compare and order given techniques with respect to rate of profit r, we must presuppose that the nominal wage rate w is exogenously given and equal in all the techniques. However, in order to presuppose that w is exogenously given and equal in all the techniques, so as to then get the rates of profit of the various techniques and perform the relevant ordering of techniques in correspondence to them, w must be a cardinally measurable magnitude. However, in order for w to be a cardinally measurable magnitude, we must normalize prices, and moreover in the same way for all the techniques. After this, we presuppose that w is equal in all techniques. We then calculate the rates of profit, which result for this w that is uniform for all the techniques and we order according to these resulting rates of profit the techniques with respect to their profitability.

Or, conversely: We assume that the rate of profit is uniform for all the techniques, we calculate the corresponding nominal wage rates of the techniques and we order them according to these nominal wage rates, accepting that the technique with the highest nominal wage rate is –for the given uniform r– the most profitable. However, we then discover that when, for given uniform r, normalization varies, the ordering of the nominal wage rates of the given techniques and consequently the ordering of the techniques themselves also varies. We ascertain, that is, that the ordering of given techniques with the w-r criterion is not unambiguous, but varies with normalization of prices, because with normalization the –for given uniform rate of profit– ordering of the nominal wage rates of the various techniques varies. [16]

Bidard, with the criterion which he introduced, tackles and 'solves', i.e. gets round this problem, as follows: He introduces conditions under which –according to him– the techniques given each time can be compared and unambiguously ordered.[17] Conditions such that –for given and common for all the techniques rate of profit– the ordering of techniques does not result from the ordering of the nominal wage rates corresponding to these techniques,

16. The same applies in the general case also when ordering given techniques with the cost minimization criterion. Here, for given uniform rate of profit, or for uniform nominal wage rate, the ordering of the costs of the commodity varies with normalization, which commodity is produced by the process, by which the techniques for comparison and ordering and consequently the ordering of the techniques themselves differ.

17. Here, we shall remind the reader of these conditions. They are the following three:
For the given and common for all the given techniques to be compared and ordered

(a) there is for each technique a production system, the surplus product and physical inputs (= means of production) of which have the same composition, that is, there is for each technique a quasi Sraffian standard system.

(b) for each of these quasi Sraffian standard systems there is at least one strictly positive real wage rate and

(c) there are, for all the above quasi Sraffian standard systems, real wage rates which have at least one common composition.

which nominal wage rates according to Bidard himself depend on normalization (see Bidard and Klimovsky 2001, p. 2), but from the ordering of the –for given uniform r– of common composition real wage rates of the given techniques. A new problem however arises for him.

If there exists common composition of real wage rates, there exists not necessarily one and only one, but there can exist, within limits, infinite common compositions. Consequently, the common composition of real wage rates can vary. But when it varies, then it is possible for the ordering of real wage rates and consequently also the ordering of techniques to vary. Thus, in the general case, the ordering of techniques according to Bidard's criterion varies with the common composition of real wage rates and consequently it is not unambiguous.

So the problem remains: When wage rates are defined as nominal wage rates, then the ordering of techniques for given r varies with normalization and, when wage rates are defined as real wage rates of common composition, then the ordering of these real wage rates and consequently the ordering of the techniques varies with varying common composition of the real wage rates.

Thus, the problem consists in the fact that wage rates are involved –either via normalization or via the common composition of real wage rates– in the comparison and ordering of techniques. And the solution to this problem consists either in eliminating wage rates from the comparison and ordering of techniques or, if we do not want such elimination, in avoiding any variation in the common direction of the real wage rates.

We shall first see how we can eliminate wage rates from the comparison and ordering of techniques and subsequently how, if we do not want such elimination, we can ensure that the common composition of the real wage rates does not vary.[18]

So how can we eliminate wage rates from the comparison and ordering of techniques? This is very simple. We consider real wage rates to be a non explicit part of physical capital. Thus, physical capital consists of the aggregate of means of production and real wages. Consequently, the rate of profit is now defined as the ratio of profits to the price of the means of production *and* real wages.

