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Concerning the Issue of the Choice of Technique in Neo-Ricardian Models of Single Production*

by
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I. Introduction

As is known, the issue of the choice of technique in Neo-Ricardian single production models is solved either in terms of the outer envelope of the w-r curves (of the available techniques) or in terms of the cost minimization criterion[1]. Thus, current discussion has now turned to solving the issue in the case of joint production models[2].

In our view, however, certain questions which are important for the case of single production remain unanswered:

1. In the case where only basic commodities[3] are produced, why do the intersection points of the w-r curves, which do not belong to the outer envelope of the w-r curves (i.e. the so-called "false switch points") depend on the composition of the numéraire[4]?
2. Why, in the case of production models which produce (and) non-basic commodities, the solving of the issue of choice in terms of the envelope of the w-r curves and the solving of the issue of the choice in terms of the cost minimization criterion do not *always* bring the same result[5]?

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1. See, for example Pasinetti (1985), Chapter 6, Bidard (1991), pp. 77-82.
2. See, primarily, Autumn (1988), Bidard (1990),Kurz and Salvadori (1995), Ch. 8.
3. For the concepts of basic and non-basic commodities, see Sraffa (1960), §§ 6-8.
4. Although the said dependence has often been pointed out in the relevant bibliography, it has not been interpreted. See, for example, Laibman/Nell (1977), pp. 882-83, Vaggi (1978), pp. 141-45 Akyüz (1978), Pertz/Teplitz (1979), pp. 252-54, Herrero/Raneda/Villar (1980), pp. 167-68, Baldone (1984), p. 278.
5. This too has been pointed out in the relevant bibliography, but has not been interpreted. For an indirect reference see, for example, Herrero/Raneda/Villar (1980), pp. 168-70 and for an express reference Bidard (1991), pp. 83-4. Lastly, Stamatis (1984), pp. 293-333 has adequately interpreted the problems associated with the choice in terms of the envelope of the w-r curves but has totally ignored the second criterion of choice and consequently has not answered this question. However, see Stamatis (1998), pp. 154-159.

3. Why, in certain production models which produce (and) non-basic commodities and for a nominal wage rate equal to zero, does the introduction of a "new" production method of a non-basic commodity, which reduces the cost of its production, not lead (irrespective of the composition of the numéraire) to an increase in the rate of profit?

The present paper answers these questions and, in order to precisely define the meaning of equally profitable techniques, proposes a method of constructing sets of equally profitable techniques.

II. Investigation of the Issue

We assume a linear and profitable technique of single production [A, l]. The matrix A, $A \equiv [\alpha_{ij}] \geq 0$ symbolises the square nXn matrix of technical coefficients, the element a_{ij} of which represents the amount of commodity i required to produce one unit of commodity j (as gross product), with i, j = 1, 2, ..., n, while the vector l, $l \equiv [lj] > 0$ symbolises the 1Xn vector of inputs in direct homogeneous labour, the component lj of which represents the amount of direct labour required to produce one unit of commodity j (as gross product).

As is known, if we introduce the usual assumptions, the prices of n commodities produced are determined by the following system of equations:

$$p = pA(1+r)+wl \tag{1}$$

where p is the 1Xn vector of the prices of n commodities produced, w is the *by assumption* uniform nominal wage rate and r is the *by assumption* uniform rate of profit.

The system (1) consisting of n equations has two degrees of freedom, and in order to determine the prices of the commodities, it is necessary to introduce some normalization equation (of prices) of the form:

$$pu = c \tag{2}$$

where u is a positive or semi-positive nX1 vector, which we shall call the *normalization commodity* and c is a positive constant, the dimension of which is: units of (fictitious) money / unit of normalization commodity, and second, the exogenous determination of the nominal wage rate or rate of profit.

We distinguish two cases:

A. *Matrix A is irreducible*

As is known, from system (1) and equation (2) the equation of the so-called w-r curve is deduced:

$$w = \frac{c}{l[B(0)+B(r)A[I-A]^{-1}r]u} = \frac{c}{lB(r)u}, \quad \forall w \in (0, w_{max}(\equiv w(0))] \qquad (3)$$

and

$$r = r_{max} \equiv (1-\lambda_m^A)/\lambda_m^A, \text{ for } w=0 \qquad (3a)$$

where I is the nxn identity matrix, $B(r) \equiv [I - A(1+r)]^{-1}$ and λ_m^A the Perron-Frobenius (maximum) eigenvalue of matrix A. The economically significant interval of the rate of profit is: $[0, r_{max}]$, because to this (and only to this) corresponds a positive prices vector p and a semi-positive nominal wage rate[6].

