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A basic difference between the orthodox and the marxian investigation of the falling rate of profit

by
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I. Introduction

The present short paper does not treat *directly* the logical cohesion, the significance and the various interpretations of the «marxian law of the tendency of the rate of profit to fall». We only focus on a basic difference between the orthodox and the marxian investigation of the falling rate of profit. This difference has, as a rule, been underestimated in the bibliography (Stamatis (1977), Ch. V (especially, p. 226) and (1984), pp. 287-88, constitutes a remarkable exception), which has lead to wrong approaches to the «marxian law». We note, lastly, that with the term «orthodox investigation» we mainly mean «neoclassical investigation» (e.g. the one that Robinson (1966) summarises at the beginning of Ch. V), but we include in it also every investigation based inconsiderately on the so-called issue of the «choice of technique» (as this has been posed by Whewell (1831), von Bortkiewicz (1906-7) and has been further developed since Sraffa (1960) and after).

In *Part II* we describe a simple one-commodity model for the investigation of the tendency of the profit rate. In *Part III* we present the relevant orthodox investigation, while in *Part IV* we present the relevant marxian and we compare it with the orthodox one. Finally, in *Part V* we summarise the conclusions of the analysis.

II. The simple model

As it is easily proven, in a one-period, one-commodity, fixed-proportions-technology model (the standard one-sector «circulating-capital» model, see, e.g. Bidard (1991), Ch. I) the following hold:

$$r = (1-\alpha-w\beta)/\alpha = (\pi_L - w)/T = (m'\pi_C))/(1+m') = g/s \quad (1)$$

$$r|_{w=0} = (1-\alpha)/\alpha = \pi_C \quad (2)$$

$$r|_{r=0} = (1-\beta)/\alpha = \pi_L \quad (3)$$

$$T = (C/L) = [(\alpha X)/(\beta X)] = \pi_L/\pi_C \quad (4)$$

$$m' = (\pi_L/w) - 1 \quad (5)$$

where r the rate of profit, $\alpha(<1)$ the capital-output ratio, $\beta(>0)$ the labour-output ratio, w the real wage rate (by *assumption* wages are paid at the end of the production period), π_L the labour productivity, T the technical composition of capital, m′ the rate of surplus value, π_C the capital productivity (or net output-capital ratio), g the rate of growth of capital, s $(0<s<1)$ the propensity to save out of profits (by *assumption* workers do not save and capitalists save and invest a fraction of their profits), C the amount of capital, L the amount of employment and X the gross output of the system.

Let us, now, assume that:

A.1. The real wage rate w $(<\pi_L)$ consists the exogenously given variable of the system.

A.2. The real wage rate w consists a strictly increasing function of time t: $w=w(t)$.

Thus, as it arises from the equations (1) to (5), the following hold (with $\dot{z} = (dz/dt)/z$ we symbolise the growth rate of z(t)):

$$\dot{m}' \gtreqless 0 \;\Leftrightarrow\; \dot{\pi}_L \gtreqless \dot{w} \quad (6)$$

$$\dot{\pi}_C \gtreqless 0 \;\Leftrightarrow\; \dot{\pi}_L \gtreqless \dot{T} \quad (7)$$

$$\{\dot{m}' > 0 \text{ and } \dot{\pi}_C > 0\} \;\Rightarrow\; \{\dot{E} \gtreqless 0 \text{ and } \dot{r} > 0\} \quad (8)$$

$$\{\dot{m}' > 0 \text{ and } \dot{\pi}_C = 0\} \;\Rightarrow\; \{\dot{E} > 0 \text{ and } \dot{r} > 0\} \quad (9)$$

$$\{\dot{m}' > 0 \text{ and } \dot{\pi}_C < 0\} \;\Rightarrow\; \{\dot{E} > 0 \text{ and } \dot{r} \gtreqless 0\} \quad (10)$$

where E the value composition of capital:

$$E = C/(wL) = T/w = (1+m')/\pi_C = (\alpha/\beta)/w \quad (11)$$

III. The orthodox investigation

The orthodox investigation is based on the introduction of the following assumption:

A.3. The technical possibilities of the system are represented by a continuously differentiable, linear homogeneous, production function.

