POLITICAL ECONOMY - 7 - AUTUMN 2000 - p.p. 79 - 80

Labor Values in Joint Production Systems

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Labor values in joint production systems can be negative. I show that this is a definitional problem. Labor value is labor content. Hence, the same good produced by two different processes must have two different values. Under this redefinition of values, the equation system that defines them always has a positive solution, provided that the technology is productive and labor is a necessary input.

There are n goods and m processes. Technology is described by matrices A, B, L with nonnegative entries. Rows correspond to goods, columns to processes.

Let $t_{isj} \geq 0$ be the proportion of input i that process j buys from process s.

Commodity (i,s) is good i produced by process s. Let $\tilde{\alpha}_{is,j} = t_{isj}\alpha_{ij}$; $\tilde{b}_{is,j} = 0$ if $s \neq j$; and $\tilde{b}_{is,j} = b_{ij}$ if $s = j$. The corresponding matrices $\tilde{A}, \tilde{B}$ have dimensions $nm \times m$. Let $M = \tilde{B} - \tilde{A}$. Labor values are then defined by a $1 \times nm$ vector Λ that solves

$$(1)\ \Lambda M = L,\ \Lambda \geqq 0.$$

I assume that the technology is productive, and that labor is a necessary input, i.e. there exists an activity vector x that

$$(2)\ Mx > 0,\ Lx > 0,\ x \geqq 0.$$

I keep x fixed from now on.

Theorem: If (2) holds, then (1) has a solution.
To see this, consider the dual linear programs:

$$(3)\ \text{Min} \quad \Lambda M x \qquad \Lambda M = L \qquad \Lambda \geqq 0$$

$$(4)\ \text{Max} \quad Ly \qquad My \leqq Mx$$

Note that (1) has a solution if and only if problem (3) has an optimal solution, because $\Lambda Mx = Lx =$ fixed number. Problem (3) has an optimal solution if problem (4) does. Problem (4) has an optimal solution if its feasible set is bounded above.

Write $M=(M_1, \ldots, M_n)'$ where each M_i is an $m \times m$ square matrix. Each M_i is a Hawkins - Simon matrix, in the sense that it has nonpositive off -diagonal elements $- t_{isj}\, a_{ij}$, $s \neq j$, $M_i x > 0$.

Hence each M_i has a nonnegative inverse. It follows that the feasible set of (4) is bounded above by x.

Note: I have borrowed the idea of redefining values from Stamatis (1983); and the idea of a proof based on duality from Sotirchos (1999).

References

Sotirchos, G. (1999): On negative labor values. A summing Up. *Political Economy*, Spring 1999, 103-138.

Stamatis, G. (1983): On negative labor values. *Review of Radical Political Economy*, 15, 81-91.