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Joint Production and "Negative Values": The weakness of Steedman's attack on the Labour Theory of Value

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Introduction: Steedman's attack on the labour theory of value

Ian Steedman, in his recent book *Marx after Sraffa* (New Left Books, London, 1977) claims to refute the labour theory of value, resting this refutation primarily on his examination of joint production, which has now become one of the most widespread forms of contemporary capitalist production. Steedman presents his examination of joint production as the inevitable consequences of the labour theory of value, consequences which allegedly reveal the absurdity and redundancy of that theory. Any analysis of joint production based on the labour theory of value (argues Steedman) confronts us with two possibilities that are factually and theoretically impossible: (1) the possibility of the existence of "negative" labour values and (2) the possibility of the coexistence of negative surplus value and positive profits. If Steedman's argument is correct, then he would indeed have every right and reason to dismiss the labour theory of value, since the existence of negative values is totally incompatible with the logic of the labour theory of value.

This paper will systematically examine the example of joint production that Steedman himself sets up and from which he derives this conclusion; we will thereby demonstrate that the results that Steedman ends up with do *not* constitute a decisive refutation of value analysis for the simple reason that the premises on which Steedman bases his deductive procedures are *not* the premises of the labour theory of value.

There are two strategies we could adopt to undermine Steedman's challenge: either we could prove that Steedman's analysis is wrong or we could present a cogent alternative way of handling the problem of joint production. Since the strategy of proving Steedman wrong imposes far fewer demands on ourselves and our readers than the task of developing an entirely new approach for handling the case of joint production in the context of value theory, we will restrict our present paper to pointing out and explaining Steedman's mistakes. This strategy will be adequate to demonstrate how, given Steedman's own premises and procedures, the core argument in his rejection of the labour theory of value, i.e. the alleged existence of negative values, must collapse. However, our analysis is not intended to be merely a negative or destructive one, for in proving Steedman wrong, we will at the same time be clearing the field for developing an adequate understanding of the problem of joint production. Thus we are interested not only in proving *that* Steedman is wrong, but also in showing *how* he is wrong, in identifying and explaining the origins of his mistaken conclusions.

Steedman himself, at the end of his book, offers his readers three possible ways of reacting to his critique of the labour theory of value. Our own reaction is to adopt the second alternative that Steedman offers: "(b) to reject *explicitly* one or more of the assumptions from which it is logically deduced" (Steedman 205). We accept Steedman's rules of the game –that the rejection of his assumptions must be made explicit– but in addition we will set up a rule of our won (which Steedman himself must surely accept): that the criterion for accepting or rejecting the assumptions from which Steedman logically derives his proposition must be the labour theory of value itself, and that the assumptions of this theory must be made explicit. Our task then is to show that Steedman's assumptions, whether explicit or implicit, are inconsistent with the explicit premises of the labour theory of value. It will follow from this demonstration that whatever anomalies or absurdities Steedman is able to deduce from his own assumptions cannot be regarded as a necessary consequence of the labour theory of value itself. The basic premise of the labour theory of value that is disregarded and violated by Steedman's premises and procedures for dealing with joint production will be presented later. First, however, we will briefly recapitulate Steedman's analysis of joint production for the benefit of any readers who are not familiar with Steedman's contribution and who are now being exposed for the first time to this type of critique of the labour theory of value.

Steedman's model of joint production

Steedman[1] sets up the problem of joint production by assuming a situation of stationary equilibrium in which two enterprises are each producing two different kinds of commodities in fixed proportions as the output of a single production process. Table I represents the different input-output processes employed by the two enterprises, each producing commodity 1 and commodity 2 in a certain fixed proportion, and each using one or other of these same commodities as an input, together with direct labour.