At the same time, because the price of the normalization commodity can be broken down (into wages and profits) as follows:

$$pu = pA[I-A]^{-1}ur + wl[I-A]^{-1}u \Rightarrow$$

$$w = (c/L_u) - (K_u/L_u)r \qquad (4)$$

it follows that the magnitude $L_u \equiv \sum_{t=0}^{\infty} lA^t u$ expresses the quantity of labour, the magnitude c/L_u $(=w_{max})$ the productivity of labour in price terms, the magnitude $K_u \equiv p(r)A[I-A]^{-1}u$ (which in the interval $[0, r_{max})$ is equal to: $\left(w\sum_{t=0}^{\infty} lA^t(1+r)^t\right)A[I-A]^{-1}u$ and for $r=r_{max}$ to: c/r_{max}) the "quantity of capital" and the magnitude $\tilde{K}_u \equiv K_u/L_u$ the capital intensity in price terms *in that* production subsystem à la Fel'dman/Sraffa[7]:

a) Which, through the technique [A, l], produces the normalization commodity as its net product, and b) In which, the price of its net product is

6. For the proof see, for example, Bidard (1991), Chapter III.
7. For the concept of subsystems, see Fel'dman (1964), pp. 176-83, Sraffa (1960), Appendix A Pasinetti (1973) and Kurz and Salvadori (1995), Ch. 3, 6.

exogenously given, constant and equal to c units of (fictitious) money. We shall call this production subsystem the normalization subsystem[8].

Lastly, let m_{ij} (*by assumption* positive) be the quantity of commodity i required *in total* to produce one unit of commodity[9] j (as gross product). The quantities m_{ij} are determined by the system (for a thorough investigation, see Mariolis (1998)):

$$M \equiv \tilde{M}A (\Rightarrow 1-m_{ii} \equiv \det[I-A]/\sigma_{ii} \textit{ and } m_{ij} \equiv \sigma_{ji}/\sigma_{ii},\ i \neq j) \qquad (5)$$

where $M \equiv [m_{ij}]$ and $\tilde{M}$ the nXn matrix, which derives from matrix M, when we replace all the elements of its principal diagonal with unit and σ_{ii}, σ_{ji} the cofactors of the elements ii and ji (respectively) of the matrix [I–A]. If we use q* to symbolise the, as is known, positive right eigenvector of A, which is connected with its maximum eigenvalue, then the following emerges from the system (5):[10]

$$[\tilde{M}-M]q^* \equiv (1-\lambda_m^A)\tilde{M}q^* \Rightarrow$$

$$\lambda_m^A < 1 \Rightarrow m_{ii} < 1 \text{ and } m_{ii} < \lambda_m^A,\ \forall i. \qquad (5a)$$

$$\exists i : m_{ii} < 1 \Rightarrow \lambda_m^A < 1 \text{ and } m_{ii} < \lambda_m^A,\ \forall i. \qquad (5b)$$

8. Thus, the normalization equation constitutes a «principle of conservation»: the shift from a equilibrium position due to the autonomous change of one of the distribution variables, will cause, with respect to direction and breadth, such a change in the other variable, that the sum of the total nominal wages and profits in the normalization subsystem remains constant and equal to c. Therefore, Laibman/Nell (1977), p. 880, quite rightly perceive the w-r curve as a contour line.
As far as we know, the concept of normalization subsystem is introduced for the first time (in Laibman/Nell (1977), pp. 880-1 there is only a simply reference), by Parys (1982), pp. 1210-11 and subsequently by Stamatis (1984), Chapter IV. See also Salvadori and Steedman (1985), Steedman (1988), pp. 92-3, D'Ippolito (1996), p. 57.
Moreover, Sraffa (1960) concludes Appendix A as follows: «At each level of the wage and the rate of profits, the commodity forming the net product of a sub-system is equal in value to the wages of the labour employed plus the profits on the means of production. And when the wage absorbs the whole net product, the commodity is equal in value to the labour that directly or indirectly has been required to produce it».
9. When for example n=2, we have:

$$m_{12} \equiv \alpha_{12} + m_{12}\alpha_{22},\ m_{11} \equiv \alpha_{11} + m_{12}\alpha_{21}$$
$$m_{21} \equiv \alpha_{21} + m_{21}\alpha_{11},\ m_{22} \equiv \alpha_{22} + m_{21}\alpha_{12}$$

10. As is easily proven (on the basis of the Perron-Frobenius theorems) -with the exception of (5a), (5b), which follow- the following also holds: $\lambda_m^A > \max\{\alpha_{ii}\},\ \forall i$.

Finally, from the systems (1) and (5) it follows that the "production cost" of each commodity can be reduced to that same commodity as follows:

$$p_i = p_i m_{ii}(v) + w\left(\sum_{t=0}^{\infty} lA^t s_i v^t\right)[1 - m_{ii}(v)] \tag{6}$$

where s_i is the i-th unit vector, $v \equiv 1 + r$ and $m_{ii}(v)$ the quantity of commodity i required *in total* to produce one unit of commodity i, on the basis of the matrix of technical coefficients: $A \cdot v$, with: $0 < m_{ii} < m_{ii}(v) < 1$, for $r \in (0, r_{max})$, $m_{ii} = m_{ii}(v) < 1$, for $r = 0$ and $m_{ii}(v) = 1$, for $r = r_{max}$.