18. The unvarying common composition of the real wage rates of the various techniques evidently means that to each of the *nominal* wage rates of these same techniques corresponds one and only one real wage rate, which has the same composition in all the techniques.

With this data we can now compare and order *any* given techniques with respect to their profitability, i.e. with respect to their rate of profit. Where, as we noted previously, firstly, real wages constitute a non explicit part of physical capital and consequently, secondly, the rate of profit of each technique is defined as the ratio of profits to the price of the means of production and real wages.

Is it possible to compare and order these techniques unambiguously with respect their profitability? Yes! Although not these techniques themselves but in the form of the Charasoffian standard systems corresponding to them and, more precisely, even in this form only those for which such systems exist.

We set out above why this comparison and ordering of these Charasoffian standard systems with respect to their profitability is unambiguous. Let us now see how we can render invariable the common composition of the real wage rates of the given techniques for comparison and ordering. We achieve this by postulating that, for the given and uniform for all the techniques to be compared and ordered rate of profit,

(a) there are quasi Sraffian standard systems corresponding to these techniques,
(b) the real wage rate of each such quasi Sraffian standard system has the same composition as the real wage rate of each other such quasi Sraffian standard system[19] and
(c) the real wage rate of each such quasi Sraffian standard system has the same composition as the surplus product and consequently as the means of production of that same system.

Each system, which fulfils –of the above three conditions– conditions (a) and (c) is evidently a corn system. And all corn systems, which fulfil condition (b) are corn systems with real wage rates of common composition. This common composition of the real wage rates of these systems is unvarying, because it is the same as the common composition of the means of production of these same systems.

We have set out above why the comparison and ordering of these systems for given and uniform rate of profit is unambiguous.

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19. Conditions (a) and (b) are evidently the conditions which, according to Bidard, are the conditions for the unambiguous ordering of techniques.

It now remains for us to explain why these techniques as such cannot be unambiguously compared and ordered with respect to their profitability. But let us first see what a technique is.

Each linear production technique is depicted by the triad [B, A, L] where B the matrix of outputs, A the matrix of inputs in means of production and L the matrix of inputs in labor.[20] When production is single production, matrix B is a diagonal matrix. A technique is usually standardized as follows: When the technique is a single production technique, then –irrespective of whether labor is homogenous or heterogeneous– we standarize the technique by presupposing that B = I, i.e. that each production process produces one and only one unit of the (unique) commodity which it produces. When the technique is a joint production technique and labor is homogenous, then we normalize the technique by presupposing that each production process uses exactly one unit of labor. When, lastly, the technique is a joint production technique and labor is heterogeneous, we can normalize the technique in any way, e.g. by presupposing that the aggregate of the quantities of heterogeneous types of labor, which the technique uses, is equal to unit.

The standardization of a technique is nothing more than the determining of the activity levels of the production processes of that technique and consequently of the unitary activity levels of the production systems, which use that technique.

A production system, which uses technique [B, A, L] is depicted by the tetrad [B, A, L, x], where x, $x \geq 0$, the vector of activity levels. Evidently there are infinite systems [B, A, L, x] which use the technique [B, A, L]. The multitude of these systems evidently remains infinite, even when we normalize x, e.g. by means of $\sum_i x_i = a$, where a is a positive constant.

Each technique is therefore an infinity of production systems. So how can two techniques be compared and ordered unambiguously with respect to their profitability? That is, how can two infinities of production systems be unambiguously compared and ordered with respect to their profitability? This is not possible. For it presupposes that all the systems, which use a certain technique have –under these conditions each time– the same profitability. This however does not happen.

20. If labor is homogenous, L is a vector. If labor is heterogeneous, i.e. if we have more than one kind of labor, then L is a matrix.

Consider the simplest case, the case of a decomposable single production technique. To this technique correspond not only systems, which use this entire technique, but also systems, which use only part of this technique. The former systems do not always have the same profitability as the latter. Or consider a separable joint production technique, i.e. a joint production technique which can produce all the commodities, which it produces when it uses all of its production processes, by using one or more but not all of its production processes. The corresponding systems, i.e. those which use all the processes, and those which use only certain processes, do not always necessarily have the same profitability.