Table I

Input 1	Input 2	Labour	Output 1	Output 2	
5	0	1	6	1	
0	10	1	3	12	
5	10	2	9	13	Totals

Steedman then assigns the coefficients l_1 and l_2 to the system he has set up. According to Steedman, these coefficients designate the socially necessary labour time "crystallized" in one unit of each commodity. (As we shall see later, they do not). In this way Steedman transforms his "physical" system[2] into a value system and obtains the following value equations:

$$5l_1 + 0l_2 + 1 = 6l_1 + 1l_2 \qquad (1)$$
$$0l_1 + 10l_2 + 1 = 3l_1 + 12l_2 \qquad (2)$$

These equations allow Steedman to solve for l_1 and l_2, with the following results:

$$l_1 = -1$$
$$l_2 = 2$$

1. Steedman's detailed presentation is given on pages 151-162 of his book (Steedman 1977). See also Wolfstetter 1977, 64ff and Flaschel 1977, 112-115.
2. Strictly speaking, Table 1 cannot represent a purely physical system, since it introduces the concept of labour, a concept which belongs to the sphere of value analysis. Hence Table 1 is really a mixed physical-and-value system, containing as it does information both about the physical sphere (physical inputs and outputs) and about the value sphere (direct labour).

Thus Steedman reaches the conclusion that minus 1 unit of socially necessary labour time is contained in commodity 1 and that 2 units of socially necessary labour time are contained in commodity 2. This conclusion makes a mockery of the labour theory of value.

Establishing that Steedman is wrong

Since Steedman has warned us not to fall into "obscurantism" [3], we shall not try to find any sense in a result which self-evidently gives no sense. Instead we shall follow our conviction that the result of a logical operation cannot be more absurd than the premises on which that operation is based. This means that, confronted with Steedman's absurd result, we now have to ask: 1) What are the absurd premises that have led to this absurd result? 2) Are these premises actually consistent with the basic premises of the labour theory of value? 3) If they are inconsistent, then in precisely what way are Steedman's premises in violation of the labour theory of value? In short, do the absurd premises stem from the labour theory of value or from Steedman's own inventive (or uninventive) mind?

In order to develop an answer to these questions, we shall first formulate a very general premise which any adherent of Ricardo or Marx or Sraffa should be able to accept.[4] In contrast to Steedman, we are here making explicit what we take to be a basic premise of the labour theory of value.

Premise

> The value of commodities produced by any process of production is equal to the sum of direct and indirect labour which is spent on the production of those commodities. If there is more than one output (i.e. more than one type of commodity) being produced by a single process, then there is no immediate or prima facie way of determining how the value of the total input is distributed among these different outputs.

Let us formalize this premise: if we call the total value of the commodities produced by the first process w_1 and correspondingly the total value of the commodities being produced by the second process w_2, and

3. Steedman 1977, 205.

4. Cf. Ricardo (ed. Sraffa), 1975, 11; Marx (1887) 1967, 38; Sraffa (1961) 1972, 56.

furthermore, if β_{ij} designates the proportions in which the total output value of the i-th process is transferred to the j-th commodity, than in the example constructed by Steedman, where there are two enterprises each carrying on a single production process (i.e. i=2) and where each process results in two commodities (i.e. j=2), the total output values of each process can be written as follows:

$$w_1 = \beta_{11} w_1 + \beta_{12} w_1 \tag{3}$$

$$w_2 = \beta_{21} w_2 + \beta_{22} w_2 \tag{4}$$

with $0 < \beta_{ij} < 1$ and $\sum_{j=1}^{2} \beta_{ij} = 1$.

Let us justify these last two expressions which set limits on the value-proportions β_{ij}. At this stage we do not know the precise proportions β_{11}, β_{12}, β_{21}, and β_{22}, but what follows from the premise we have just formulated is that 1) the sum of all value proportions of a single process must add up to unity, since the value of the commodities produced is equal to the sum of direct and indirect labour that goes into their production; and 2) no commodity can have a greater value than the sum of all direct and indirect labour used up in its production ($\beta_{ij} \leq 1$), which –together with condition (1)– implies that no value proportion can be negative ($0 \leq \beta_{ij}$). Thus for each of the processes in Steedman's model, which produce two commodities (j=2) as a result of a single process (i), the value proportions must add up to 1 ($\beta_{j=1}^{2}{}_{ij} = 1$) and must lie within a range from zero to unity ($0 < \beta_{ij} < 1$).

