POLITICAL ECONOMY - 4 - SPRING 1999 - p.p. 5 - 29

A Reappraisal of Adaptive Expectations

by
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1. Introduction

At least since the early 1980s, the concept of rational expectations is the dominant expectational hypothesis in macroeconomic theory. On the basis of the paradigm that all economic behaviour derives from the decisions of optimizing agents, rational expectations are the natural way to model expectations since they are optimal by definition. So, the original impetus behind the idea of rational expectations was not provided by considerations that it may take a step forward towards greater realism, but a primary reason for it to become so popular was precisely because it removed the need to conduct an empirical inquiry into actual processes of the formation of expectations (cf. Simon 1984).

In this paper we wish to do some justice to simple error-learning rules and especially to adaptive expectations, the predecessor of rational expectations. Because of their apparent suboptimality, employing these mechanisms in theoretical work has become something of a heresy. We collect several types of arguments to be found in the literature which suggest that rule-of-thumb behaviour may not be so imprudent or even 'irrational' as it is usually made out to be. Our interest in a rehabilitation of adaptive expectations is connected to a desire in macroeconomics to abandon the equilibrium framework of rational equilibrium models with their excessive information requirements, and to turn to the alternative of studying dynamic process that incorporate some features of imperfect adjustments originating with, and giving rise to, economic disequilibrium. Adaptive expectations, in a pure or modified form, may be reconsidered as a tractable and convenient device of modelling the revision of expectations in this kind of economies.

The paper is organized as follows. We begin in the next section by compiling evidence from empirical investigations and laboratory experiments which implicitly or explicitly reject the rational expectations hypothesis[1]. In

1. Here and in the following, the references quoted are by no means exhaustive.

addition, we offer a largely neglected argument on a theoretical level against rational expectations, which takes into account the costs of arriving at more sophisticated forecasts. Section 3 is concerned with simple adaptive mechanisms of expectations formation and empirical studies establishing that they are a wide-spread phenomenon. The discussion in this and also in later sections places special emphasis on expectations about inflation. Theoretical support of adaptive expectations is reported in Section 4. The aim is to show that they need not be incompatible with optimizing behaviour. Section 5 examines a close relationship between adaptive expectations and extrapolative forecasts obtained from regression estimates of the prevailing trend. In a simulation experiment, both methods are tested to predict a stochastically perturbed oscillatory motion. This little exercise serves to illustrate that though being biased, in many applications the resulting forecast errors may nevertheless be reasonably tolerated.

The forecasts discussed in Sections 3 - 5 refer to the immediate future. By contrast, in Section 6 reference is made to a longer time horizon. A theoretical argument is summarized which seeks to demonstrate that in this framework delayed adjustments are the best an imperfect decision maker can do; that is, an agent whose decisions are not already perfect by hypothesis. We then consider the aspect of return-to-normal expectations. They lead us to propose a straightforward flexibilization of the speed of adjustment in adaptive expectations which may prove useful in macrodynamic modelling. Section 8 concludes.

2. Evidence Against Rational Expectations

A first issue in evaluating rational expectations is the behaviour of actual human beings and the question whether it is compatible with that hypothesis. It is an unresolved methodological problem if direct testing of the rational expectations hypothesis is a suitable and worthwhile activity, but we share the view that a theory which is said to be based on microfoundations should survive empirical testing at the level of individual units (cf. Lovell 1986, pp. 110f). On the other hand, if one follows the 'instrumentalist' methodological statement that "the only real test [of the RE hypothesis] ... is whether theories involving rationality explain observed phenomena any better than alternative theories" (Muth 1961, p. 330), then this should also mean that rational expectations cannot *a priori* claim exclusiveness.

The most immediate implication of rational expectations is that the forecast errors made by individuals are unbiased. There is ample evidence from psychological experiments denying this. For example, Alpert and Raiffa (1982) report of experimental subjects tending to be *overconfident,* which makes them take on more risk. *Overreaction* was found by Tversky and Kahneman (1982): in making inferences, too little weight was put on base rates and too much weight on new information. Andreassen and Kraus (1988) observed that individuals tend to *extrapolate* past time series, which could lead them to *chase trends.* Further references to psychological evidence on systematic judgement errors made by experimental subjects may be taken from De Long *et al.* (1991, p. 5). Concentrating on the field of cognitive and social psychology, Earl (1990) gives a survey which "suggest[s] to mainstream economists that there are gains to be had from seeking help from psychology" (p. 718; for a short discussion of still existing misperceptions about inflation see p. 747).