It is only possible to compare only one system of the infinite systems of a technique with a 'corresponding' system of another technique. Only 'corresponding' systems of given techniques and not the techniques themselves can be compared and ordered with respect to their profitability.

In what does this 'correspondence' consist? This 'correspondence' is introduced by the criterion used each time for the comparison and ordering of techniques. None of the above-mentioned five criteria for the comparison and ordering of techniques is a criterion for the comparison and ordering of techniques, but each of them is a means of introducing a –different for each criterion– 'correspondence' between systems, which use different techniques. Because the criterion for comparing and ordering techniques is one and only one: profitability. Each of the five criteria for the comparison and ordering of techniques, which we analyzed above, constitutes a statement or restatement of the criterion of profitability. In this statement or restatement of the criterion of profitability however, a principle is tacitly introduced of 'correspondence' between the systems –the systems and not the techniques–, which it ultimately compares and orders with respect to their –directly or indirectly stated– profitability. So ultimately, each of these five 'criteria' for the comparison and ordering of techniques tells us what kinds of systems, which use the given techniques for comparison and ordering, it compares and orders with respect to their profitability.

The w-r criterion tells us that with this criterion, instead of the given techniques, what are compared and ordered with respect to their profitability are the –for each given normalization of prices and each given r or w– normalization subsystems corresponding to these techniques.

The cost minimization criterion tells us that with this criterion, instead of the given techniques, what are compared and ordered are the –for each given normalization of prices and each given r or w– normalization subsystems corresponding to these techniques with respect to their cheapness in producing the commodity, which is produced by that production process, by which the given techniques differ, where it is postulated that 'greater cheapness' means 'greater profitability'.[21]

Bidard's criterion tells us that with this criterion, instead of the given techniques, what are compared and ordered are the –for each given r– quasi Sraffian standard systems corresponding for the given r to these techniques, which systems have real wage rates of common composition.[22]

The first of the two criteria for the comparison and ordering of 'techniques', which we presented in this paper, tells us that with this criterion, instead of the given techniques, what are compared and ordered are the Charasoffian standard systems corresponding to these techniques, with respect to their rate of profit and consequently with respect to their profitability.

The second of the two criteria for the comparison and ordering of 'techniques', which we presented here, tells us that with this criterion, instead of the given techniques, what are compared and ordered are the corn systems corresponding to these techniques, which systems have real wages of common composition.

Thus, the above five 'criteria' for the comparison and ordering of 'techniques' all have one common characteristic: They are not criteria for the comparison and ordering of techniques with respect to their profitability, but ultimately define a –different each time– 'correspondence' between those systems, which use the given techniques, in order to subsequently compare and order them with respect to their profitability.

The original intention was to *unambiguously* compare and order given techniques. However, given that, as we have seen, the comparison and ordering of techniques is impossible and only the comparison and ordering of 'corresponding' systems, which use the given techniques, is possible, the following issue arises: Which of the 'corresponding' systems, defined by each of the above five 'criteria' for the comparison and ordering of given 'techniques' can be unambiguously compared and ordered with respect to their

21. This is one of the restatements of the criterion of profitability, to which we alluded previously.
22. Here too we have a restatement of the criterion of profitability.

profitability? Evidently those which are defined by the last two of the five 'criteria'.

Final conclusion: We have shown that the comparison and ordering of linear techniques with respect to their profitability is impossible. We also showed that only systems can be compared and ordered with respect to their profitability. Lastly, we showed that of all the systems (five kinds in total) which can be compared and ordered with respect to their profitability, only the Charasoffian standard systems and the –for each given r– corn systems with real wage rates of common composition can be unambiguously compared and ordered with respect to their profitability. The other three kinds of systems, i.e. (a) the normalization subsystems, which are compared and ordered according to the w-r criterion for given r with respect to w, (b) the normalization subsystems, which are compared and ordered with the cost minimization criterion with respect to cheapness in the production of the commodity, which is produced by the process, by which these normalization subsystems differ, and (c) the quasi Sraffian standard systems, which are compared and ordered with Bidard's criterion with respect to the size of their real wage rates of common composition, cannot be unambiguously compared and ordered.

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