For the sake of simplicity, let us accept that the abovementioned production function takes the following «intensive form»:

$$\pi_L = T^e \Rightarrow$$

$$\pi_C = T^{e-1} \quad (12)$$

$$\beta = [(1-\alpha)/\alpha^e]^f \quad (12a)$$

where e is constant (0<e<1) and f=1/(1–e). As it is known, from the «cost minimization conditions» we derive:

$$w = (1-e)\pi_L = (1-e)T^e \quad (13)$$

$$r = e\pi_C = eT^{e-1} \quad (14)$$

$$(dw/dr) = -T \quad (15)$$

Consequently, the orthodox investigation comes to the following results:

1. $\dot{m}' = 0$, because $\dot{\pi}_L = \dot{w}$. Concretely it holds:

$$m' = (rC)/(wL) = -(r/w)(dw/dr) = e/(1-e) \quad (16)$$

2. $\dot{r} < 0$, because $\dot{m}' = 0$ *and* $\dot{E} > 0$. Concretely it holds:

$$-\dot{r} = \dot{E} = -\dot{\pi}_C = (1-e)\dot{T} = (\dot{w}/m') = \dot{\alpha}/(1-\alpha) \quad (17)$$

3. If $\dot{L} = 0$, then s is endogenously determined:

$$g = (\dot{w}/e) \Rightarrow s = g/(e\pi_C) \quad (18)$$

and if $\dot{w}$ is constant:

$$\dot{s} = -\dot{r} \quad (19)$$

Generally (if $\dot{L} = 0$) it holds:

$$\dot{w} = \dot{Y} = \dot{U} = e\dot{C} < \dot{X} < \dot{C} \quad (20)$$

where Y=(1–α)X the net product and U=rC the surplus product of the system.

IV. The marxian investigation

In contrast with what is, as a rule, maintained (e.g. Robinson (1966), Ch. V, Sweezy (1970), Ch. VI, Abraham-Frois (1991), pp. 478-82, Bidard (1991), pp. 66-9), it can be proved (Stamatis (1977), Ch. II, III, IV, §1-2) that Marx introduces *initially* the following «assumptions»[1] (A.1. and A.2. are given):

A.3.* $\{\dot{T} > \dot{\pi}_L > 0, \forall t\}$ $(\Rightarrow \{\dot{\pi}_C < 0, \forall t\})$. For the sake of simplicity, let us accept that:

$$\pi_L(t) = [T(t)]^e \;\Rightarrow\; Y(t) = [C(t)]^e [L(t)]^{1-e} \;\Rightarrow$$

$$\pi_C(t) = [T(t)]^{e-1} \tag{12*}$$

$$\beta(t) = \{[1-\alpha(t)]/[\alpha(t)^e\}^f \tag{12a*}$$

where e is constant (0<e<1) and f=1/(1–e).

A.4.* $\{\dot{m}' > 0, \forall t\}$. Thus, from A.2., A.3.*, it follows:

$$\dot{T} > \dot{\pi}_L > \dot{w} > 0, \forall t \tag{21}$$

Given A.2., A.4.* the assumption A.3.* constitutes the *necessary condition* of the appearance of a strictly decreasing r (see conditions (8), (9), 10)). Indeed, if *for example* we introduce the function:

$$\pi_L = w^h \tag{22}$$

where h is constant (h>1) and w>1 (⇒ m'>0), then r will be equal to:

$$r = (w^h - w)/w^i, \quad i = h/e \tag{23}$$

and its evolution will be determined by w(t), e and h.

1. As we are going to see later, they are assumptions that have economic meaning and, mainly, they are not arbitrary assumptions-axioms. In fact (Stamatis (1977), Ch. II, III, VIII), the point is for those basic conclusions, to which the marxian study of the technical base and of the operation of the developed capitalist mode of production came. Finally, we note that Steedman (1977), Ch. 9, determines those conditions under which «a rising rate of surplus value, a rising value composition of capital and a falling rate of profit are mutually consistent» (p. 117). Thus, he is excluded by those who claim that Marx was based on the assumption of a constant m'. However, he does not achieve to conceive the cohesion and the importance of the marxian investigation.

