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Κριτικά Σημειώματα - Critical Notes

Dynamic and Static Values:
A Counter-reply to Duménil and Lévy

by
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Gérard Duménil and Dominique Lévy (D&L, from now onwards) have answered my (partial) rejoinder on what they call the productivity paradox in sequential values and on the formal theory of values of jointly produced commodities in a way that contributes, in my view, to obscure rather then clarify issues.[1] Hereby, I thus wish to stress some basic point.

Productivity

D&L assert that

> Giussani, first, confirms our finding that economizing on labor or material inputs can lead, in the sequential-value approach, to a *rise* in the value of the good produced. (pp.179-180 of Duménil and Lévy, 1999)

Subsequently, they add

> Giussani discusses the conditions on initial values and on the forms of technical change to which the paradox is subject. He does not question the existence of this paradox, which is only observed in particular cases. Giussani's discussion could be helpful to Freeman, if it led to new assumptions that prevent the productivity paradox. Giussani suggests one such condition: «[...] that the only rational choice for the initial condition of the dynamic value equation would be is the solution of the simultaneous systems for $t=0$», i.e. traditional values. Will Freeman adopt it? This would be equivalent to saying that the relevance of the sequential-value approach is limited to the vicinity of the traditional solution. (p.180)

1. See: Duménil and Lévy (1997) and (1999), Giussani (1998).

The reader will remind that D&L by setting initial conditions (commodity unit values for $t=0$) for the value difference equation at such a level that the initial productivity is higher that the subsequent one are able to claim they have find out a paradox in the sequential treatment of values (the "productivity paradox"). By the same token, an astronomer by setting the initial position of the Moon in the differential equation for her circular movement *outside* the path of her orbit could pretend of having found a strange "Moon paradox" made of a very bizarre kind of *initial* movement of the planet.

D&L *do not* justify their (arbitrary) choice of an initial relatively higher productivity, or any other choice whatsoever for the unavoidable initial conditions. But, when dealing with dynamical systems one always has to do with the problem of setting some initial conditions and, thus, of justifying them[2]. I have indeed justified the choice I propose (in the reply D&L avoid mentioning my rationale) quite independently of the existence the productivity paradox, an existence that I strongly deny – actually I have not confirmed just anything of the sort pretended by D&L. In reality, D&L chose an *ad hoc* initial condition just in order to show the productivity paradox, and not for other reasons, but, since all formal natural science is built upon differential systems, the possibility of producing all kinds of (empty) paradoxes such as the alleged productivity paradox is virtually limitless.

Stating, as D&L do, that choosing for the initial values (at $t=0$) the solutions of the corresponding simultaneous equations amounts to acknowledging that "the validity of the sequential-value approach is limited to the vicinity of the traditional solution" is mere nonsense. What D&L call the "traditional solution" and the dynamical system of equations for commodity values are trivially *the same* if technical coefficients are kept constant over time, in which case the problem does not simply arise out. When technical coefficients are made change over time the "traditional solution" no longer exists -both for values *and* prices of production- while the sequential systems yield solutions that *limitlessly* must go away from the chosen initial values (the traditional simultaneous solutions for $t=0$) as $t \to \infty$. There is no "vicinity" to any possible kind of predefined solution at all since the very same concept of

2. If there is no conceivable choice for the initial conditions of a dynamic system, and if the choice of the initial conditions is irrelevant as to the subsequent whole dynamical path of the variables, then the *initial influence* (i.e. the local movement engendered by the choice of the initial conditions *before* the variables reach their otherwise invariant path) is equally irrelevant and cannot be used as an argument.

"vicinity" implies one or more equilibrium states (fixed points) which simply do not exist if technical coefficients change continuously as (known) functions of time. Thus, the assertion of D&L is simply deprived of any content: they probably exchange a continuous unidirectional (irreversible) process of change for some type of cyclical motion as is in most models of cyclical growth, having perhaps in mind something like the Volterra-Lotka type of differential equations employed in the popular Goodwin model, which of course has absolutely nothing to do with the dynamical formalism for values and/or prices.

Values, Prices and Technical Change

D&L are ready to admit that

> The gravitation of prices under the assumption of technical change is an interesting but difficult problem. (p.181)

However

> concerning values, no gravitation is at issue. At a given point of time, values only depend on the average technology during the period. They obviously change with the technology. (p.181)

Here we have a clear *non sequitur*. From the statement "at a given point of time values only depend on the technology during the period" it does not follow at all that "values must be calculated within the standard simultaneous-algebraic framework" as D&L imply. "Depending on the given technology" is of course very far from being identically equal to saying "calculated by the simultaneous methodology". Within *both* frameworks, simultaneous *and* sequential, values of course change with changes in technology (i.e. with changes in labour and input/output coefficients) albeit in different ways. The problem is exactly that in the traditional linear algebra formalism values can only change in a very strange manner indeed. Changes in a given point of time are not and cannot be linked to changes in subsequent or prior points of time. It is a peculiar feature of the algebraic framework that the time movement of change of unit values is just a collection of discrete vectors separated by void intervals. It is a fictitious type of change since, to occur, variations in unit values at time t need variations in technology, that is in unit values of used inputs at time t *so creating a logical (and physical as well) impasse*. The bizarre circumstance is that D&L are keen to acknowledge this ... but only as far as prices and not values are concerned: according to them prices are (also) a

