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An Examination of the so-called New Solution to the Transformation Problem

by
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In this paper we shall examine the correctness of the so-called new solution to the problem of transforming labour values into production prices.

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As is known, according to the new solution, in each production system and for each set of positive prices of commodities, the price of the net product is proportional or equal to the labour value of the net product and at the same time, the price of the surplus product (= profit) is proportional or equal to the labour value of the surplus product (= surplus value), where the ratio of proportionality is in both cases the same. Thus, on the condition that production is single production, according to the new solution the following holds

$$mpY=\omega Y=\omega(I-A)X \tag{1}$$

and

$$mpU=\omega U=\omega[I-(A+d\ell)]X, \tag{2}$$

for each p,

$$p>0 \tag{3}$$

and for each X,

$$X>0, \tag{4}$$

where p is the 1xn vector of prices, ω the 1xn vector of labour values, A, $A\geq 0$, the indecomposable nxn matrix of inputs of means of production per unit of produced commodity, ℓ, $\ell>0$, the 1xn vector of inputs of direct labour per unit of produced commodity, d, $d\geq 0$, the nx1 vector of the real wage rate, $d\ell$, $d\ell\geq 0$ the nxn matrix of real wages per unit of produced commodity, $(A+d\ell)$, $(A+d\ell)\geq 0$ the indecomposable nxn vector of inputs of means of production *and* real wages per unit of produced commodity, X, $X>0$, the nx1 vector of activity levels and -because production is, by assumption, single production- at the same time the $n \times 1$ vector of gross product,[1] Y the $1 \times n$ vector of net product, U the $n \times 1$ vector of surplus product and m, $m>0$ the aforementioned coefficient of proportionality, which the new solution calls value of money.[2]

For the sake of simplicity, we assumed above that A and consequently $(A+d\ell)$ is indecomposable. If we allowed $(A+d\ell)$ and therefore A the possibility of being decomposable, then $X>0$ would not necessarily hold for X, but rather $X \geq 0$.

According to the above, the following holds for Y

$$Y=X-AX=(I-A)X \tag{5}$$

and for U the following holds

$$U=Y-d\ell X=(I-A)X-d\ell X= \\ =[I-(A+d\ell)]X, \tag{6}$$

where $d\ell X$ is the $n \times 1$ vector of total real wages.

1. This, of course, does not hold in the case in which production is joint production.
2. (1) follows directly from

$$m=\frac{\ell X}{pY}, \tag{α}$$

by which the new solution sets the value of money m. (2) results as follows: The new solution sets the value of one unit of labour power VLP (value of labour power) by means of

$$VLP=wm, \tag{2α}$$

where w is the wage rate in money terms.
For profit Π the following holds

$$\Pi=pY-w\ell X, \tag{c}$$

where ℓX is the total labour power.
For surplus value S the following holds

$$S=\ell X-(VLP)\ell X$$

and, taking into consideration (a), (2a) and (c):

$$S=mpY-wm\ell X=m(pY-w\ell X)=m\Pi.$$

This latter relation is the same as (2). So, when (1) and (2) hold, then (2a) necessarily also holds. In addition, when (1) and (2a) hold, then (2) necessarily also holds. Lastly, when (2) and (2a) hold, then, as a consequence of $w=pd$ and $Y=d\ell X+U$, (1) necessarily also holds. So, one could say that the new solution consists in the assertion that (1) and (2) hold generally or in the assertion that (1) and (2a) hold generally or in the assertion that (2) and (2a) hold generally.

We assume, self-evidently, that the production technique $[A, \ell]$, which is used by the production system $[A, \ell, X]$ is –for at least one positive or semi-positive real wage rate d, $d \geq 0$– surplus productive, i.e. that it is capable of producing –for at least one positive or semi-positive real wage rate– each positive or semi-positive surplus product U, $U \geq 0$, using one or more or all the production processes at positive activity levels and no production process at a negative activity level and consequently producing a positive or semi-positive gross product X, $X \geq 0$, and specifically here, where A and therefore $(A+d\ell)$ are, by assumption, indecomposable, using all the production processes at positive activity levels and consequently producing a positive gross product X, $X > 0$.