The *necessary and sufficient condition* of the appearance of a strictly decreasing r is the following (it derives from (1), (4), (5)):

$$\dot{r} < 0 \;\Leftrightarrow\; [(\dot{\pi}_L - \dot{w})/m'] < (\dot{T} - \dot{\pi}_L) \tag{24}$$

If we introduce (12*) and symbolize with $\overline{m}'$ the rate of surplus value associated with the orthodox investigation (see (16)), then the condition (24) is written:

$$\dot{r} < 0 \;\Leftrightarrow\; \dot{\pi}_L[1-(m'/\overline{m}')] < \dot{w} \tag{25}$$

From the condition (25) we conclude that: a) if $m' \geq \overline{m}'$, then r must fall, b) if $m' < \overline{m}'$, then r may decrease and c) if $\dot{w}_L, \dot{\pi}_L$ are constant, then r (if it does not strictly decrease) it will begin, at one time, to decrease, because m′ will take that value, which ensures the validity of the condition (25) (*Example.* If $e=0.99$ ($\Rightarrow \overline{m}'=99$), $\dot{w}=0.05$, $\dot{\pi}_L=0.1$, then (25) is satisfied for $m'(t)>49.5$. Thus, if $m'(0)=1$, r(t) takes its maximum value at $t \cong 64.58$).

We have already proved that the system of assumptions A.1., A.2., A.3.*, A.4.* does not exclude, but it also does not guarantee the appearance of a strictly decreasing[2] r. Is this system, however, cohesive? If it is cohesive, then which is its economic meaning?

It is obvious that if Marx provides A.3.* with exactly the same content that A.3. has, then the marxian investigation is not cohesive. The assumption A.3. entails, through the «cost minimization conditions», an unchangeable m′ and thus A.4.* is in contradiction to A.3.*. So, we are lead to two possibilities: a) A.3.* and A.3. are exactly the same. Therefore, the introduction of A.4.* violates the «cost minimization conditions» and, thus, the marxian investigation does not have an economic meaning. b) A.3.* and A.3. are not exactly the same (despite their algebraic identity). If, however, they are not the same (and as we are going to show, they are not), which is the meaning of the marxian system?

Marx also introduces an additional assumption (see Stamatis (1977), Ch. V, §3-4):

A.5.* $\{\dot{s}>0, \forall t\} \rightarrow \{\dot{m}'>0, \forall t\}$. Namely, the strict increase in the rate of surplus value *presupposes* the strict increase in the propensity to save out of profits. From this assumption and from (1) it obviously follows that r definitely

2. See also Stamatis (1976), pp. 107-9, (1977), Ch. IV, §3 and Steedman (1977), pp. 126-7.

falls when $\dot{g} \leq 0$, while r may fall when $\dot{g} > 0$ (in the second case it falls when, and only when, $\dot{s} > \dot{g}$).

The marxian system of «assumptions» has a concrete economic content, because it is reconstructed as follows:

1. Workers are in a position to increase strictly w (A.1., A.2.).

2. Through the introduction of new production methods, capitalists are in a position to grow π_L faster than w (A.4.*). If these new methods are not characterised by a lower π_C, then r rises (see (8), (9), (10)).

3. *However,* every given rate increase in π_L *presupposes* a higher rate increase in T (A.3.*. According Marx, this consists a fundamental feature of the technical base of the developed capitalist mode of production. Exactly for this reason he calls the production methods that consist the said technical base «spezifisch kapitalistische Produktionsmethoden» and the corresponding mode of production «spezifisch kapitalistische Produktionsweise»[3]). Thus, the successive introduction of new production methods leads to the successive decrease in π_C (and possibly in r). Finally, the capitalists *do not introduce always* such a new production method. If w is increased from w_0 to w_1 and if we symbolise with $r^*(w_1)$ the profit rate that will correspond to the new method, with $r(w_1)$ the profit rate that will correspond to the previous method and with $r(w_0)$ the initial profit rate, then the new method is introduced when, and only when, it holds:

$$r^*(w_1) \geq r(w_1) \tag{26}$$

and irrespectively of if it holds (see, e.g. *Diagram 1,* where[4] $r^*(w_1) < r(w_0)$):

$$r^*(w_1) \gtreqless r(w_0) \tag{27}$$

3. For this important issue: Stamatis (1977), Ch. II, III. Empirical research indicates (see the results that Stamatis (1977), Ch. VIII, cites and elaborates) that since the first decades of the 20th century and after, the technical base of the capitalist mode of production is characterised by the relation: $0 < \dot{T} \leq \dot{\pi}_L$. Therefore, since the first decades of the 20th century and after the necessary condition of the appearance of a falling profit rate has vanisched.