The phenomenon of judgement biases is not confined to laboratory studies of non-experts. Overly optimistic forecasts and, generally, overreaction on the part of professional investors on financial markets was extensively studied by De Bondt and Thaler (1985, 1987, 1990). Likewise, extrapolation is a key feature of the popular stock market models discovered by the questionnaires described in Shiller (1990). Similar results with respect to predictions of the exchange rate were obtained by Frankel and Froot (1986). A particular sort of systematic errors is the subject of Saunders (1993). He finds out that the weather in New York City has a long history of significant correlation with major Wall Street stock indexes, an effect which also appears to be robust with respect to a variety of market 'anomalies'.

Also macroeconomic time series which are far less volatile than the movements on stock and foreign exchange markets involve serious difficulties. Even the many statisticians who have long been working on the problem of seasonal adjustments of GNP, M1 and other economic indicators have not learned enough from prior experience to achieve rational forecasts: the official preliminary data on these magnitudes turn out to deviate in a systematic manner from the revised time series that eventually appear (Lovell 1986, pp. 118f). In sum, the empirical evidence is sufficiently strong to allow us to conclude that expectations are a rich and varied phenomenon that is not adequately captured by the concept of rational expectations.

Taking for granted that one cannot dismiss as peripheral the experimental and empirical results which contradict the rational expectations hypothesis, one has to address the problem if there are theoretical reasons why people violate this principle. A good introduction to what are the most relevant points, in particular with respect to the neutrality results of rational expectations models, is Friedman (1979). Here we concentrate on another argument that has not received much attention in the literature. We believe, however, that it is no less central to the issue. To begin with, recall the basic assumption of the concept of rational expectations that economic agents utilize efficiently whatever information is available and, in addition, that the information which is actually available to them is also sufficient to permit them to form their expectations as if they know the structure of the whole economic process itself (to within a set of additive white-noise disturbances). Even if this availability assumption is fulfilled, the information may easily become so complex that it has to be doubted if agents can entirely cope with the difficulties of their correct interpretation. The problem still remains if agents are capable of handling the bulk of information and transforming it into the objectively unbiased conditional expectations of the time series to be predicted.

Let us accept the assumptions about the availability and use of information, but not the accompanying one, namely, that acquiring and processing all the data is costless. The existence of such a cost, which may be called an optimization cost, has far-reaching theoretical consequences. If optimization is costly and if cheaper, sub-optimal expectations formation procedures are available, the decision maker is faced with the problem of whether or not to optimize. There would be no difficulties if this choice could itself be made optimally, with full knowledge of the costs and benefits involved, but typically the latter are *not* known in advance. Hence, in order to determine the optimal degree of optimizing, a larger optimization problem must be solved which will have its own optimization cost. It is readily seen that attempts of this kind to fold optimization cost into a conventional optimization problem will inevitably be caught in an infinite regress (cf. Conlisk 1988, pp. 214f).

In our view, the most appropriate theoretical approach in this situation is to start with behavioural heterogeneity of agents and admit optimizers, who have to pay a cost, as well as a non-optimizers, who adopt less expensive rules of thumb. Then, within a fully specified dynamic setting, suppose that the group performing better in the recent past wins some converts from the other group. Thus allowing the population composition to evolve endogenously under the evolutionary pressure of differential payoffs, the basic question is: will one group (the non-optimizers?) be completely competed away, or is there scope for coexistence even in the long-run. A problem of this kind was first addressed by Conlisk (1980); his model provides an example in which naive agents can indeed survive. The result is not confined to the specific linear economy considered there. Isolating some of the most important mechanisms involved, the analysis in Franke and Sethi (1992) shows that naive expectations (simple adaptive expectations incidentally, as in equation (2) below) survive under very general conditions. It may also be noted that this appoach of evolutionary dynamics not only argues against rational expectations, but also against the paradigm of the representative agent.

