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Production of Goods by means of Goods or Capitalist Production of Commodities?

by

*Georg Stamatis**

In current theory of linear systems of production, certain things are ineffably and arbitrarily assumed concerning the definition of the real gross product, the real net product and the surplus product, which promote a view of the capitalist economy as a system that produces goods by means of goods and which do not help very much in considering the aforesaid economy as it really is, i.e. a system that produces commodities by means of paid labour.

What follows aims to prove this point.

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Assuming a system of production [B, A, L, x], which uses the productive production technique [B, A, L], for which, as is well known, the following holds

$$(B - A)^{-1} \geq 0 \tag{1}$$

and which produces the positive gross product X,

$$X = Bx\,(\geq 0), \tag{2}$$

in which the following symbols apply:

B, $B \geq 0$, the nxn matrix of outputs,

A, $A \geq 0$, the nxn matrix of inputs of means of production,

L, $L \geq 0$, the kxn matrix of inputs of k different types of direct labour,

when the system operates at unitary activity levels, and

x, $x \geq 0$, the activity levels of the system.

Assuming also D, $D \geq 0$, the nxk matrix of k different real wage rates (one for each of the k different types of direct labour). All the columns of each of the matrixes B, A, L and D are positive or semi-positive.

* Panteion University, Department of Public Administration, Athens, Greece.

The above system may be either "closed", i.e. not having trade exchanges with other systems, or 'open', i.e. having trade relations with other systems.

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In the theory of linear systems of production, certain things are ineffably and arbitrarily assumed and most importantly without any awareness as to whether they are or are not possible and their consequences, concerning the definition of the real gross product X, the real net product Y and the surplus product U, the feasibility and correctness of which we shall examine below.

More specifically, it is assumed that the real gross product X is given and positive or semi-positive

$$X = Bx \tag{2}$$

and

$$X \geq 0. \tag{2a}$$

Also, the real net product Y is defined as

$$Y = Bx - Ax = (B - A)x \tag{3}$$

and the surplus product U as

$$U = Bx - Ax - DLx = (B - \bar{A})x, \tag{4}$$

with

$$\bar{A} = A + DL, \tag{5}$$

where DL is the nxn matrix of real wages when the system is operating at unitary activity levels.

The following holds by definition for the k x n matrix Ω of labour values of n produced commodities

$$\Omega B = \Omega A + L \;\Rightarrow\; L = \Omega(B - A). \tag{6}$$

Each column of Ω is positive or semi-positive. Consequently, $\Omega \geq 0$.

Thus, given (6), we take for $(B - \bar{A})$:

$$B - \bar{A} = B - A - D\Omega(B - A) = (I - D\Omega)(B - A) \tag{7}$$

and for $(B - \bar{A})^{-1}$:

$$(B-\bar{A})^{-1} = [(I-D\Omega)(B-A)]^{-1}$$
$$= (B-A)^{-1}(I-D\Omega)^{-1}. \tag{8}$$

Assuming also that

$$(I-D\Omega)^{-1} \geq 0 \tag{9}$$

and consequently, given (1),

$$(B-\bar{A})^{-1} = (B-A)^{-1}(I-D\Omega)^{-1} \geq 0. \tag{10}$$

Relation (10) means that the technique [B, A, L] is for the given real wage rates D surplus productive, i.e. that it can produce each exogenously given positive or semi-positive surplus product.

Given (1), (2) and (10), we take from (3) and (4) respectively

$$X = B(B-A)^{-1}Y \tag{3a}$$

and

$$X = B(B-\bar{A})^{-1}U. \tag{4a}$$

Bearing in mind (1), (10) and (2a), it emerges from (3a) that Y is not necessarily positive or semi-positive, $Y \geq 0$, but may contain –in addition to positive or positive and zero– also negative quantities of commodities, and from (4a) that U is not necessarily positive or semi-positive, $U \geq 0$, but may contain - in addition to positive or positive and zero - also negative quantities of commodities.

These "paradoxes" do not reflect reality, but are the consequences of relations (3) and (4) and of $X \geq 0$ (relation (2a)), which is assumed by relations (3) and (4).

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We shall show, firstly, that (2a) holds only in closed but not open systems and, secondly, that the relations (3) and (4) are not only arbitrary, but also impossible, i.e. that they hold only under conditions, which in reality are never given, and therefore they cannot be and are never fulfilled. We shall assume firstly that the given system is closed and subsequently that it is open.