The technique is surplus productive, when the Perron-Frobenius eigenvalue of the matrix $(A+d\ell)$ is (positive[3] and) smaller than unit and consequently, in the general case, when

$$[I-(A+d\ell)]^{-1} \geq 0$$

and here, where A and consequently $(A+d\ell)$ is indecomposable, when

$$[I-(A+d\ell)]^{-1} \geq 0. \tag{7}$$

Proof:

For X the following holds

$$X = AX + d\ell X + U \;\Rightarrow$$
$$[I-(A+d\ell)]X = U \;\Rightarrow$$
$$X = [I-(A+d\ell)]^{-1}U. \tag{8}$$

Given (7), the following results from (8)

$$U \geq 0 \;\Rightarrow\; X > 0$$

i.e. that for each positive or semi-positive surplus product, the gross product is positive.[4]

3. The Perron-Frobenius eigenvalue of $(A+d\ell)$ is positive, because $(A+d\ell) \geq 0$.
4. The converse, i.e. $X > 0 \Rightarrow U \geq 0$, does not always hold. This means that –although the production technique is for each real wage rate, for which (7) is fulfilled, surplus productive– the surplus product may contain, apart from positive or semi-positive and zero quantities, also negative quantities of commodities. These latter are those quantities of means of production *and* wage commodities, which are missing from the system in order for

Because the production technique is, by assumption, surplus productive, it is for all the more reason also productive. That is, the Perron-Frobenius eigenvalue of A is (positive and) smaller than unit. Because of $A \geq 0$, the maximum eigenvalue of A is positive. Because of $\ell > 0$, $d \geq 0$ and consequently also $d\ell \geq 0$, at least n elements of A are smaller than the corresponding elements of $(A+d\ell)$. Consequently, the Perron-Frobenius eigenvalue of A is smaller than the Perron-Frobenius eigenvalue of $(A+d\ell)$ and is thus also smaller than unit. Therefore, the following holds in general

$$(I-A)^{-1} \geq 0$$

and here, where A is indecomposable,

$$(I-A)^{-1} \geq 0. \tag{9}$$

This means that the technique is productive, i.e. that it is capable of producing each positive or semi-positive net product Y, $Y \geq 0$, using one or more or all the production processes at positive activity levels and none at a negative activity level and therefore producing a positive or semi-positive gross product X, $X \geq 0$, and here, where A is indecomposable, using all the production processes at positive activity levels and therefore producing a positive gross product X, $X > 0$.

For X the following holds

$$X = AX + Y \quad \Rightarrow$$
$$(I-A)X = Y \quad \Rightarrow$$
$$X = (I-A)^{-1}Y.$$

Given (9), we get from the above relation

$$Y \geq 0 \quad \Rightarrow \quad X > 0$$

i.e. that an X, $X > 0$ corresponds to each positive or semi-positive Y[5].

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it to be viable. We assume that the system is supplied with these quantities of commodities from its respective stocks of these commodities and consequently it is viable. It is clear that the surplus product cannot contain only negative or only zero, or only negative and zero quantities of commodities, because this would be contradictory to (7).

5. The converse, i.e. $X > 0 \Rightarrow Y \geq 0$, does not always hold. This means that –although the technique is by definition productive– the net product may contain, apart from positive or

In certain versions of the new solution, the labour values ω are defined as Marxian labour values. But in other versions they are defined in another way (to which we shall refer below). The same is more or less true for prices p, for in certain versions of the new solution, prices p are defined as production prices. In other versions however, they are defined as any positive market prices whatsoever. We shall first examine the validity of (1), (2), (3) and (4) on the assumption that labour values ω are defined as Marxian values and prices p are defined as production prices and subsequently, on the assumption that prices p are not necessarily production prices and labour values ω are not defined as Marxian values but are determined in another way.

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So, assuming that labour values ω are defined as Marxian values and prices p are defined as production prices.

Then for labour values ω, as is known, the following holds,

$$\omega = \ell(\mathrm{I}-\mathrm{A})^{-1} \tag{10}$$

and, because of $\ell > 0$ and (9),

$$\omega > 0. \tag{11}$$

And for production prices p, as is known, the following holds,[6]

$$\begin{aligned} &\mathrm{p} = \mathrm{pA} + \mathrm{rpA} + \mathrm{pd}\ell + \mathrm{rpd}\ell \;\Rightarrow \\ &\mathrm{p} = (1+\mathrm{r})\mathrm{p}(\mathrm{A}+\mathrm{d}\ell) \;\Rightarrow \\ &\mathrm{p}[\mathrm{I}-(1+\mathrm{r})(\mathrm{A}+\mathrm{d}\ell)=0, \end{aligned} \tag{12}$$

where r is the uniform rate of profit and d the exogenously given real wage rate.

positive and zero quantities, also negative quantities of commodities. These negative quantities of commodities are those quantities of means of production that are missing from the system so that it can be viable. Consequently, they are parts of the negative quantities of commodities that are contained in the surplus product (see also footnote 4). The net product cannot, of course, contain only negative or only zero or only negative and zero quantities of commodities, because this would be contradictory to (9).