Equations (3) and (4) therefore formalize what we accept as a basic premise of the labour theory of value: that total output value equals total input value, thereby implying that the maximum value of a single commodity cannot be more than the total value of all the inputs that went into its production. The total value w_1 of the commodities produced by process 1 is defined in equation (3) and the total value w_2 of the commodities produced by process 2 is defined in equation (4). We will now sum the elements of these two equations vertically in order to obtain definitions for w^1 and w^2. w^1 and w^2 (with numerical superscripts) represent the total values (produced by both processes taken together) of commodity 1 and commodity 2 respectively. Thus each process produces a total value of w_1 and w_2 respectively, and both together produce a total value w^1 of commodity 1 and a total value w^2 of commodity 2.

$$w^1 = w_1\beta_{11} + w_2\beta_{21} \tag{5}$$

$$w^2 = w_1(1-\beta_{11}) + w_2(1-\beta_{21}) \tag{6}$$

Before we can apply our equations (3) through (6) to Steedman's example in order to formulate a closed value circuit, we must first take account of that part of the output of any production period which does not reenter the value-formation process of the next period. In other words, we have to take account of that part which is consumed productively or unproductively in every production period by the workers and the owners of the means of production. In Steedman's model, we can derive this part of the product by subtracting the total inputs that go into the production of commodities 1 and 2 from the total output produced by commodities 1 and 2. As a result of this subtraction, we obtain the following figures for the portion consumed in the production process itself:

	Output		Input		Workers' & Capitalists' Consumption
Commodity 1	9	–	5	=	4
Commodity 2	13	–	10	=	3

We can see from these figures, which here represent a purely physical system, that 5/9 of the total output of commodity 1 and 10/13 of the total output of commodity 2 return into the production circuit: the remainder (4/9 and 3/13 respectively) evaporates in the course of production because it is consumed by the workers and capitalists. These proportions can equally well apply to the value system, for there is nothing inherent in them that restricts them to the physical system only. Consequently we can say that the mass of value of commodity 1 which is used for the production of new commodities in the next production cycle is 5/9 w^1, and that the mass of value of commodity 2 which likewise reenters the production process is 10/13 w^2. If we substitute from equations (5) and (6) our definitions of w^1 and w^2 respectively, then we can express the value inputs of commodity 1 and commodity 2 as follows:

value inputs of commodity 1: $\frac{5}{9}w^1 = \frac{5}{9}(w_1\beta_{11} + w_2\beta_{21})$ (7)

value inputs of commodity 2: $\frac{10}{13}w^2 = \frac{10}{13}(w_1(1-\beta_{11}) + w_2(1-\beta_{21}))$ (8)

Now that we have defined the value structure of the inputs (equations 7 and 8) and since we have already defined the value structure of the outputs (equations 3 and 4), we are at last in a position to formulate Steedman's reproduction model in the form of the following closed value system:

Input 1		Input 2	Labour		Output 1 Output 2	
$\frac{5}{9}(w_1\beta_{11} + w_2\beta_{21})$	+	0	+1	=	$w_1\beta_{11}+w_1(1-\beta_{11})$	(9)
0	+	$\frac{10}{13}(w_1(1-\beta_{11}) + w_2(1-\beta_{21}))$	+1	=	$w_2\beta_{21}+w_2(1-\beta_{21})$	(10)

Thus we now have an indeterminate value system consisting of two equations with four unknowns (w_1, w_2, β_{11} and β_{12}). If we solve the two value equations (9) and (10) for w_1 and w_2, we get the following algebraic expressions for the total labor values of the commodities produced by process 1 and process 2 respectively:

$$w_1 = \frac{27 + 155\beta_{21}}{27 - 15\beta_{11} + 40\beta_{21}} \tag{11}$$

$$w_2 = \frac{207 - 155\beta_{11}}{27 - 15\beta_{11} + 40\beta_{21}} \tag{12}$$

If we now substitute these algebraic expressions (given in equations 11 and 12) for w_1 and w_2 respectively in equations (5) and (6), which define the total values of each of the two commodities, w^1 and w^2, being produced by the two processes taken together, we obtain the following expressions for w^1 and w^2.