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So, assuming that the given system is closed. If this is the case, then the primarily given magnitude is not the real gross product X, but gross production Φ, which is obviously positive or semi-positive,

$$\Phi \geq 0. \tag{11}$$

The nx1 vector of gross production Φ contains also the nX1 vector C, $C \geq 0$, of intermediate outputs. Consequently, in order to take from gross production Φ the real gross product X, we must deduct from Φ the intermediate outputs C. We can do this only because the intermediate outputs C are, by virtue of the definition of Φ, part of Φ. The fact that C is part of Φ means that the following holds

$$\Phi \geq C.$$

Consequently, for the gross product X we get

$$X = \Phi - C \geq 0. \tag{12}$$

So, when the system is closed, (2a) indeed holds, as assumed by the usual theory of linear systems of production.

Let us now see what happens with (3) and (4). The vector Ax represents by definition the used up means of production of the period. Assuming F, $F \geq 0$, the vector of stocks of the system in means of production and consumption commodities at the beginning of the period. Then Ax constitutes that part of F which was used up during the period.

Consequently

$$Ax \leqq F. \tag{13}$$

However, (3) presupposes ineffably and arbitrarily that these used up means of production were fully replaced at the end of the period. This presupposition clearly entails the presupposition that these same used up means of production Ax were reproduced during the period and consequently constitute part of the real gross product X, i.e. that $X \geq Ax$. Because, only when this presupposition is fulfilled can one –as assumed by (3)– deduct Ax from X to get Y.

The said ineffable presupposition is however not only arbitrary, i.e. it does not necessarily always hold, but also impossible, i.e. it cannot hold and in fact never holds. This is because the used up means of production are either not replaced at all or are not replaced in full, but only in part. Otherwise, if, that is, they were always replaced in full, the system would use in each period t also the means of production of all the previous periods t–1, t–2, ..., 2, 1, 0 – which is impossible, for the system at some point stops using e.g. wooden ploughs or wooden ploughs and metal ploughs or wooden ploughs and metal ploughs and tractors and uses only tractors. Consequently, Ax is part of X only in part and not in whole. Therefore, the definition of Y by virtue of (3) is impossible and consequently erroneous.

The only thing we know is that

$$\bar{Y} = pX - \lambda, \tag{14}$$

where $\bar{Y}$ is the nominal net product, p, p > 0, the 1xn vector of prices and λ the nominal depreciation of the used up means of production,

$$\lambda = pAx.$$

With all the more reason, the definition of U by virtue of (4) is impossible and consequently erroneous. It is impossible and consequently erroneous, firstly, for the aforementioned reason, for which the definition of Y by virtue of (3) is impossible and consequently erroneous, and, secondly, for the additional reason that (4) arbitrarily presupposes that the real wages of the period DLx are wholly part of Y and consequently of X. This presupposition is arbitrary and may therefore be erroneous. For the real wages of period DLx may only partly be part of Y and consequently of X and partly part of F or wholly part of F. When they are partly or wholly part of F, the aforesaid presupposition is erroneous. Therefore, the definition of U by virtue of (4) is –as a definition of general validity– erroneous.

The only thing we know is that for the nominal surplus product $\bar{U}$, i.e. for profit, the following holds

$$\bar{U} = \bar{Y} - pDLx = pX - \lambda - pDLx, \tag{15}$$

where

$$pDLx = wLx,$$

$$w = pD$$

and

w, w > 0, the 1xk vector of k different nominal wage rates (one for each of the k different types of labour power).

But is there a correct definition of Y and U? Let us see if there is. Assuming K, K ≥ 0, the nx1 vector of consumption of capitalists in the given period of production.

However, consumption K of capitalists does not necessarily wholly constitute part of Y and consequently of X, but may partly be part of Y and consequently of X and partly part of F.

Therefore, unlike in the usual theory of (closed) linear systems of production, the following does not hold

$$X = Ax + DLx + K + S, \tag{16}$$

$$Y = X - Ax = DLx + K + S \tag{17}$$

and

$$U = K + S, \tag{18}$$

where S is the nx1 vector of net saving (= net investment). Instead, the following holds

$$(F + X) - (Ax + DLx + K) = F + S = F_1, \tag{19}$$

where F_1, $F_1 \geq 0$, the nx1 vector of the system's stocks of means of production and consumption commodities (wage commodities and luxury commodities) at the end of the given and at the beginning of the next period.

From (19) of course, by deleting F from both sides, one may get (16), but (16) does not always hold, because Ax is not always part of X. Conversely: Although (16) does not always hold, (19) always holds, because Ax + DLx + K is always part of F + X. For, if it were not, it would be impossible for the given system to exist, as it exists by assumption.

The following also holds

$$F + S = F_1 \tag{21}$$

where F, $F \geq 0$, the nx1 vector of the system's stocks of means of production and consumption commodities at the beginning of the given period.

The vector S may be either positive or semi-positive or contain, apart from positive or positive and zero, also negative quantities of commodities or, lastly, contain only negative or only zero or negative and zero quantities of commodities.