4. Thus the transition of the system from the point (w_0, $r(w_0)$) to the point (w_1, $r^*(w_1)$) is *logically* separated into two transitions: the transition from the point (w_0, $r(w_0)$) to the point (w_1, $r(w_1)$) (along which the production method does not change) and the transition from the point (w_1, $r(w_1)$) to the point (w_1, $r^*(w_1)$) (which is ruled by the well-known Okishio's theorem (1961)).

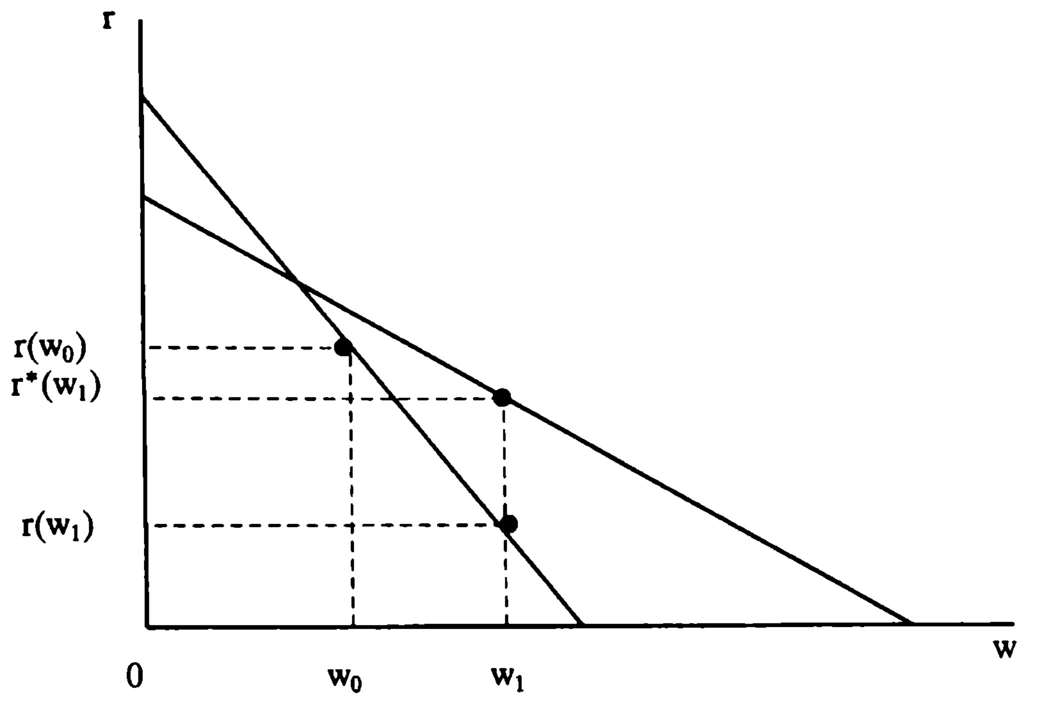

Diagram 1

When $\dot{r} < 0$, then (26) can be written as:

$$[(\partial r / \partial w)(dw/dt)(1/r)] \leq \dot{r} < 0 \quad (26a)$$

From (1), (5) and (26a) we obtain:

$$[(-\dot{w})/m'] \leq \{[(\dot{\pi}_L - \dot{w})/m'] + (\dot{\pi}_L - \dot{T})\} \Rightarrow$$

$$\dot{T} \leq [\dot{\pi}_L(\pi_C/r)] \quad (26b)$$

or (equivalently)

$$(dw/dr) \leq -T \quad (26c)$$

If we introduce (12*) and symbolise with $\bar{\pi}_L$ the labour productivity associated with the orthodox investigation (see (13)), then the condition (26b) is written:

$$\{\bar{m}' \geq m'\} \Leftrightarrow \{\bar{\pi}_L \geq \pi_L\} \quad (26d)$$

Consequently, A.1., A.2., A.4.* and $\dot{r} < 0$ are –not only mutually consistent, but also– *economically significant,* when, and only when (Stamatis (1977), disregards this point):

$$\dot{\pi}_L < \dot{T} < [\dot{\pi}_L(\pi_C/r)] \quad (A.3.^{**})$$

and the necessary and sufficient condition (24) ((25)) of the fall of r, under A.1., A.2., A.3.*, A.4.*, must be written as:

$$\dot{r} < 0 \;\Leftrightarrow\; [(\dot{\pi}_L - \dot{w})/m'] < (\dot{T} - \dot{\pi}_L) < (\dot{\pi}_L/m') \qquad (24a)$$

$$\dot{r} < 0 \;\Leftrightarrow\; (\dot{\pi}_L - \dot{w}) < [\dot{\pi}_L(m'/\overline{m}')] < \dot{\pi}_L \qquad (25a)$$