$$w^1 = \frac{27 + 155\beta_{21}}{27 - 15\beta_{11} + 40\beta_{21}} \cdot \beta_{11} + \frac{207 - 155\beta_{11}}{27 - 15\beta_{11} + 40\beta_{21}} \cdot \beta_{21} \tag{13}$$

$$w^2 = \frac{27 + 155\beta_{21}}{27 - 15\beta_{11} + 40\beta_{21}} \cdot (1-\beta_{11}) + \frac{207 - 155\beta_{11}}{27 - 15\beta_{11} + 40\beta_{21}} \cdot (1-\beta_{22}) \tag{14}$$

which can be simplified to

$$w^1 = \frac{27\beta_{11} + 207\beta_{21}}{27 - 15\beta_{11} + 40\beta_{21}} \tag{15}$$

$$w^2 = \frac{234 + 182\beta_{11} - 52\beta_{21}}{27 - 15\beta_{11} + 40\beta_{21}} \tag{16}$$

Equation (15) now defines the total value of commodity 1 being produced by both processes (w^1) and equation (16) the total value of commodity 2 (w^2), with the only unknowns being the two value proportions (β_{ij}).

We can now obtain expressions for the average per unit value of each of the two species of commodities (still in terms of the unknown value proportions) by dividing equations (15) and (16), each of which expresses total value output, by the physical output of each commodity given in Table 1, i.e. 9 units of commodity 1 and 13 units of commodity 2. We thereby obtain expressions for l_1 and l_2 as the coefficients assigned by Steedman to designate the socially necessary labour-time "crystallized" in one unit of commodities 1 and 2 respectively:

$$l_1 = \frac{3\beta_{11} + 23\beta_{21}}{27 - 15\beta_{11} + 40\beta_{21}} \tag{17}$$

and

$$l_2 = \frac{18 - 14\beta_{11} - 4\beta_{21}}{27 - 15\beta_{11} + 40\beta_{21}} \tag{18}$$

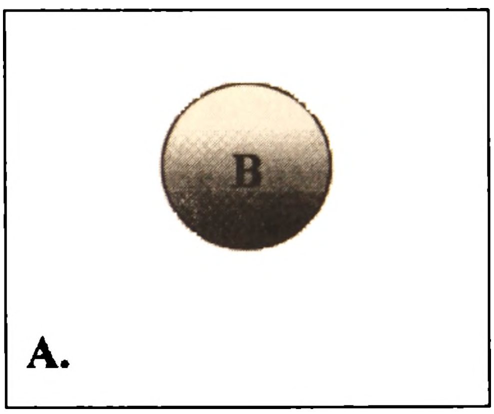
According to the *Basic Premise* of the labour theory of value, no value proportion β_{ij} is ever negative nor does it ever exceed unity. Hence, although we cannot know the precise values of the coefficients, l_1 and l_2, until we know the precise values of the value proportions β_{11} and β_{21}, we *can* establish the limits within which these coefficients vary, by assigning to the value proportions their maximum and minimum values ($0<\beta_{ij}<1$). These boundaries are given in Table II:

Table II

	$\beta_{11}=0$ & $\beta_{21}=0$	$\beta_{11}=1$ & $\beta_{21}=1$	$\beta_{11}=1$ & $\beta_{21}=0$	$\beta_{11}=0$ & $\beta_{21}=1$
l_1	0	0,5	0,25	0,34328
l_2	0,5	0	$0,\overline{3}$ ..	0,20895

Thus it is immediately clear that the coefficients l_1 and l_2 cannot take on a value that is greater than 0.5 or less than 0. Table II makes irrefutably clear the fact that Steedman's results (l_1=-1, l_2=2) fall into the realms of the impossible: both of them lie outside either of the outermost limits given in Table II (i.e. 0 and 0.5). Consequently Steedman's solution to the problem he sets up presupposes value proportions β_{ij} that are inconsistent with the basic premise of the labour theory of value formulated on page 5 above.

The point that we have so far succeeded in establishing may be diagrammed as follows:



Legend: A = Steedman's solution
B = correct solutions according to the labour theory of value

There is an indefinite set of solutions to the value problem in the case of joint production, some of which are correct and some of which are incorrect. The box in the above diagram represents the totality of both correct and incorect solutions. One of these correct or incorrect solutions is Steedman's, which is represented by point A in the diagram. According to the labour theory of value, the only correct solutions lie in the subset of possible solutions represented by the area B in the diagram. Since Steedman's solution lies outside this area, it has already been judged by the labour theory of value as incorrect. For Steedman to insist on the wrongness of this solution is tantamount to telling us that the labour theory of value is valid. Why then does he think that he is proving that the labour theory of value is wrong?