As deduced from the equation (3), if the method of producing commodity j ($1 \leq j \leq n$) is marginally changed and the nominal wage rate (rate of profit) is not changed, then:

$$dr \geq 0 (dw \geq 0) \Leftrightarrow (dl)B(\bar{r})u + lB(\bar{r})(dA)(1+\bar{r})B(\bar{r})u \leq 0$$

$$\Leftrightarrow [\bar{w}(dl_j) + \sum_{i=1}^{n} \bar{p}_i (d\alpha_{ij})(1+\bar{r})]B(\bar{r})u \leq 0$$

and because $B(\bar{r})u > 0$, *for each* positive or semi-positive u, it follows that:

$$dr \geq 0 (dw \geq 0) \Leftrightarrow \bar{w}(dl_j) + \sum_{i=1}^{n} \bar{p}_i (d\alpha_{ij})(1+\bar{r}) \leq 0 \qquad (C_{A.1})$$

If the nominal wage rate is constant and equal to zero, then[11]:

$$p = pA(1 + r_{max}) \Rightarrow$$

$$dr_{max} \geq 0 \Leftrightarrow \bar{p}(dA)q^*(1 + r_{max}) \leq (dp)[I - A(1 + r_{max})]q^*$$

$$\Leftrightarrow \bar{p}(dA)q^*(1 + r_{max}) \leq 0$$

$$\Leftrightarrow \left(\sum_{i=1}^{n} \bar{p}_i (d\alpha_{ij})\right) \leq 0 \qquad (C_{A.2})$$

On the basis of the preceding investigation, we can observe the following:

1. With respect to the effect of marginal changes of a_{ij} and l_j on the nominal wage rate (rate of profit), for an exogenously given and constant value

11. As is known, the conditions $(C_{A.1})$ and $(C_{A.2})$ constitute cost minimization conditions. See, also, Abraham-Frois (1991), pp. 469-71, 482-83 and Bidard (1988).

of the rate of profit (nominal wage rate), as deduced from the equation (3), the following holds:

$$dr \gtreqless 0 (dw \gtreqless 0) \Leftrightarrow [\bar{w}(dl) + p(\bar{r})(dA)(1+\bar{r})]B(\bar{r})u \gtreqless 0 \qquad (C'_{A.1})$$

Because the sign of the components of the 1xn vector inside the brackets does not depend on the normalization equation and because the nx1 vector $B(\bar{r})u$ is positive, it follows that the sign of the inner product of these vectors *depends* (in the general case) *on the composition of the normalization commodity u* and that the *necessary conditions* for the existence of this dependence are the following two: a) The technical conditions of production must change in at least two production processes, and b) The change in the technical conditions of production in at least one production process must not follow (must conflict with) the cost minimization criterion. That is, it must not fulfil the condition ($C_{A.1}$).

Although this case -as noted in the *Introduction* of this paper- has often been pointed out in the relevant bibliography, its significance has been underestimated precisely because its existence presupposes the *second* of the two aforementioned *necessary conditions*. However, this case must be interpreted because: a) It shows that the w-r curve *is not* a curve which unambiguously characterises each production technique (if it was a curve which unambiguously characterised each production technique, then the intersection points of *any* two w-r curves would *always* be independent of the composition of the normalization commodity), b) By definition, it cannot be ruled out in the case of so-called «merger production»[12].

From the equations (3) and (4), one may easily conclude that when two w-r curves are collated, in reality a comparison is not being made of two production techniques with respect to their profitability, but rather a comparison is being made of those subsystems (with respect to their profitability) which use these two production techniques in order to produce each chosen normalization commodity u as their net product. Put differently: when the rate of profit is exogenously given, that subsystem is chosen which minimizes the quantity $lB(\bar{r})u$. That is, which minimizes the $\bar{v}$ -value of the normalization commodity[13] u.

12. For the concept of merger production, see Burmeister (1974), pp. 445-47.

13. Obviously, with $\bar{v}$ -values ($\omega^{\bar{v}}$) we define the values of commodities which correspond to

Example 1:

$$A^{\alpha}=\begin{bmatrix}0.2 & 0.1\\ 0.1 & 0.2\end{bmatrix},\ l^{\alpha}=[1,1],\ A^{\beta}=\begin{bmatrix}0.1 & 0.08\\ 0.5 & 0.1\end{bmatrix},\ l^{\beta}=[0.57,1.16]$$

and assuming that $\bar{r}=0.0165(<r^{\alpha}_{max}=r^{\beta}_{max}=0.7/0.3)$. The $\bar{v}$ -values of the commodities $\omega_i^{\bar{v},d}$, $i=1,2$ and $d=\alpha,\beta)$ are equal to: $\omega_1^{\bar{v},\alpha}=1.4387$, $\omega_2^{\bar{v},\alpha}=1.4387$ and $\omega_1^{\bar{v},\beta}=1.4387$, $\omega_2^{\bar{v},\beta}=1.4215$. Let us now consider the techniques $\alpha'\equiv[A^{\alpha}\bar{v},l^{\alpha}]$ and $\beta'\equiv[A^{\beta}\bar{v},l^{\beta}]$ and let us assume that one unit of direct labour is available. The production-possibility curves which correspond to the techniques α', β' are given by the following equations (where: $X\equiv[X_1, X_2]^T$ is the vector of the levels of operation of the processes of the technique and $Y\equiv[Y_1,Y_2]^T$ is the vector of the corresponding net product):

$$\alpha':\ 1X_1+1X_2=1=1.4387Y_1+1.4387Y_2 \qquad (7)$$

$$\beta':\ 0.57X_1+1.16X_2=1=1.4387Y_1+1.4215Y_2 \qquad (8)$$

while their relative position is shown in *Diagram 1*:

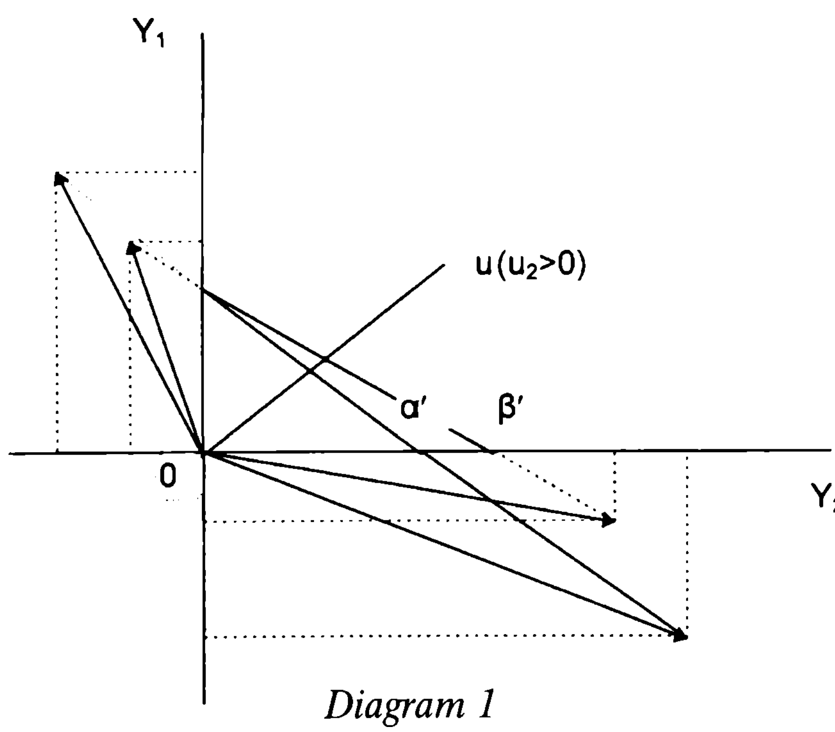

Diagram 1

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the technique: $[A\bar{v},l\ ]$ and which are determined by the system: $\omega^{\bar{v}}\equiv\omega^{\bar{v}}A\bar{v}+l$. Ceteris paribus, therefore, it may be said that, for a definite value of r, w expresses the $\bar{v}$ productivity –in price terms– of labour in the normalization subsystem.

For precisely this reason, only if we normalize the prices using the equation:

$$pu = p_1 u_1 + p_2 0 = c \tag{9}$$

do the techniques α, β appear as being equally profitable, while each normalization equation with: $u_2 > 0$, shows technique β as being the most profitable[14].

2. Let the nominal wage rate be constant and equal to zero. As emerges from the systems (5), (6) and their breakdown, in such a case the following holds (for a thorough investigation, see Mariolis (1998)):

$$\begin{aligned}
&m_{ii}(v_{max}) = 1 \Rightarrow \\
&M(v_{max}) = \tilde{M}(v_{max}) \Rightarrow \\
&\tilde{M}(v_{max})[I - Av_{max}] = 0 \Rightarrow \\
&m_{ij}(v_{max}) = p_j^* / p_i^* \Rightarrow \\
&m_{ij}(v_{max}) \cdot m_{ji}(v_{max}) = 1, \ \forall i,j.
\end{aligned} \tag{10}$$

where $v_{max} \equiv 1 + r_{max}$, $M(v_{max}) \equiv [m_{ij}(v_{max})]$ and p* the solution of the system of prices for w = 0 (and consequently p* is the left eigenvector of matrix A, which is connected with its maximum eigenvalue).

So, let w = 0 and let there be two matrices of technical coefficients A^α, A^β, *which differ only in one column*. As may easily be deduced from the above, the following will hold: $r_{max}^\alpha = r_{max}^\beta$, if and only if:

$$\exists\, i : m_{ii}^\alpha(v_{max}^\alpha) = 1 = m_{ii}^\beta(v_{max}^\alpha) \tag{11}$$

14. Thus, for example, if we normalize the prices using the equation: $p_1 = 1$, we get:

$$w^\alpha = 1 - 0.3v, \quad w^\beta = \frac{-0.03v^2 - 0.2v + 1}{0.57 + 0.523v}$$

which are intersected for $\bar{r} = 0.0165$ (and $r = r_{max}$), while if we normalize using the equation: $p_1 + p_2 2.5 = 1$, we get:

$$w_\alpha = 0.2857 - 0.0857v, \quad w_\beta = 0.2881 - 0.0864v$$

which are not intersected for $\bar{r} = 0.0165$ (they are intersected only for $r = r_{max}$).
As the reader may readily ascertain, that which is set forth here and that which is set forth in Bidard (1990), pp. 841-45, *in substance* do not differ.