Thus, first, we may say that the fall of r is «specifically marxian», when, and only when, A.3.** holds (a «non-specifically marxian» fall of r is either «orthodox» or «trivial», i.e., connected with an unchangeable production method) and, second, we must review the numerical *example* (and the comments), which has been given after the derivation of condition (25). The later is depicted in the following *Diagram 1a* (where: i) e=0.99, $\dot{w} = 0.05$, $\dot{\pi}_L = 0.1$, m′(0)=1, ii) $m'(t) = \overline{m}' = 99$, at $t \cong 78.24$, iii) $\bar{r}(t)$ the profit rate associated with the orthodox investigation, iv) the «specifically marxian» fall of r appears in the interval 64.58<t<78.24 and the «orthodox» one for t ≥ 78.24 (therefore, for t ≥ 78.24, it holds: $\dot{w} = \dot{\pi}_L = 0.05$) and v) the dashed extension of r(t) does not really exist):

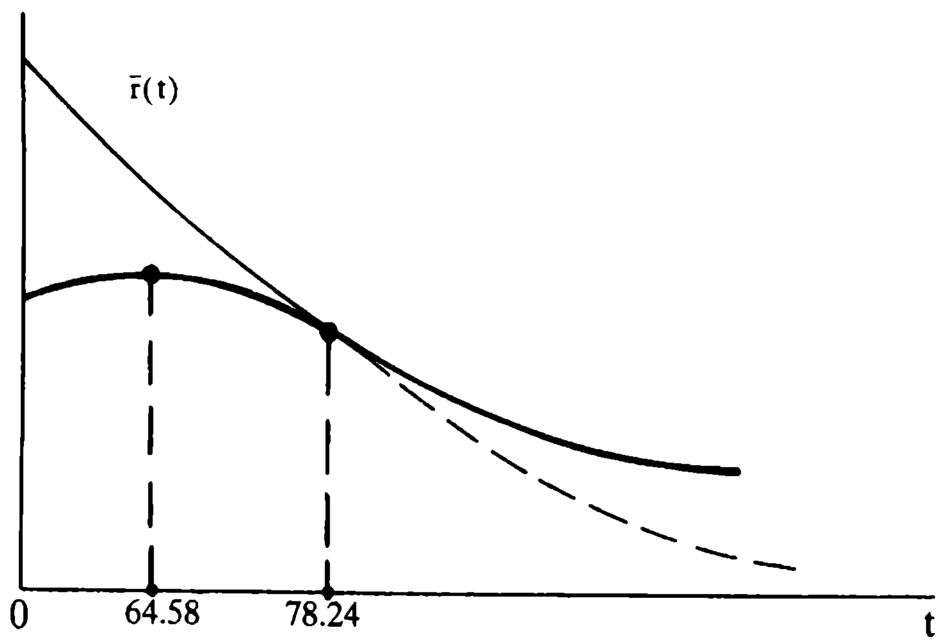

Diagram 1a

4. Capitalists are in a position to grow π_L faster than w, only if they strictly increase s (A.5.*). Namely, only if they strictly increase s, they achieve these increase in T, which ensure the strictly increase in m′.

Consequently, the marxian investigation proves that, *when* e.g. $\dot{g} > 0$, the fall of r:

a) Is due to the abovementioned *point 3,* provoked by the abovementioned *point 1* and contains: $\dot{s} > \dot{g} > 0 < \dot{m}'$, $\forall t$ (*point 4 and* 2).

b) Has the following meaning: only through a strictly increasing s the capitalists can partially reverse the decreases in r, which are provoked by a strictly increasing w.

c) Expresses the existence of a contradiction between the capitalists' *objective* (: strict increase in r) and the corresponding *mean* they have in their availability (: introduction –by means of capital accumulation– of new methods, which increase π_L, but reduce π_C). It is a contradiction, which activates in every increase in w.

d) If $\dot{L} = 0$, contains (as it easily proven) the following relations (compare with (20)):

$$\dot{w} < \dot{Y} < \dot{U} < \dot{C} < \dot{I} \tag{28}$$

$$\dot{Y} < \dot{X} < \dot{C} < [\dot{Y}(Y/U)],\ \dot{X} \gtrless \dot{U} \tag{28a}$$

where I the net investment of the system.