Why Steedman (but not the labour theory of value) is wrong

We have so far shown that Steedman's result is not consistent with a basic premise of the labour theory of value. This means that Steedman must have introduced some premises of his own, either explicitly or implicitly, in order to arrive at a result which this theory would dismiss as incorrect. In other words, for the present we know nothing more than that Steedman gave an answer to the problem he posed and that this answer has proved wrong by the standards imposed by the labour theory of value. According to this theory, we cannot determine the precise proportions in which the total output value of a given process is transferred to the partial outputs of this process (β_{ij}), but we are given certain explicit limits on the value transfer from the production inputs to the commodities produced. Though Steedman himself does not grant explicit recognition to the concept of value-proportion (β_{ij}), this concept is nevertheless implicit in his procedure. We therefore have to ask: what are Steedman's implicit premises which allow him to operate as though he has determined these proportions? Having made Steedman's premises explicit, we may then ask: are they consistent with the labour theory of value (for if so, the labour theory of value falls victim of its own rigorous standards; and if not, then Steedman's account of joint production has nothing to do with the validity of that theory). Our immediate task then is to identify the premises which allow Steedman to determine his value proportions and thereby to "solve" his system.

These premises may be uncovered by comparing Steedman's table on page 153 which depicts his "physical" system with his depiction of the value system on the following page (p. 154), also reproduced on our page 4. Here we see that the per unit values of the coefficients l_1 and l_2 apply equally to the physical outputs of equation 1 and to the physical outputs of equation 2. Or, to put it another way, the individual values of these specific commodities produced by two different enterprises using two different production processes are treated as equivalent to (identical with) the average values of each set of commodities. What this means is that Steedman implicitly relies on a set of additional equations, which in our notation can be formulated as follows:

Steedman's unstated claim for commodity 1 is

$$\frac{\beta_{11}w_1}{6} = \frac{\beta_{21}w_2}{3} \tag{19}$$

and for commodity 2,

$$\frac{(1-\beta_{11})w_1}{1} = \frac{(1-\beta_{21})w_2}{12} \tag{20}$$

These two additional equations, by means of which Steedman closes his system may be converted into two equations, each expressing the ratio of the total output value of process 1 (w_1) to the total output value of process 2 (w_2), in terms of the value proportions of the two processes (β_{11} and β_{21}),

$$\frac{w_1}{w_2} = 2\frac{\beta_{21}}{\beta_{11}} \tag{21}$$

and

$$\frac{w_1}{w_2} = \frac{(1-\beta_{21})\cdot 1}{(1-\beta_{11})\cdot 12}, \tag{22}$$

Since these two equations express one and the same thing $\left(\frac{w_1}{w_2}\right)$, we may equate them to obtain the relationship between the value proportions of the two processes, implicit in Steedman's assumption that identical individual commodities, though produced by different processes, have identical value. Equating (21) and (22) gives us:

$$2\frac{\beta_{21}}{\beta_{11}} = \frac{(1-\beta_{21})\cdot 1}{(1-\beta_{11})\cdot 12} \tag{23}$$

which allows us to express β_{11} in terms of β_{21} as follows:

$$\beta_{11} = \frac{24\beta_{21}}{1+23\beta_{21}} \tag{24}$$

This is Steedman's implicit assumption about the value proportions of the two production processes in his system, which allows him to close that system. It would also close ours, *if* we accepted it; but whether or not we accept it depends on whether or not it is consistent with the labour theory of value. So let us see what the consequences would be if we try to integrate Steedman's assumption into our basic premise of the labour theory of value.