This is by no means paradoxical, but something quite commonplace and known: The positive quantities of commodities that are contained in S mean positive investment in those commodities, i.e. an increase in the stocks of those commodities, the zero quantities of commodities that are contained in S mean zero investment in those commodities, i.e. unchanged stocks of those commodities, and the negative quantities of commodities that are contained in S mean disinvestment in those commodities, i.e. a decrease in the stocks of those commodities.

For Φ, the following evidently holds

$$\Phi = C + A^*x + (DL)^*x + K^*, \tag{22}$$

where C the intermediate outputs, A^*x, $A^*x \geqq 0$, the means of production produced during the given period, $(DL)^*x$, $(DL)^*x \geqq 0$, the wage commodities produced during the given period and K^*, $K^* \geqq 0$, the luxury commodities produced during the given period. Consequently, for X we get:

$$X = \Phi - C = A^*x + (DL)^*x + K^*. \tag{23}$$

For the net product Y, the following holds

$$Y = (DL)x + K + S, \tag{24}$$

where clearly S,

$$S = (A^*x - Ax) + [(DL)^*x - (DL)x] + (K^* - K). \tag{25}$$

If we substitute (25) in (24) we get

$$Y = (A^*x - Ax) + (DL)^*x + K^*. \tag{26}$$

The definition of Y by virtue of (26) is however impossible and therefore erroneous, because, as we have already shown, Ax is not a part of A^*x and consequently of X (compare relation (23)), so as to be able to be deducted from A^*x (or from X).

For U we get from (25)

$$U = X - (DL)x = (A^*x - Ax) + [(DL)^*x - (DL)x] + K^*. \tag{27}$$

So, with all the more reason the definition of U by virtue of (27) is not possible. Therefore, the only magnitudes which we can define are the nominal net product $\overline{Y}$ and the nominal surplus product, i.e. the profit $\overline{U}$.

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Let us now look at the case of open systems. Here, things get worse. For in open systems we cannot even define the real gross product X.

What is directly given and positive or semi-positive in these systems is the nx1 vector of gross production Φ, $\Phi \geq 0$.

Because the system is open, to its inputs now belong –apart from the inputs of used up means of production Ax from the system's stock F of means of production and consumption commodities and the intermediate inputs C– also the imports Im, Im $\geqq 0$, where Im is a n×1 vector.

In order to be able to get X from Φ, we would have to deduct from Φ the vectors Ax, C and Im. However the system necessarily produces only C. As explained previously, it does not necessarily produce Ax, so that Ax does not constitute part of Φ. Also, it is quite clear that it does not produce Im, so that Im also does not constitute part of Φ. If however we deduct from Φ along with C also Ax and Im, then evidently the X that emerge,

$$X = \Phi - C - Ax - Im,$$

may contain also negative components. Thus, (2a) does not hold in open systems.

And because one cannot define X, with all the more reason as well as for the reasons that we set out above it is not possible to define Y, let alone U.

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The manner in which the issue is treated, as introduced in that set out above, is far from restrictive. On the contrary. It frees us from the restrictions set by (10) and consequently by (1). For there are systems of joint production which, without fulfilling (1) and consequently nor (10), produce a positive or semi-positive real product Y and a positive or semi-positive surplus product U and which therefore for $p > 0$ produce not only a positive nominal net product $\bar{Y}$, $\bar{Y} > 0$, but also a positive profit $\bar{U}$, $\bar{U} > 0$.[1]

Our treatment of the issue allows the inclusion of these latter systems. Because the only thing that it presupposes is, for $p > 0$, that profit $\bar{U}$ is positive, regardless of whether (1), (2a), (3), (4) and consequently (3a) and (4a) hold.

The usual way of defining Y and U by virtue of (3) and (4) facilitates the presentation of production as "the production of goods by means of goods",[2] i.e. as an exclusively technical, not also social, process of production and

1. See Gérard Duménil and Dominique Lévy, "Value and Natural Prices Trapped in Joint Production", *Journal of Economics*, vol. 47 (1987), No 1, pp. 15-46.
2. Sraffa of course speaks of "production of commodities by means of commodities". The 'commodities' however of which he speaks are not commodities but simply goods that in some mysterious way are exchanged without the intermediation of money.

exchange of commodities by means of money. It is therefore typically neoricardian.[3] On the contrary, the ascertainment that the net product and the surplus product, and in 'open' systems even the gross product, can only be defined as magnitudes in money terms, undermines the neoricardian attempt to present production as a purely technical process and, by pointing to nominal magnitudes, points to money as the paramount 'nexus rerum' (Marx) of capitalist society.

3. This does not mean of course that every neoricardian analysis of the technical base of social production promotes only the misconception of the latter as an exclusively technical process.