Departing from these thought experiments and moving closer to reality, we may negate altogether the existence of agents who are sufficiently competent to always react optimally (i.e., optimally if optimization costs were neglected). Instead, there are several goups of agents with different forecasting procedures, whose cost increases with the degree of sophistication. The forecasting competitions conducted by Makridakis *et al.* (1984) yielded a considerable evolutionary fitness of rule-of-thumb behaviour also under these circumstances. In their tournaments, straightforward and informationally undemanding adaptive forecasting methods regularly outperformed more complicated and informationally demanding techniques.

3. Empirical Evidence in Favour of Simple Adaptive Mechanisms

One of the simplest rules of forming expectations is the principle of adaptive expectations. Letting $x = x(t)$ be a dynamic variable and $x^e(t)$ its expected value, it says that x^e is revised upwards, but only partially so, if the actual value of x exceeds the value that has been currently expected. Correspondingly, the revision is downwards if $x < x^e$. In continuous time, the basic version of adaptive expectations is described by

$$\dot{x}^e = \beta (x - x^e) \tag{1}$$

(the dot denotes the time derivative). β is a positive constant which is called the speed of adjustment, while $1/\beta$ indicates the adjustment lag. A simple reasoning is that if the right-hand side of (1) happened to remain constant during the adjustment process, then it would take exactly $1/\beta$ time units for the solution of this hypothetical differential equation to close the initial gap between x and x^e. Another view on the parameter β is given in Section 5, which deals with extrapolative forecasts[2].

To be more precise about the meaning of the variable x^e, the general concept is that $x^e(t)$ is the value of x which, at time t, is expected to prevail at some future date, or over some future period (may be as a time average). The usual interpretation, however, refers to the immediate future. That is, expectations are told to be formed at the beginning of the short period and x^e is the expected value of x over the rest of this period. In a continuous-time framework, the period is in fact infinitesimally short. Myopic perfect foresight then means that $x^e(t) = x(t)$ for all t. Under the assumption of smooth time paths this identity is obtained as the limiting case when the adjustment speed β tends to infinity. To point out the conceptual equivalence of, so to speak, infinitely fast adaptive expectations and myopic perfect foresight, occasionally $\beta = \infty$ is written for this situation.

In discrete time where the 'short period' is of definite length $h > 0$, expectations are formed at time t and x^e_t is the predicted value of x for the ensuing time interval $(t, t+h)$. The updating of expectations then reads

$$x^e_{t+h} = x^e_t + h\,\beta\,(x_t - x^e_t) = h\,\beta\,x_t + (1 - h\,\beta)\,x^e_t \tag{2}$$

Naturally, equation (1) is obtained if h shrinks to zero. With a fixed positive number h (normally $h = 1$, of course), the discrete-time adjustment process is only meaningful if the adjustment lag $1/\beta$ does not fall short of the period length h, so that in this setting the possibility of identifying myopic perfect foresight with $\beta = \infty$ breaks down.

Before going on, it may be mentioned that equations (1) and (2) implicitly assume that the variable x exhibits no long-run trend. They can be easily generalized to a growth context by including a growth rate g^* which represents the perceived trend of long-run growth. (1) and (2) are thus modified to

$$\dot{x}^e = g^* x^e + \beta\,(x - x^e) \tag{3}$$

$$x^e_{t+h} = (1 + h\,g^*)\,x^e_t + h\,\beta\,(x_t - x^e_t) \tag{4}$$

g^* may be a constant or itself be governed by (slow) adjustments of, e.g., the adaptive expectations type.

2. We are not interested in the interpretation that x^e is set up as a geometric distributed lag of a history of the actual time series x(t), which is the form of the explicit solution of the differential equation (1) (see, e.g., Sargent 1979, p. 112).

Although the notion of x^e as one-period ahead forecasts is predominant in macroeconomics, it should be taken with some care. In Section 6 we call into question if this point of view is really so meaningful. In the meantime the usual interpretation may do, which in particular has the advantage that the high values of the adjustment speed β suggest to come close to the seemingly more rational concept of myopic perfect foresight.