market phenomenon, hence requiring a sequential formalism, values are instead only another name for present technology (what means 'simultaneously calculated' as we have seen), they say. It goes without saying that D&L do not provide any foundation whatsoever for this that amounts to be just an axiom they simply choose to follow.[3] Neither they answer the crucial question: what about a simple commodity economy (with no rate of profit) able to yield technical change? One only has (changing) values (or some other, still unknown, category) upon which to base (changing) exchange values, but they can not be treated as time variables since they are just an expression stemming out of the 'present' (i.e. simultaneously determined) technology. Hence you cannot have commodity exchange values.

D&L's theory can be briefly summarized by the help of the three following points:

1. Values are simultaneous as they only belong to the technology of the present time
2. Phenomenologically observable prices are the ever changing market prices, which thus require a dynamical (maybe sequential) formalism. Market prices gravitate around production prices.
3. Sraffa's production prices are the long run equilibrium solution of the dynamical system for market prices.[4]

We have already seen about the first point. The third point is intended to be the key to the second one: nonetheless, as such it is simple illusion. Current prices change because of changes in the supply/demand ratios *and* because

3. D&L's axiom of values as mere functions of present state technology is made out of two distinct points: a) values only serve to tell us something about the technology; b) given a) then values must be calculated as solution of algebraic systems. This are just an arbitrary choice made by D&L, whose utility is very unclear if not for fostering further credit to the Sraffa-type valueless theory of production prices. Within a noncommodity economic system, thus deprived of exchange values, commodity values could not of course exist, what would prevent them from telling anything about technology. It should also be stressed that labour values cannot anyway be good functions of sectoral technologies in the most general case where the range of individual technologies within a given sector of production is wide. Point b) does not in any case follow from a) since 'present' does not mean "simultaneous' and a 'present state' always necessarily functions with elements inherited from the past.
4. On p.181 D&L say: «Giussani refers to "the claim [...] that the Sraffian theory and static (or algebraic) formalism simply capture the notion of (long run) equilibrium values or prices". This is probably the case».

changes in technology; for a long-run variable to be the gravitation path of a fluctuating magnitude it is required that it may change too, i.e. for some system of long run prices to be the gravitation axis for market prices it is needed that those price change as technology (input/output and labour coefficients) varies. Systems of the Sraffa-type simply rule out changes as they are algebraic formalism with no possible link between an alleged state of equilibrium (a given set of input/output and labour coefficients) at a given point of time and another (alleged) state of equilibrium at a subsequent point of time. To reach such type of *changing* long-run equilibrium systems it is exactly necessary to alter the Sraffa-type system by converting it from an eigenvalue/eigenvector problem into a nonhomogenous dynamical system (requiring a vector of inputs of direct labour time). Far from being "long-run equilibrium solutions for systems of oscillating market prices", the Sraffa-type system of production prices can only be conceived of as a *short run equilibrium solution of a single state system of market prices*.[5] Once again, D&L get confused by their spontaneous reference to standard dynamical systems used to model (mainly keynesian and postkeynesian) cycle theories, where the distinction between cyclical movement and fixed point solution is obviously clear cut given the inner static nature of those theories. But, it is even useless to recall that in long run analysis one has varying coefficients which become functions of time, what prevents the possibility of any fixed equilibrium. Within this framework, it is rather trivial that the TSS or other possible kinds of dynamical systems of *production* prices (that is having profit rates uniform in each point of time) cannot be considered a dynamical system *of actual prices* what I have actually never done – but just as a heuristic device to display the narrow limits of the blind ally known as (the sacred indeed) von Bortkiewicz-Sraffa-Steedman tradition and, hopefully, also to be a bridge towards a wider sphere of research, what is barred by the absurd and irritating sacredness of the above mentioned tradition.

5. Sraffa himself thought of his system of production prices *not* as an equilibrium solution of something else of a more concrete nature, but just as a heuristic device, with very little reference to real phenomena but greatly powerful to internally criticize the orthodox theory (i.e. the neoclassical production function). Ironically enough, it has been a great number of marxists, quite often the readiest and quickest in society to get the latest word of cultural fashion as most of left people are, who have extended the Sraffian construction much beyond its intended limits.

Some econometrics?