In order to do this, we must first select from the equations that we earlier derived (5 through 18) from this basic premise (formalized in equations 4 and 5), an expression that can easily be inserted into the equations that we have now derived from Steedman's two additional assumptions. We will select equations (11) and (12), which express the total output values of the two processes, w_1 and w_2, respectively, and if we divide them by one another, we obtain the following expression for $\frac{w_1}{w_2}$:

$$\frac{w_1}{w_2} = \frac{27 + 155\beta_{21}}{207 - 155\beta_{11}} \tag{25}$$

This equation makes explicit the condition inherent in the labour theory of value for the relationship between total output value and value proportion, a condition which has to be met by *any* value system, including ours and Steedman's. We can now test the validity of Steedman's two additional assumptions by using them to solve the unknown value proportions β_{11} and β_{21} and seeing if these solutions fall within the limits of β_{ij} imposed by the labour theory of value. We begin by substituting one of Steedman" additional assumptions, equation (21), in equation (25), thereby getting rid of the two unknowns, w_1 and w_2:

$$2\frac{\beta_{21}}{\beta_{11}} = \frac{27 + 155\beta_{21}}{207 - 155\beta_{11}} \tag{26}$$

Simplifying this equation, we get

$$\beta_{11} = \frac{414\beta_{21}}{27 + 465\beta_{21}} \tag{27}$$

This equation combines the general condition inherent in the labour theory of value, which is therefore binding for any value system, with the particular assumption that Steedman introduces into his system, the assumption that identical commodities, though produced by different processes, have identical values. We can now make use of Steedman's other assumption, formulated in equation (24), to get rid of the unknown β_{11}:

$$\frac{24\beta_{21}}{1+23\beta_{21}} = \frac{414\beta_{21}}{27+465\beta_{21}} \tag{28}$$

and obtain the following solution for β_{21}:

$$\beta_{21} = -1/7$$

This result gives us the value-proportion of production process 2, if Steedman's additional assumptions are accepted. By substituting this result for β_{21} in equation (27), we obtain the following result for the value-proportion of Steedman's process 1:

$$\beta_{11} = 1.5$$

We can now summarize our findings as follows: Firstly we can clearly see (what we already suspected from our earlier critique of the results Steedman obtained for his coefficients l_1 and l_2) that the value proportions ($\beta_{21}=-1/7$; $\beta_{11}=1.5$), implicitly assumed by Steedman in his use of two hidden premises and made explicit in our equations (19) and (20), contradict the basic premise of the labour theory of value. For they lie outside the boundaries inherent in this premise, according to which β_{ij} must lie within the range from zero to unity ($0<\beta_{ij}<1$). Thus Steedman's procedures violate any notion of the labour theory of value. At the same time, we can see from our reconstruction of how Steedman arrived at his incorrect result,[5] the precise way in which his procedure went wrong and violated the labour theory of value. His mistake lies in his assumption that the individual and average values of commodities coincide. In Steedman's own examination of his joint production example, this assumption remains hidden and implicit. We have made it explicit in equations (19) and (20) above.

These two equations contain the unwarranted assumption that the amount of direct and indirect labour which is stored up in one unit of a

5. An easy way of checking whether or not our value equations are consistent with those given by Steedman is to insert into equations (11) and (12) the value proportions we have computed here ($\beta_{11}=1.5$, $\beta_{21}=-1/7$). The reader will then obtain (using our equations) the same results as Steedman, namely $w_1=-4$, and $w_2=21$, derived from equations (1) and (2). Thus what is at issue is not any discrepancy between the method of computation we have chosen and the method of computation chosen by Steedman, but a basic flaw in Steedman's reasoning.

commodity is necessarily the same for all commodities of the same species. In certain cases, it is indeed legitimate to treat the values of individual commodities as equivalent and to assign to them all the average value of the species as a whole. But in the example of joint production that Steedman has set up, this simplification is illegitimate precisely because it leads him to violate the basic premise of the labour theory of value, which states that no commodity can contain more value (i.e. more "crystallized" labour) than the total value (i.e. the sum of living labour and dead labour) expended upon its production.