A great deal of empirical research has been devoted to the specific issue of the formation of inflationary expectations. An important reason is certainly the availability of data on such forecasts. In particular, J.A. Livingston, the financial journalist, has conducted a semi-annual survey in the US since 1947 in which respondents from a variety of occupations have given their wage and price predictions. For a detailed desription see Turnovsky (1970), Turnovsky and Wachter (1972), Gibson (1972)[3]. Confining oneself to the concept that present and past realizations of the price inflation rate π are the only determinants of inflationary expectations π^e, a most elementary approach to be tested is

$$\pi^e_{t+1} = \alpha_0 + \alpha_1 \pi^e_t + \alpha_2 \pi_t \qquad (\alpha_0, \alpha_1, \alpha_2 = \text{const})$$

Clearly, with $h=1$, $\pi^e_{t+1} = x^e_{t+1}$, $\alpha_0 = 0$, $\alpha_2 = \beta$, $\alpha_1 = 1-\beta$, equation (2) and this specification would be equivalent.

Employing this model to explain the Livingston and other data has typically produced insignificant estimates of α_0 and estimates of α_1 and α_2 which sum to unity; see, for example, Turnovsky (1970) and Lahiri (1976). On the basis of the Livingston data, also Figlewsky and Wachtel (1981), after concluding that the hypothesis of rational expectations "does not appear to provide an adequate explanation of actual inflation expectations in the post-war period" (p. 4), worked out that an adaptive expectations model best describes the price expectations formation process. However, their study permits the expectations to vary across individuals and time. An experimental study by Williams (1987), which utilized repetitive-stationary market environments, indicated that the market dynamics "leading to a rational

3. Carlson (1977) pointed out that the Livingston data may be contaminated by measurement errors and, on the basis of this criticism, (re)adjusted the series. Since then, also the Carlson series is often used in empirical research.

equilibrium" (p. 16) are likely to be governed by an adaptive process that is inconsistent with Muthian rationality[4].

If we focus on aggregate forecasts, already a simple juxtaposition of the results from Turnovsky (1970) and Lahiri (1976) suggests that a rigid adaptive expectations scheme may be a useful device to begin with, but the estimates do not seem too reliable in detail[5]. Jacobs and Jones (1986) present a multilevel adaptive expectations scheme that provides for expectations about a trend in inflation rates and distinguishes transitory from permanent shifts in the price level. Simulation of this model (over the period 1947 – 1975) replicated the expectations adjustment process closely, and the adaptation coefficients displayed remarkable stability over long periods of quite different inflation experience. To give a concrete figure, when fitted to the Livingston survey data the model explains 0.89 of the variation in expected inflation rates for thirteen-month forecasts, with two-thirds of the residuals lying under 0.5 per cent per year (p. 276).

Models of adaptive expectations use past observations of the variable to be forecasted as their only 'input'. Whether disregarding additional sources of information is 'inefficient' depends not only on the cost of collecting and processing other data, but also on the cost of *selecting* the data that have to prove relevant for the specific forecasting problem. This cost may be considerable if there is an overload of information or competing (working) hypotheses. Again, the benefits of such a data selection process are not known before it has actually been carried through. In the case of inflationary expectations there is some evidence that it might not be worthwhile to make this investment. A necessary condition for the utilization of nonnegligible cost information sets to increase forecasting accuracy is that such sets serve as leading indicators. In order for a time series to qualify as a leading indicator it

4. Additional evidence that the Livingston forecasts are not Muthian rational is given by Gramlich (1983) and, improving on a specification problem, Bryan and Gavin (1986). In another study, however, Schroeter and Smith (1986) conclude that while the Livingston CPI predictions are not rational, the PPI forecasts from this survey do pass rationality tests. Mullineaux (1978) obtains that for the expected inflation series constructed by Carlson from the Livingston data, a weak form of Muthian rationality cannot be rejected.
5. There are substantial differences in the coefficients α_1 and α_2, which read $\alpha_1 = 0.226$, $\alpha_2 = 0.781$ over the period 1962 – 1969 in Turnovsky (1970), and $\alpha_1 = 0.534$, $\alpha_2 = 0.426$ over the period 1952 – 1970 in Lahiri (1976).

must 'cause' inflation in the sense of Granger. Examining measures of monetary and fiscal policy (the growth rates of monetary aggregates and the high-employment budget surplus, respectively), Feige and Pearce (1976) find that the incremental information contained in these measures is not useful for predicting inflation when information contained in past inflation rates has already been efficiently exploited (p. 506). They conclude that "an economically rational agent would not employ these series to help him forecast inflation" (p. 518).