D&L's comment in a very strange way on a simple example of change of values as produced by changes in the input/output coefficient that I inserted in my rejoinder to them to show the very high degree of elasticity of changes in simultaneous values as functions of changes in technology:

> Giussani performs some econometrics to illustrate the fact that values in the traditional definition may vary as function of technology. Indeed, if technology is modified randomly in period $t+1$ with no relationship to its features in t, values vary considerably. (In a one-good model, if the quantity of the good required for the production of 1 unit of the good varies between 0 and 1, the value of the good varies between the labour coefficient and infinity). The consequences of Giussani's exercise are unclear. (p.181)

The exercise, mentioned by D&L, is totally different from what they describe. I have compared the magnitudes of changes in traditional simultaneous values with the magnitudes of changes in sequential values as produced by the same variation in technology, that is to say in the input/output coefficient of a one-good system of production. To this aim I have let the input/output coefficient vary within the interval (0,1) according to a uniformly distributed random function[6]. I, of course, did not want to show that values, be them simultaneous, sequential or whatever, changed as effects of variations in technology, which thing is trivially evident, but only to grasp the *quantitative extent of the difference* between the change in simultaneous values and the change in sequential values in response to changes in input-output coefficients. With a starting coefficient of direct labour time $L_0 = 10$ varying at a constant time rate of change $gL = -0.1$, I did not perform any "econometrics", as D&L strangely assert, rather calculated a very standard table of elementary descriptive statistics, just to compare the time behaviours (going from $t=0$ to $t=20$) of the two types of values, simultaneous and sequential, calculated in each point of time from the same coefficients. The result was the table 1, that I reproduce here below, through which it was discernible the extremely more elastic movement of the traditionally calculated (SSM) values in relation to that of the sequential (TSS) values.

6. See Giussani (1998).

Statistics	*TSS*	*SSM*	*TSS/SSM*
Coeff. of Variation %	31.32324	111.5033	0.280918
Average	8.977639	15.61463	0.57495
Standard Error	0.613648	3.799354	0.161514
Median	8.868042	10.81315	0.820116
Standard Deviation	2.812088	17.41083	0.161514
Variance	7.907837	303.1369	0.026087
Kurtosis	-0.48787	4.145489	-0.11769
Asymmetry	-0.17595	2.198409	-0.08004
Interval	10.5771	61.37895	0.172325
Min	3.071105	1.887836	1.626786
Max	13.64821	63.26679	0.215725
Sum	188.5304	327.9073	0.57495

Table 1
Summary of Descriptive Statistics of TSS and SSM Variables as in Graphic 1

Why SSM values respond so violently (see their coefficient of variation and variance) to variations in the input-output coefficients? Precisely because changes at a given point of time $t+1$ are in no way related to changes at t, each system being an isolated self-sufficient (simultaneous) construction with no possible evolution over time. Very mistakenly indeed, D&L impute this so highly erratic movement of SSM values to the random function ruling technological change within (0,1). Nonetheless, TSS values are submitted to the same random function but exhibit much less changes: being presupposed by the choice of a random function to make the input-output coefficient vary, the erraticity of changes in values cannot be what is at stake here. What is shown by means of this simulation is not the existence of a kind of erratic movement of values but the *relative extent* of an already assumed erratic change: a degree of elasticity which is not due to randomness but to the formalism employed, the random function ruling changes in the input-output coefficient within (0,1) being just a device to lay bare this property of the simultaneous calculus as different from the sequential formalism. Hence, it seems to me that the content of the exercise should be clear enough; as to the "consequences" of it, anyone is of course free of drawing whatever consequence s/he likes best.

Joint Production

In their reply D&L choose to avoid the issue of joint production by stating that I have not dealt with the existence of negative values and of increasing values for old machines in sequential systems with fixed capital -their "second paradox". Since systems with fixed capital are just a particular case of joint production systems, in the final section of my rejoinder I had preferred to talk of the general case, graver than the particular one, something that D&L seems simply to ignore.

Joint production is a another black hole where it is clearly shown that the famed internal consistency of the sraffian theory is nothing more than a myth or a metropolitan tale, something that has already be proved several times and in a number of various ways. But there is more to add. So far, joint production has been and still is an unsolvable problem for *all kinds* of formal theory of (production) prices and values but with some difference among them, this is what I have argued in my rejoinder, leaving of course no room to 'absolve' or 'defend' the sequential approach. To show this I have presented a very simple double, TSS and SSM, system of joint values with changing coefficients. After a given time interval of evolution, the traditional SSM values become negative while the corresponding TSS value stay positive forever, so showing that even if you start from a simultaneous joint production system yielding a positive vector of values you can never be certain about the exit after a period of changes, on the very contrary you can be absolutely certain that your system will sooner or later fatally enter a negative price area.

In the rejoinder to D&L, I had also re-proposed the usual known counterargument to the Steedman-invented-paradox of negative values in joint production[7] that D&L simply uncritically reiterate and extend to the sequential case[8], i.e. that Steedman's (and D&L's) values are not Marx's values but individual values arbitrarily calculated as (social) average values – that I called spurious or false values. Once the Steedman-D&L paradox is shown to be fruit of arbitrariness, values in joint production far from being negative become indeterminate leaving room for possible extensions or some kind of

7. *Medice, cura te ipsum!*

8. In another previous paper (see Duménil and Lévy, 1989) D&L do criticize the Steedman paradox of negative values for the simultaneous case: nonetheless in their reply they accept and repeat it for the nonsimultaneous formalism: it is evident that *quod licet Iovis non licet bovis.*

modification in the theory and/or the corresponding formalism: something that is absolutely and hopelessly excluded as far as the basic sraffian theory of production prices is concerned which is simply inconsistent, notwithstanding this theory being an acceptable and respectable form of academic opposition.

References

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