6. We assume that wages in their entirety are paid in advance at the beginning of the production period and that the means of production expended are used up entirely during the production period.

If, instead of the real wage rate d, the nominal wage rate w is exogenously given,

$$w=pd, \tag{13}$$

then for the production prices, the following holds

$$\begin{aligned} p&=pA+rpA+w\ell+rw\ell \Rightarrow \\ p&=(1+r)pA+(1+r)w\ell \Rightarrow \\ p[I-(1+r)A]&=(1+r)w\ell \Rightarrow \\ p&=(1+r)w\ell[I-(1+r)A]^{-1}. \end{aligned} \tag{12α}$$

In this case, d is not exogenously given but represents every d that fulfils (13).

As is known, there is for (12) a positive solution in the form of

$$r=\frac{1-\lambda}{\lambda}$$

and

$$p>0$$

where λ is the Perron-Frobenius eigenvalue of $(A+d\ell)$.

With the exception of a scalar, p is fully determined. Consequently, p represents the relative production prices. Also, because it is positive, p fulfils (3).

According to the version of the new solution which defines labour values ω as Marxian values and prices p as production prices, (1), (2), (3), (4) and (12) should hold simultaneously. The system of equations comprising (1), (2) and (12) which, according to the version of the new solution being examined, holds for each p, $p>0$ and for each X, $X>0$, is however in the general case overdetermined and consequently does not always have a solution. This is because it consists of $n+2$ equations with $n+1$ unknowns (the n prices of commodities and the rate of profit). So, when these $n+2$ equations are independent of one another, the system is overdetermined and has no solution. Consequently, the relations (1) and (2), the general simultaneous validity of which is asserted for positive production prices and for each X, $X>0$ by the version of the new solution being examined here, do not hold in the general case simultaneously.

Not only (1) but also (2) is a normalisation equation of the vector of production prices p. When do these two normalisation equations hold simultaneously? Only when they are linearly dependent on each other! And when are they linearly dependent on each other?

Mainly in the following two cases:

Case 1: In this case

$$Y=\alpha U, \quad \alpha>0, \tag{14}$$

i.e. Y and U are collinear, that is, the net product and the surplus product have the same composition. Here, the system of production [A, ℓ, X] is for the given real wage rate d a quasi-charasoffian standard system, i.e. a system whose net product Y and surplus product U and consequently also the total real wages $d\ell X$ have the same composition, or a charasoffian standard system, i.e. a system whose net product Y, gross product X, means of production AX, surplus product U and total real wages $d\ell X$ have the same composition.[7]

So, in the case where (14) holds, (1) and (2) do not hold for each X, $X>0$, i.e. for each production system, which for a given positive or semi-positive real wage rate uses a surplus productive and therefore productive technique, but only for those X, $X>0$, and their multiples, for which the corresponding production system is a quasi-charasoffian standard system or a charasoffian standard system.

Case 2: In this case

$$mp=\omega, \tag{15}$$

i.e. the production prices of all commodities p are analogous to Marxian labour values. But when (15) holds, then (1) and (2) do not hold, as asserted by the new solution, simultaneously for each p, $p>0$. This is because (15) (and consequently (1) and (2)) does not *always* hold for each p, $p>0$. When does (15) hold for all p, $p>0$, and consequently for all $0<r<R$, where R is the maximum rate of profit, with the consequence that (1) and (2) hold simultaneously for each p, $p>0$, and consequently for each $0<r<R$, as asserted by the new solution? As is known, only when the value compositions

7. The charasoffian standard system is a Ricardian corn model, which however does not produce only one commodity (corn), but more than one commodity. See Stamatis 1999.

of capital are equal in all sectors of production, i.e. when ℓ is an eigenvector of A. However, this holds for only certain productive production techniques. Only if this held for all productive production techniques would (1) and (2) hold simultaneously for each p, $p>0$, as asserted by the new solution.

We have proved that the version of the new solution examined above, i.e. the version of the new solution which defines labour values ω as Marxian values and price p as production prices, is erroneous.[8]

At the same time, we indirectly proved that the version of the new solution which defines labour values ω as Marxian values and prices p as market prices, which are not necessarily proportional or equal to production prices, is also erroneous. The proof consists in the following: Although market prices are not necessarily proportional or equal to production prices, it is possible for them to be proportional or equal to the latter. But when market prices are proportional or equal to production prices, then it is as though prices p have been defined not as market prices but as production prices. With respect to this latter case however, we showed above that (1), (2), (3) and (4) do not hold generally.