Example 2:

$$A^{\alpha} = \begin{bmatrix} 0.5 & 0.1 \\ 0.2 & 0.4 \end{bmatrix}, \; r^{\alpha}_{max} = 0.4/0.6, \; A^{\beta} = \begin{bmatrix} 0.5 & \alpha^{\beta}_{12} \\ 0.2 & \alpha^{\beta}_{22} \end{bmatrix}$$

$$m^{\beta}_{11}(v^{\alpha}_{max}) = 1 \;\Leftrightarrow\; 10\alpha^{\beta}_{12} + 5\alpha^{\beta}_{22} = 3$$

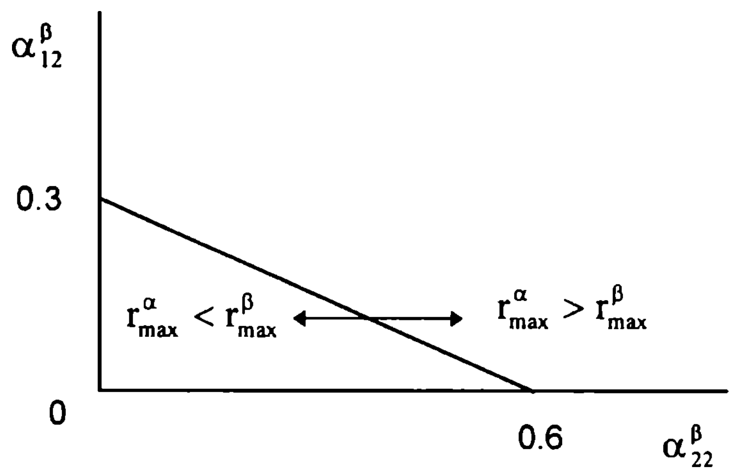

Diagram 2

Consequently, the two techniques are equally profitable if, for example, the following holds: $\alpha^{\beta}_{12} = \alpha^{\beta}_{22} = 0.2$ or $\alpha^{\beta}_{12} = 0$ and $\alpha^{\beta}_{22} = 0.6$. However, the second of these cases not only belongs to the category of reducible techniques –a category which we shall examine later– but also presents particular (as we shall see) interest[15].

If however, the matrices of technical coefficients differ with respect to more than one column, then the condition (11) is not sufficient:

Example 3: Let the data A^{α} of the *Example 2* and

$$A^{\beta} = \begin{bmatrix} 0.4 & 0.1 \\ 0.3 & 0.45 \end{bmatrix}. \text{ Consequently, } r^{\alpha}_{max} = r^{\beta}_{max}, \textit{ but } (p^*_1/p^*_2)^{\alpha} \neq (p^*_1/p^*_2)^{\beta}.$$

In this case (i.e. not «adjacent» techniques), the following hold: $r^{\alpha}_{max} = r^{\beta}_{max}$ *and* $(p^*_1/p^*_2)^{\alpha} = (p^*_1/p^*_2)^{\beta}$, if and only if:

$$\exists\, i : m^{\alpha}_{ij}(v^{\alpha}_{max}) = m^{\beta}_{ij}(v^{\alpha}_{max}), \; \forall j \tag{12}$$

15. See *Example 6* in the present paper.

Example 4: Let the data A^{α} of the *Example 2* and

$$A^{\beta}=\begin{bmatrix} 0.1 & 0.15 \\ 1 & 0.3 \end{bmatrix}. \text{ Consequently, } r_{max}^{\alpha}=r_{max}^{\beta} \textit{ and } (p_1^{*}/p_2^{*})^{d}=2,\ d=\alpha,\beta.$$

So eventually, the investigation of the case w=0, through the coefficients m_{ij} (v_{max}), shows: a) How it is possible to construct sets of equally profitable techniques, which differ with respect to *one* or *more production processes*. It therefore illustrates the meaning of: equally profitable techniques, and b) That it is not possible to produce a general rule for classifying techniques, which is based solely and exclusively on the calculation of the coefficients m_{ii} (and not on the calculation of m_{ij} (v_{max}))[16]: In *Example 2*, for $\alpha_{12}^{\beta}=\alpha_{22}^{\beta}-0.2$, the following holds: $m_{11}^{\alpha}<m_{11}^{\beta}$, $m_{22}^{\alpha}>m_{22}^{\beta}$, $m_{12}^{\alpha}<m_{12}^{\beta}$, $m_{21}^{\alpha}=m_{21}^{\beta}$, while in *Example 4* the following holds: $m_{11}^{\alpha}>m_{11}^{\beta}$, $m_{22}^{\alpha}<m_{22}^{\beta}$, $m_{12}^{\alpha}<m_{12}^{\beta}$, $m_{21}^{\alpha}<m_{21}^{\beta}$.

B. *Matrix A is reducible*

We assume that matrix A takes the form:

$$A=\begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix},\ l=\left[l_{I}, l_{II}\right]$$

where $[A_{11}, l_{I}]$, $[A_{12}, A_{22}, l_{II}]$ is the technique of the k ($1 \leq k < n$) basic and of the n–k non-basic processes respectively. We also assume that the matrix A_{22} is irreducible and that the following holds: $\lambda_m^{A_{11}} \gtreqless \lambda_m^{A_{22}} \Rightarrow R_{I} \lesseqgtr R_{II}$, where $R_{I}\equiv(1-\lambda_m^{A_{11}})/\lambda_m^{A_{11}}$, $R_{II}\equiv(1-\lambda_m^{A_{22}})/\lambda_m^{A_{22}}$, are the maximum rates of profit of the basic and non-basic processes[17]. Lastly, with $p = [p_{I}, p_{II}]$ we shall symbolise the vector of prices of k basic (p_{I}) and n–k non-basic (p_{II}) commodities and with $u = [u_{I}, u_{II}]^{T}$ the normalization commodity.

16. That is, a rule which emanates *directly* from the «physical data» of production. As the reader with no doubt realise (by means of system (6)) this holds primarily when: w>0.
17. As the maximum rate of profit of basic (non-basic) processes, we set that value of the rate of profit which emerges for w=0 (for w=0 and for prices of basic commodities equal to zero) and to which positive prices correspond for all the basic (non-basic) commodities.