With respect to less technical procedures than the Box-Jenkins methodology employed by Feige and Pearce, remember the enhanced adaptive expectations model by Jacobs and Jones (1986), whose results exclusively rely on observations of past prices. Mullineaux (1980) and Noble and Fields (1982) report that the Livingston (and other) forecast data are informationally efficient in the sense that forecasts are uncorrelated with a set of pertinent past information[6].

4. Theoretical Support of Adaptive Expectations

The adaptive expectations hypothesis was put forward originally as a plausible rule for updating and revising expectations in the light of recently observed errors. It nevertheless does not need to conflict with optimizing behaviour. Perhaps the best-known fact, established by Muth (1960), is that adaptive expectations in equation (2) are optimal in the sense of yielding minimum mean square prediction errors if the variable to be forecasted follows a stochastic process that has an integrated autoregressive moving average representation ARIMA(0,1,1). There are other examples of such a statistical optimality property[7], but clearly it does not hold in general.

A second and more important characteristic of adaptive expectations is a relationship to Bayesian learning. An illuminating example is provided by Lawson (1980). He supposes that the variable in question is composed of a normal or permanent component, where agents believe that it remains constant or has just undergone a step change, and a transitory noise term with expected value zero. Using simple conditional probability theory the predicted

6. The latter characterization is taken from Williams (1987, p. 2, fn 2).
7. See the models developed by Taylor (1975) and Mussa (1975). There it is in particular the specific money supply process which implies that the optimal forecasts of the inflation rate are similar to an adaptive expectations assumption.

value of the variable can then be expressed as the adaptive expectations formula (2). The speed of adjustment, however, is endogenous and may vary over time (see p. 307). Lawson then works out that his model conforms with the empirical evidence (pp. 312 – 316)[8]. Another contribution in this direction is Caskey (1985). He reconsiders the Livingston expectations data and assumes that this panel followed Bayes' Rule in updating their believes (where the observations include the prices as well as other variables), and that it believed the underlying parameters of the inflation process were constant over the estimation period. Under these conditions reasonable initial beliefs can be found such that the outcome of the thus defined Bayesian process is fairly close to the Livingston forecasts. If it is taken into account that the Livingston data may alternatively be explained by adaptive expectations mechanisms, Caskey's result also shows that the principle of adaptive expectations may be quite consistent with optimal forecasting behaviour of Bayesian type.

The Bayesian updating procedures just proposed are based on the notion that agents believe to live in a stationary stochastic environment. If they do not trust in this assumption or consider the lag lengths involved to be too long, they may act like many econometricians in practical work do. That is, they may think the best they can do is to employ a linear model which relates the variable to be forecasted to a vector of predetermined variables, and then form the corresponding minimum variance expectations. Friedman (1979, pp. 34 – 37) demonstrates that the predictions derived from these least-squares estimations are similar in form to the familiar rules of adaptive expectations, in particular if old observations are discounted or a rolling sample period is adopted. This result of optimal least-squares learning therefore provides further evidence that the adaptive model may be a useful approximation to more sophisticated procedures. Friedman also mentions that the least-squares estimator of the slope parameters is the Bayes estimator when there is no prior information about these parameter values. Moreover, if economic agents apply a quadratic loss function to prediction errors, their use of least-squares estimations is fully consistent with the spirit of rational expectations (p. 30, fn 5).

8. This refers to an augmented version of the model. It includes an incremental growth of inflation (i.e. trended inflation) that is supposed to follow a random walk. The author emphasizes that his model is similar to those which have long existed in control theory and the operations research literature.

There are many empirical studies which obtain significant explanations of inflationary expectations such as the Livingston data by regressing them on lagged values of actual inflation and a couple of other predetermined variables. On the other hand, it has been pointed out above that these expectations can already be satisfactorily explained by elementary adaptive mechanisms. As a side result, Friedman's analysis helps to understand why the explanatory power of the simple adaptive expectations approach is not much worse than that of the more advanced procedures. Consciously or not, application of the former implies approximation of the latter. The goodness of the approximation depends, of course, on the concrete specifications.