We have proven that the marxian investigation is cohesive and has an economic meaning. It is left to determine explicitly the relations between the marxian and the orthodox investigation. So, we summarise the fundamental similarities and their difference based on the previous analysis:

Similarities: a) The transition to a new production method is «rational». This is expressed with (13) in the orthodox investigation and with (26) in the marxian. b) The transition to a new production method is realized by means of the capital accumulation[5]. c) A higher value of π_L is associated with a lower value of π_C.

Difference: In the orthodox investigation the set of production methods is *a priori* given. A.3. entails the existence of infinite r-w lines, which are a priori given and whose the relative position is predetermined. The system always moves along the outer envelope of the r-w lines («profit-wage frontier») and

5. This holds for the investigation of *Part III* and not for every investigation, which is based on the issue of the «choice of technique».

thus: a) $\dot{r} < 0, \forall t, b$, b) s is endogenously and unambiguously determined, while we may have $\dot{s} > 0$. In the marxian investigation the A.3.* *does not* imply the existence of infinite r-w lines, which are a priori given. A.3.* and A.4.* mean that at a given moment of time one, and only one, r-w line exists, of which, however, the relative position (with respect to the previous and next r-w line) is predetermined. Exactly for this reason A.3.* and A.4.* do not ensure that $\dot{r} < 0$ (see, e.g. *Diagram 2,* where $\dot{r} > 0$ or $\dot{r} = 0$):

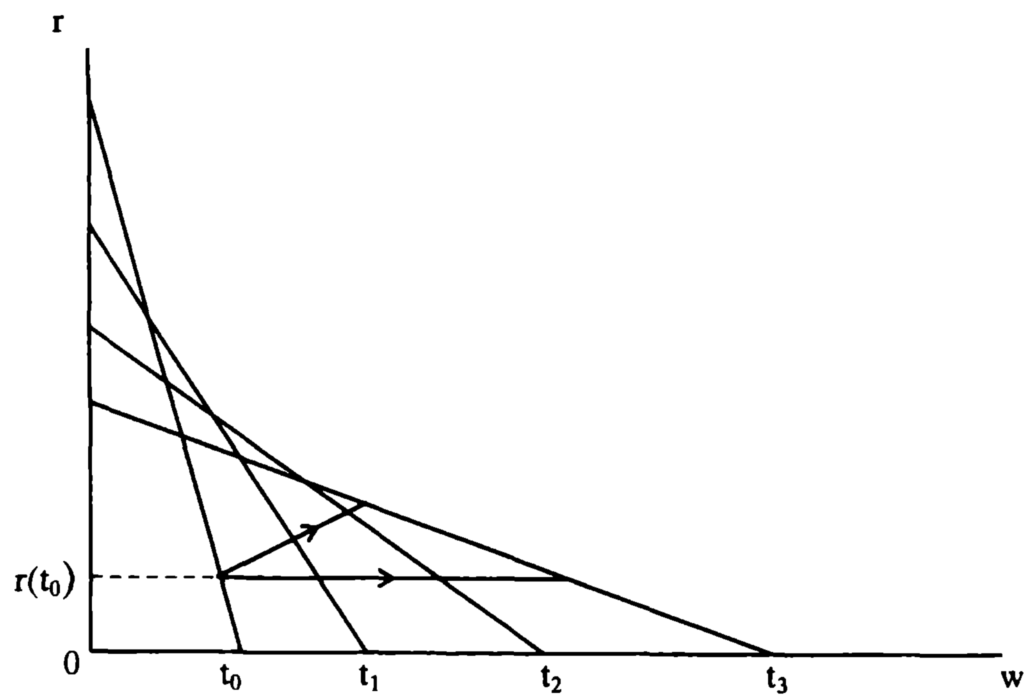

Diagram 2

Also, A.3.*, A.4.*, A.5.* mean that: a) the production method (or the r-w line) that exists at a given moment of time it results, by means of capital accumulation, from the production methods (or the r-w lines) that existed at previous moments of time, b) this accumulation is characterised by the strict increase in s and c) the evolution of s must lead to $\dot{m}' > 0, \forall t$.