In view of these clarifications, we may observe the following:

1. In the event that $\bar{r} < \min\{R_I, R_{II}\}$ and the technique of non-basic processes changes:

$$dr > 0 (dw > 0) \Rightarrow [\bar{w}(dl_{II}) + (\bar{p}_I(dA_{12}) + \bar{p}_{II}(dA_{22}))(1+\bar{r})] B_{22}(\bar{r}) u_{II} < 0$$

$$\Rightarrow [\bar{w}(dl_{II}) + (\bar{p}_I(dA_{12}) + \bar{p}_{II}(dA_{22}))(1+\bar{r})] < 0 \qquad (C_{B.1})$$

where $B_{22}(\bar{r}) \equiv [I_2 - A_{22}(1+\bar{r})]^{-1}$ and $B_{22}(\bar{r}) u_{II} > 0$, for each positive or semipositive u_{II}. However, if the condition $(C_{B.1})$ is valid and $u_{II} = 0$, then the income distribution variable will not increase[18]. Therefore, whether or not each change in the technique of non-basic processes, *which is consistent with the cost minimization criterion*, (i.e. which satisfies the condition $(C_{B.1})$) leads to an increase in the income distribution variable, depends on the composition of the normalization commodity. This "strange" attribute of reducible techniques can be interpreted by the fact that the w-r curve depends on the magnitudes of the normalization subsystem: for the income distribution to change as a result of a change in the production conditions of a commodity, this commodity must enter (directly or indirectly) into the production of the normalization commodity (:necessary condition, because there is the case of the "switch point"). Obviously, only when the technique is irreducible, is this condition *always* given.

In the final analysis, this explains precisely why, when the technique is reducible, the determination of the most profitable technique through the envelope of the w-r curves and the determination of the most profitable technique through the cost minimization criterion arrive at the same result, if and only if: $u_{II} \geq 0$.

Example 5: In the event that n = 2, commodity 1 is basic and two alternative production methods are available for the production of commodity 2. For $\bar{r} < \min\{R_I, R_{II}\}$, in terms of *Diagram 1*, it is possible for us to have:

18. For this reason, in deducing $(C_{B.1})$ (and *in contrast* with $(C_{A.1})$) we do not use the symbol of equivalence ($\Leftrightarrow$).

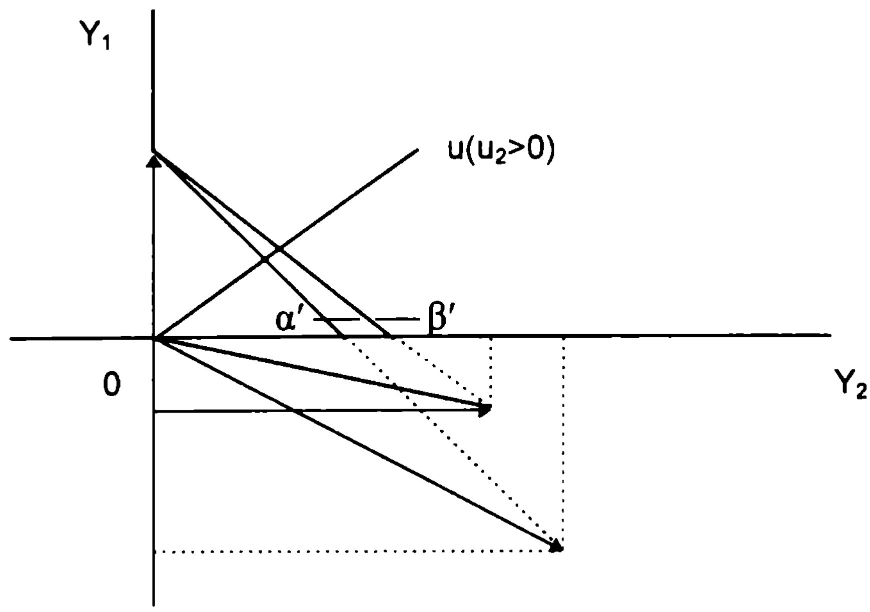

Diagram 3

If $u_2 = 0$, then in terms of the envelope of the w-r curves the techniques α, β appear as being equally profitable, while in terms of the cost minimization criterion technique β is chosen. *However, if technique a is initially given, then the introduction of technique β will have no effect on the income distribution variables*. If, however, $u_2 > 0$, then both criteria show technique β as being the most profitable. *Moreover, if technique a is initially given, the introduction of technique β will lead, for constant w (constant r), to an increase in r (increase in w).*

2. The case in which the nominal wage rate is constant and equal to zero is somewhat complex and in order to examine it we must first determine the value of the rate of profit and the prices vector which correspond to the value w = 0. We distinguish the following cases[19] (we assume that the w-r curve is not linear):

2.1. Let $R_I<R_{II}$:
- $u_{II} = 0 \Rightarrow r_{max}=R_I$, $p>0$.
- $u_{II} \geq 0 \Rightarrow r=R_I$, $p>0$ *and* $r_{max}=R_{II}$, $p_I=0$, $p_{II}>0$.