5. Adaptive Mechanisms and Extrapolative Forecasts

A most straightforward least-squares estimations approach is to extrapolate past observations of the variable x. Agents may try this one first and only search for a better alternative if the losses resulting from the forecast errors appear to be too heavy (in relation to the presumed costs and benefits of searching for and/or selecting a more extensive model). If these extrapolative regression forecasts have a constant rolling sample period underlying, then finer details of their relationship to simple adaptive expectations can be spelled out. To this end, consider a discrete-time framework and let h be the length of the adjustment period, T the length of the sample period. Suppose that, at the end of period [t, t+h), agents fit a straight line through the last 1 + T/h logarithmized values of x (logarithms allow one to properly deal with long-run exponential growth in linear models). The formal regression equation is

$$\mathrm{Ln}\, x_\tau = \alpha_{t,0} + \alpha_{t,1} \cdot \tau, \qquad \tau = t-T, \ldots, t-h, t$$

With $\alpha_{t,0}$ the estimated intercept and $\alpha_{t,1}$ the slope, the one-period ahead forecast, denoted by x^τ_{t+h}, is given by

$$\mathrm{Ln}\, x^\tau_{t+h} = \alpha_{t,0} + \alpha_{t,1} \cdot (t+h) \tag{5}$$

A constant rolling sample period means that one period before, at the end of period [t − h, t), the same regression was performed on the basis of observations $x_{t-T-h}, \ldots, x_{t-2h}, x_{t-h}$. It gave rise to x^τ_t, the one-period ahead forecast for period [t, t + h). The corresponding slope coefficient being $\alpha_{t-h,1}$, it is shown in Franke (1992c) that the old and the new forecast are connected by the approximate equation

$$Ln x^{\tau}_{t+h} \approx Ln x^{\tau}_{t} + h\alpha_{t-h,1} + \frac{4h}{T+h}[Ln x_t - Ln x^{\tau}_{t}]$$

This formula provides an updating rule which bears some similarity to the adaptive expectations equation (4), with a constant speed of adjustment $4/(T+h)$ and an endogenous term $\alpha_{t-h,1}$. Since the latter estimates the present growth trend of x, extrapolative regression forecasts can be conveniently incorporated in small macrodynamic models if $\alpha_{t-h,1}$ in the above equation is replaced with a (constant) rate of perceived long-run growth, g^*. Using also the approximation properties of the logarithmic function[9], this yields the following adaptive mechanism,

$$x^{\tau}_{t+h} \approx (1+hg^*)x^{\tau}_{t} + \frac{4h}{T+h}(x_t - x^{\tau}_{t}) \tag{6}$$

Going to the limit, $h \to 0$, the continuous-time formulation reads

$$\dot{x}^{\tau} \approx g^* x^{\tau} + (4/T)(x - x^{\tau}) \tag{7}$$

The correspondence to equations (3) and (4) is obvious. Hence, the adaptive expectations formula may alternatively be viewed as aproximately representing the one-period ahead extrapolative forecasts that are obtained from linear regressions of x on time with a constant rolling sample period T. The speed of adjustment β is then linked to T by the equation

$$\beta = 4/T \quad \text{in continuous time,} \qquad \beta = 4/(T+h) \quad \text{in discrete time}$$

This interpretation of the adjustment speed is also helpful in assessing the numerical values that may be assigned to β in computer simulations of macroeconomic models. Clearly β rises when in the regression approach more of the older observations of x are discarded as misleading and so the length of the rolling sample period decreases.

Whether agents really regard the imperfections associated with the methods of adaptive expectations or extrapolative regression forecasts as tolerable will depend on the particular applications. In order to get an impression of the order of magnitude of the forecast errors, we put the rules (2) and (5) to the test of a little simulation experiment. Let x, the series to be

9. Namely, $Ln\, x^{\tau}_{t+h} - Ln\, x^{\tau}_{t} = Ln\,(x^{\tau}_{t+h}/x^{\tau}_{t}) = Ln\,[1 + (x^{\tau}_{t+h} - x^{\tau}_{t})/x^{\tau}_{t}] \approx (x^{\tau}_{t+h} - x^{\tau}_{t})/x^{\tau}_{t}$, and similarly with $Ln\, x_t - Ln\, x^{\tau}_{t}$.