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The second version of the new solution assumes that prices are not necessarily production prices but market prices and that labour values ω are not defined as Marxian values by virtue of (10). In this version, the new solution indirectly assumes the validity of (15). Because only then, i.e. when (15) holds, do (1) and (2) hold simultaneously for each p, $p>0$ and for each X, $X>0$.[9]

Is this second version of the new solution correct? This question cannot be answered for the following reasons. This second version too of the new solution expressly aims to transform labour values into production prices and –because it puts market prices in the place of production prices– to transform

8. This version of the new solution is erroneous also for an additional reason. Production prices are not always positive. Here, they are always positive, because we assumed that A and consequently $(A+d\ell)$ are indecomposable. If $(A+d\ell)$ and consequently also A are indecomposable, then it is possible for indeterminate or zero or negative production prices to appear.

9. At the same time, (14) may or may not hold. Consequently, irrespective of whether (14) holds or not, when (15) holds, (1) and (2) hold simultaneously.

labour values into market prices. At the same time however, it does not define the labour values beforehand, in order to be able subsequently to assert that it is transforming them into market prices. Instead, by virtue of (15) it axiomatically sets labour values proportional or equal to market prices. It thus leads the problem that it itself raises and wants to solve ad absurdum. For instead of transforming, as it wants, labour values into market prices, by virtue of (15) it defines labour values as being proportional or equal to market prices, i.e. it determines that labour values as proportional or equal to the given market prices.

In my view and that of Mariolis, labour values are indeed equal to market prices. However, this needs to be founded.[10] But as we saw, in the said second version of the new solution, the proportionality of labour values and market prices is not founded, but rather set axiomatically by virtue of (15) as a condition for the supposed simultaneous validity of (1) and (2) for each p, $p>0$, and for each X, $X>0$. And most importantly, the said second version of the new solution axiomatically presupposes the validity of (15), i.e. the determination of labour values through market prices as values proportional or equal to market prices, within the framework of the problem of transforming labour values into prices and, moreover, into market prices. But thus, the said version of the new solution by virtue of the problem which it sets (: transformation of labour values into market prices) and the answer it gives to this problem (: labour values are not transformed into market prices, but rather market prices determine labour values) leads itself ad absurdum. For either the problem which it raises is meaningful, in which case the answer that it gives to the problem is contradictory to the meaning of the problem and is consequently meaningless, or conversely, the answer is meaningful, in which case the problem is meaningless. So, the second version of the new solution is neither correct nor erroneous but absurd.

We may therefore conclude the following from the above analysis:

Regardless of how the new solution defines labour values ω and prices p,

10. This foundation is established by Stamatis 1998c and 1999a and Mariolis 1998, 1999 and 2000, according to which Stamatis and Mariolis assume the existence of heterogeneous labour. In contrast, the new solution assumes, without stating so, that labour is homogeneous. Otherwise it could not calculate labour values by means of (10).

its assertion that (1) and (2) hold for each X, $X>0$, and for each p, $p>0$, is correct only when the following holds

$$mp=\omega \tag{15}$$

i.e. when all the prices p are always proportional to labour values ω. That is, when the price of each commodity is proportional to the labour value of that commodity and the coefficient of this proportionality is the same for all the commodities. But if the coefficient of this proportionality is the same for all the commodities, then it is clearly also the same for all the possible bundles of commodities, i.e. for each pair of bundles of commodities Z, $Z>0$, and H, $H>0$, the following holds

$$\left.\begin{array}{ll} & \beta pZ=\omega Z \\ \text{and} & \\ & \gamma pH=\omega H \\ \text{with} & \\ & \beta=\gamma=m. \end{array}\right\} \tag{15α}$$

So, if this coefficient of proportionality is the same for each pair of bundles of commodities, then it is the same *also* for the pair of bundles of commodities "net product" and "surplus product" and consequently (1) and (2) necessarily hold.

Given that this is the way things are, the assertion of the new solution that *only* the coefficient of proportionality between the price and the labour value of the net product Y and the coefficient of proportionality between the price and the labour value of the surplus product U are equal, i.e. its assertion that *only* (1) and (2) hold –and not (15) and therefore (15a)– for each X, $X>0$, and for each p, $p>0$, is quite clearly erroneous.

If, in contrast, the new solution began with (15) and consequently (15a) –but it does not begin with (15) and consequently (15a)– then its aforesaid assertion would be platitudinous.

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