Example. Let us assume that (we introduce t as a discrete variable):
i) w(t=0)=10, w(1)=11, w(2)=12.10, w(3)=14.52, w(4)=15.10,
ii) L(t)=1, iii) $Y=C^{0.8}$.

If C(0)=100, s(0)=0.50, s(1)=0.70, s(2)=0.90, s(3)=0.987, then the «marxian evolution» of the system is depicted in *Table 1* ($\tilde{r}$ sympolises the evolution of r for[6] s(t)=1, while s′ symbolises those values of s, which entail an

6. Thus, we can see that (in this *example*) even if capitalists accumulated in each period all

unchangeable m′ between two successive periods of the evolution that is already depicted in the first four rows of the *Table 1* (s(t)>s′, ∀t)).

t	0	1	2	3	4
C	100	114.91	138.35	173.92	220.76
r	0.298	0.291	0.286	0.273	0.271
m′	2.981	3.045	3.266	3.269	3.968
α	0.715	0.721	0.728	0.737	0.746
$\tilde{r}$	0.298	0.293	0.287	0.274	0.270
s′	0.424	0.434	0.897	0.184	–

Table 1

On the contrary the «orthodox evolution» of the system is depicted in *Table 2* (the value C(0) ≅ 132.957 is *endogenously* determined).

t	0	1	2	3	4
C	132.957	149.780	168.731	211.919	222.553
r	0.301	0.294	0.287	0.274	0.271
m′	4.000	4.000	4.000	4.000	4.000
α	0.727	0.732	0.736	0.745	0.747
s	0.421	0.431	0.892	0.183	–

Table 2

Lastly, it is clear that A.1., A.2., A.3.*, A.4.*, A.5.*, do not ensure that $\dot{r} < 0$. In our point of view, however, this does not invalidate the contribution of the marxian investigation. Marx investigated extensively (logically and historically) the technical base of the developed capitalist mode of production (namely, the «large-scale mechanized industry») and resulted in the following conclusions: a) The said technical base is characterised by the existence of production methods, which increase π_L and decrease π_C. b) Exactly for this

their profits, r would decrease. Lastly, $r(4) > \tilde{r}(4)$ arises, precisely because the comparatively higher decrease in π_C negates (gradually) the comparatively higher increase in π_L.

reason, it is absolutely possible for the strict increase of m′ to be expressed by a strictly decreasing r. c) Each fall of r is reduced to the abovementioned feature of the technical base of the developed capitalist mode of production and contains a strictly increasing s. d) Each fall of r expresses a contradiction between the capitalists' objective and the mean they have in their availability.

V. Conclusions

Four conclusions are deduced from the preceding exposition:

1. The marxian investigation is cohesive and has economic meaning.
2. The orthodox investigation and the marxian have a basic difference. The marxian investigation is founded on the logical and historical (and not on the abstract and axiomatic) study of its object. Precisely for this reason it *ends* in the map of the real evolution of the system.
3. The «marxist» view that the capitalist mode of production is *generally* characterised by a falling rate of profit (or that the marxian investigation of the falling rate of profit is directly connected with the issue of the economic crises) and the prevailing view that the «marxian law» arises from simplificated or/and thoughtless assumptions, are wrong[7].
4. The well-known position that the marxian investigation has failed because it was based on the wrong (indeed) solution that Marx gave to the so-called «transformation problem» is obviously out of the subject (of course in a model with heterogeneous capital we have to take into account the so-called «Wicksell effects»[8]). Moreover, it can be proved (Stamatis (1979), Ch. II, (1984), Ch. I and IV, Mariolis (1998), (1998a)) that the famous independence of the profit rate from the production conditions of the «non-basic» and the «luxury» commodities, is the result of misleading algebraic manipulations.

7. See also Stamatis (1977), Ch. VIII, §4, IX-XI and Van Parijs (1980), pp. 7-9.
8. For the thorough investigation of the «price Wicksell effects», see Stamatis (1988), pp. 85-91, Mariolis / Stamatis (1997).

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