2.2. Let $R_I = R_{II}$:
- $u_{II} = 0 \Rightarrow r=R_I$, $p_I>0$, while the prices of the non-basic commodities tend to infinity $\left(\lim_{r \to R_I} p_{II}(r) = \pm\infty\right)$.
- $u_{II} \geq 0 \Rightarrow r_{max}=R_I$, $p_I=0$, $p_{II}>0$.

19. See, also, Egidi (1975), pp. 11-13.

2.3. Let $R_I > R_{II}$:

- $u_{II} = 0 \Rightarrow r = R_I$, $p_I > 0$, while p_{II} has at least one negative component. The p_{II} is positive in the interval $[0, R_{II})$ and for $r = R_{II}$ tends to infinity.
- $u_{II} \geq 0 \Rightarrow r_{max} = R_{II}$, $p_I = 0$, $p_{II} > 0$.

As a consequence of the existence of zero or indeterminate prices, only the first case, for the value $r = R_I$, appears as regular. In this case also, however, if the method of production of a non-basic commodity changes (in conformity with the cost minimization criterion), then: irrespective of the composition of the normalization commodity, the rate of profit will not change, but the relative prices of the commodities may change[20]. This means –a fact which invalidates the so-called "non-substitution theorem"– that to an exogenously given value of the nominal wage rate ($w = 0$) or rate of profit ($r = R_I$), adjacent techniques may correspond which must be considered equally profitable, even though they are characterised by different positive prices vectors (see also Kurz and Salvadori (1995), pp. 151-2, 155-6, Exercise 8.7).

The following example presents all the "strange" features, which are associated with reducible techniques also for the value $w = 0$:

Example 6: Let us assume that a technology production of two commodities, which for the production of the basic commodity (commodity 1) includes only one production method: $\alpha_{11} = 0.75$, $\alpha_{21} = 0$, l_I, while for the production of the non-basic commodity, includes four alternative production methods. The said technology may be summarised as follows (assuming that l^{γ} does not constitute an eigenvector of A^{γ}):

$$\tilde{A} = \left[\begin{array}{l||c|c|c|c} \alpha_{11} = 0.75 & 4 & 1 & 0.5 & 0.2 \\ \alpha_{21} = 0 & 0 & 0 & 0.5 & 0.75 \end{array}\right]$$

$$\tilde{l} = \left[\begin{array}{l||c|c|c|c} l_I & l_{II}^{\alpha} & l_{II}^{\beta} & l_{II}^{\gamma} & l_{II}^{\delta} \end{array}\right]$$

Consequently, for $\bar{v} = 4/3$, we have ($d = \alpha, \beta, \gamma, \delta$):

20. Just as the symbol of equivalence cannot be used in deducing ($C_{B.1}$), the symbol of equivalence, again, cannot be used in deducing a condition corresponding to ($C_{A.2}$). This is because the right eigenvector of matrix A, which is associated with the eigenvalue $\lambda_m^{A_{11}}$, has its k first components positive and *the remaining n-k components equal to zero.*

$m_{11}^{d}(\bar{v}) = 1, m_{21}^{d}(\bar{v}) = 0, m_{12}^{\alpha}(\bar{v}) = 5.33, m_{22}^{\alpha}(\bar{v}) = 0, m_{12}^{\beta}(\bar{v}) = 1.33, m_{22}^{\beta}(\bar{v}) = 0,$

$m_{12}^{\gamma}(\bar{v}) = 2,\ m_{22}^{\gamma}(\bar{v}) = 0.67,\ \lim_{v \to \bar{v}} m_{12}^{\delta}(v) = \pm\infty,\ m_{22}^{\delta}(\bar{v}) = 1.$

If we normalize the prices with the equation: $p_I = 1$, then for $r = 1/3$ the same value of w ($w = 0$) corresponds to all four techniques, but not the same prices vector: $p_{II}^{\alpha} = 5.33$, $p_{II}^{\beta} = 1.33$, $p_{II}^{\gamma} = 2$, while p_{II}^{δ} is *not defined*. Although, according to the usual definition, the point: ($r = 1/3$, $w = 0$) does not constitute a "switch point", it remains a fact that for the value $r = 1/3$ and given initially one of the techniques α, β, γ, *each introduction of an alternative production method does not lead to an increase in the nominal wage rate*. Therefore, for $r = 1/3$, the techniques α, β, γ, must be considered equally profitable. Lastly, for $r = 1$, a negative w emerges in all the techniques, and therefore the comparison would be economically meaningless.

If we normalise the values with the equation: $p_{II} = 1$, then for $r = 1/3$, the same value of w ($w = 0$), corresponds to all four techniques, but not the same prices vector: $p_{I}^{\alpha} = 0.188$, $p_{I}^{\beta} = 0.752$, $p_{I}^{\gamma} = 0.5$, while p_{I}^{δ} is *determined* and equal to zero[21]. Consequently, for $r = 1/3$, the techniques α, β, γ *and* δ must be considered equally profitable. In addition, for the value $r = 1$, the technique γ is the only profitable one, since it gives: $w^{\alpha}<0$, $w^{\beta}<0$, $w^{\gamma}=0$ with $p_{I}^{\gamma} = 0$, $w^{\delta} > 0$ with $p_{I}^{\delta} < 0$.