Discussion: The hidden problem of differential rent

The attractive simplicity of Steedman's example is deceptive, for in fact it represents a highly complex situation that throws up not merely one, but two problems. The one problem is the problem that Steedman explicitly addresses: the situation of joint production, where several commodities are produced by a single production-process. The other problem arises from the fact that the same bundle of commodities is being produced and marketed by different producers. The latter problem would occur even if joint production was not involved, i.e. even if the different producers were devoting their production-processes to turning out only one line of product. Steedman does not expressly deal with the second problem, nor does he even seem to be aware of its inclusion in the model he sets up. For if he were, he would have seen that his attempt to resolve the problem(s) he poses is bound to fail, because of the vary nature of his approach. If he had recognized the second problem for what it really is, namely the problem of different producers using different production-processes to create one and the same species of commodity, and examined it in its simplest form, namely the case where several producers produce and sell a single commodity, then he would have immediately realized that this is a problem that cannot be solved by formulating a set of simultaneous equations to determine the values of the respective commodities. For such an approach presupposes that within one species of commodity, there can only be one value and it is precisely this presupposition that we *cannot* make (except by specifying an additional series of particular conditions), if we want to presuppose at the same time that these producers are each using a different production process. Steedman sets up his model by explicitly presupposing the latter (that each producer is using a different process), but then proceeds to examine this model by means of simultaneous equations, thereby slipping in the former contrary presupposition (that these two different processes share a common value formula[6]).

In summary, as soon as we make explicit the second problem inherent in the model that Steedman sets up, and as soon as we isolate and examine that problem in its simplest form (different single-product processes producing the same species of commodities), then we –and Steedman– can immediately see that *the coexistence of different individual values is an absolute necessity.*

In *Capital III*, Marx examines the case of different production-processes turning out the same species of commodities and recognizes the necessary coexistence of different individual values. He introduces and applies the concept of differential rent, in order to construct a methodological framework for determining those individual values, avoiding the fallacy of treating value as though it were equivalent to the equilibrium price. Marx himself did not examine the problem of joint production posed by Steedman, where two producers, each producing the same bundle of commodities as the output of a single process. But any attempt to solve Steedman's problem must take into account the existence of differential rent.

Here we will simply restrict ourselves to pointing out that Steedman's simultaneous equations can do nothing to dispel the necessary coexistence of different individual values in the case that he has set up; they can only mystify the real issue, by implicitly violating the logic of the labour theory of value. All that Steedman can succeed in doing with his simultaneous equations is to brush under the carpet the indeterminacy of the system that he has set up. He thereby combines what are really two separate problems, that cannot be solved without closer examination and reflection, to create the false impression of a single and apparently solvable problem.

Needless to say, the progeny of this forced and illegitimate marriage of two inadequately presented problems is a premature bastard masquerading

6. Another way of characterising Steedman's mistake is to say that he falls into the trap of treating value as though it were an equilibrium price, illegitimately transforming the fact that all commodities belonging to the same species can have only one price into the unwarranted claim that all commodities belonging to the same species can only have one price.

as the legitimate heir of the labour theory of value: a blunder that arises from Steedman's own faulty premises and procedures and that, despite all of Steedman's protests to the contrary, has nothing to do with the validity or invalidity of the labour theory of value.

Conclusion

We have already made it clear that we did not intend in this paper to present a solution to the problem of how, in the situation of joint production, the total value of the ingredients of the production-process are transferred to and distributed among the commodities produced. But willy-nilly we have taken a decisive step in this direction. For we now know 1) that further attempts along Steedman's lines are futile; and 2) that any solution to the problem has to take account of the possibility that the values of individual commodities which belong to the same species diverge (this is the problem that Marx addresses with his concept of differential rent). Now that we know this, we can further recognize that the crucial problem is still the problem of the indeterminacy of the system. Thus the next step in constructing a cogent alternative way of handling the problem of joint production within the context of the labour theory of value is to figure out how we can deal with the problem of indeterminacy. This means investigating the missing links that we have identified in this paper instead of ignoring them by devices such as Steedman's illegitimate and hidden assumption of the equivalence of individual values. Before we go on to complete this task, please let us have your comments and reactions to this paper.

Bibliography

Flaschel, P., Zu technologischen Sichtweisen des quantitativen Wertproblems in Gleichgewichtsmodellen - eine Alternative, *Mehrwert* 13, 1978, 111-139.

Marx, K., *Capital*, Vol. I, International Publishers, New York, 1967.

Ricardo, D., *Principles of Political Economy and Taxation*, Volume I (ed. P. Sraffa), Cambridge University Press, Cambridge, 1975.

Steedman, I., *Marx after Sraffa*, New Left Books, London, 1977.

Wolfstetter, E., *Wert, Profitrate und Beschäftigung*, Frankfurt, 1977.