One may therefore conclude the following from this example:

a) To an exogenously determined value of one of the income distribution variables, and *irrespective of the composition of the normalization commodity*, techniques may correspond which must be considered equally profitable, even though they are characterised by different prices vectors.

Obviously this results from assuming the existence (and conservation) of a uniform rate of profit and a uniform nominal wage rate for the basic and non-basic processes: Let technique α be initially given and $r = 1/3$ ($\Leftrightarrow w=0$). The introduction of technique β or of γ, though satisfies the cost minimization criterion, does not lead to an increase in w (or in r) precisely because this has already been determined by the basic process and precisely because the

21. At this point the reader can easily understand how, within the framework of reducible techniques, the equations (10) and the condition (11) are modified.

aforementioned assumption has been introduced to the model. Thus, the change in the relative price of the commodities necessarily appears as the sole consequence of the change in the production method of the non-basic commodity, as shown in *Diagram 4* below:

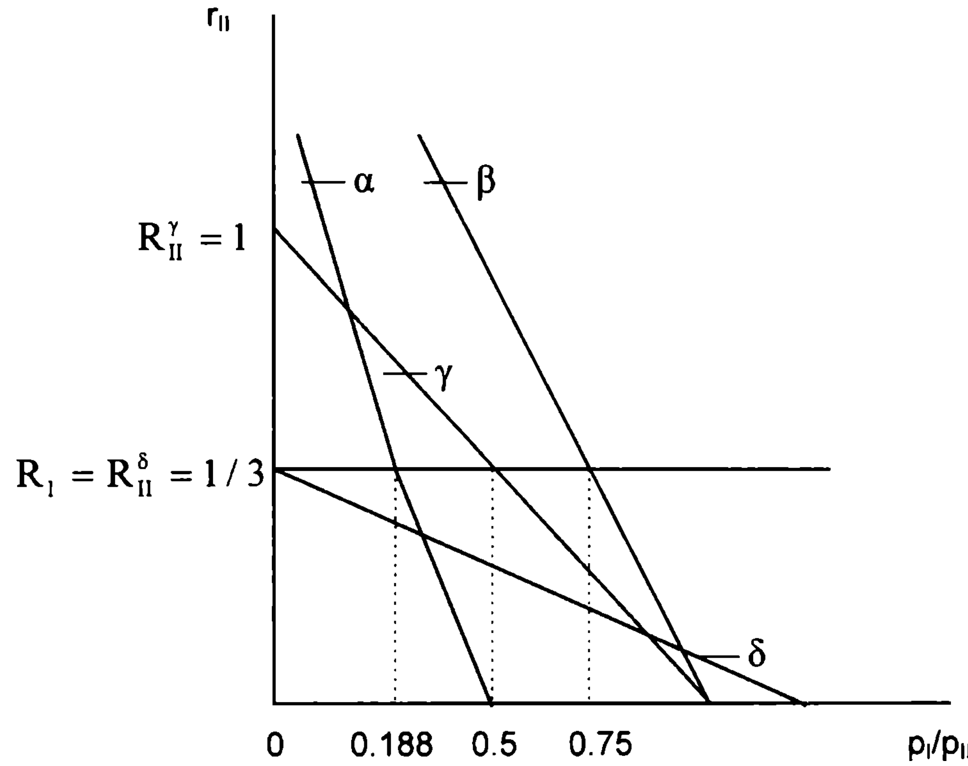

Diagram 4

It could therefore be said that the aforementioned assumption does not only lead to the appearance of zero prices or prices which tend to infinity[22], but also to this "strange" property of reducible techniques.

b) The classification of techniques depends on the composition of the normalization commodity, because by changing the composition of the normalization commodity, the economically significant interval of the rate of profit changes.

22. See Sraffa (1960), §35, n.1, §39, n.1, Appendix B and, for example, Pasinetti (1985), pp. 113, 223-24. As is in any case apparent from *Diagram 4*, to technique δ (γ) corresponds a uniform rate of profit equal to R_{II}=1/3(=1) and a semi-positive prices vector, if and only if the price of the basic commodity becomes equal to zero. Naturally, this can only be possible if the normalization commodity includes the non-basic commodity.

III. Conclusion

The present paper showed that with respect to the issue of the choice of technique in Neo-Ricardian single production models, certain problems appear which arise:

a) From the fact that the so-called comparison of techniques in terms of the outer envelope of the w-r curves in reality constitutes a comparison of those subsystems à la Fel'dman/Sraffa which produce the normalization commodity as their net product. This fact has no consequences for the case of the usual single production of basic commodities, but is of particular importance both in the case of so-called «merger production» and in the case of reducible techniques. Moreover, in this latter case, it explains why the application of the two available criteria for choosing the technique do not always bring the same result.

b) From the assumption of the existence of a uniform rate of profit and a uniform nominal wage rate for basic and non-basic processes. For a nominal wage rate equal to zero, this assumption –when it does not lead to prices for certain commodities which are not economically significant– leads to a rate of profit which appears as being independent of the technique used by the non-basic processes. Thus, to the said combination of nominal wage rate/rate of profit, techniques may correspond which are characterised by different prices vectors.

Naturally, these are not problems which pertain to economic reality per se, but to its representation. Therefore, their existence –without prejudicing the Neo-Ricardian criticism of the traditional Neo-Classical theory– illustrates the inner limits of Neo-Ricardian models as cohesive representations of economic reality.

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