predicted, favour systematic forecast errors in that it oscillates in a rather regular way. Algebraically, let Ln x be a trendless sine wave perturbed by serially correlated random shocks,

$$\mathrm{Ln}\, x_t = \alpha \sin(\phi t) + u_t$$

$$u_t = \rho u_{t-h} + \varepsilon_t$$

where the disturbances ε_t are drawn from a normal distribution with zero mean and standard deviation σ. The parameter values are as follows,

$$\alpha = 5/100 \qquad \phi = 2\pi/8 \qquad \rho = 0.75 \qquad h = 1/12 \qquad T = 1 \qquad g^* = 0$$

So we have a monthly series with an amplitude of ± 5 per cent, an average cycle period of 8 years, and relatively high autocorrelation in the random shocks, while the lenght of the sample period is one year. The corresponding value of β in (2) is $\beta = 4/(T+h) = 3.69$. Setting the initial value of x^e equal to the first regression forecast x^τ and allowing for a transition period of a full cycle, Figure 1 is a representative example of the evolution of the regression forecasts and the adaptive expectations if x is a purely deterministic sine wave. In Figure 2 the motion of x is stochastic, the standard deviation of the random perturbations being one-fifth of the amplitude of x, i.e., $\sigma = 2/100$.

Figure 1

Extrapolative forecasts and adaptive expectations: deterministic case

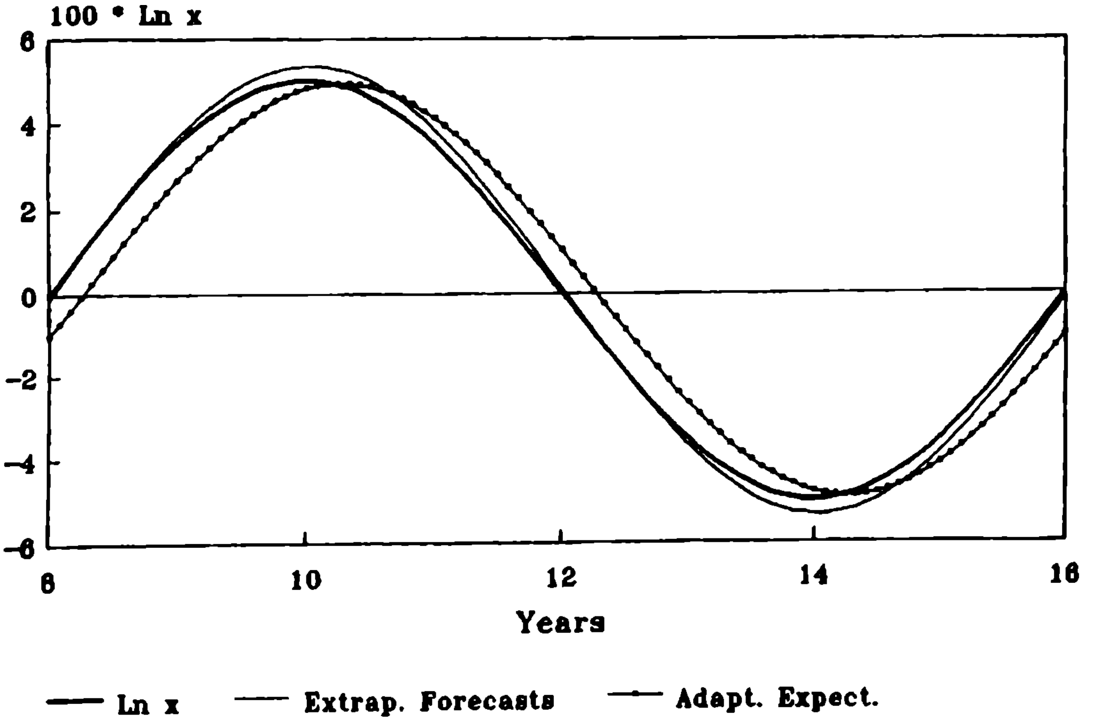

Figure 1 plainly displays the weaknesses of the two naive forecasting methods. Adaptive expectations are chasing the series, catching up shortly after the turning points and then chasing it again. In contrast, the regression forecasts overpredict the series near the turning points. Though these deviations are systematic, they are limited in size. Figure 2 indicates that with random shocks imposed on the oscillations, the shortcomings are less severe and the differences between the two procedures tend to be washed out.

Figure 2

Extrapolative forecasts and adaptive expectations: stochastic case ($\sigma = 2/100$)

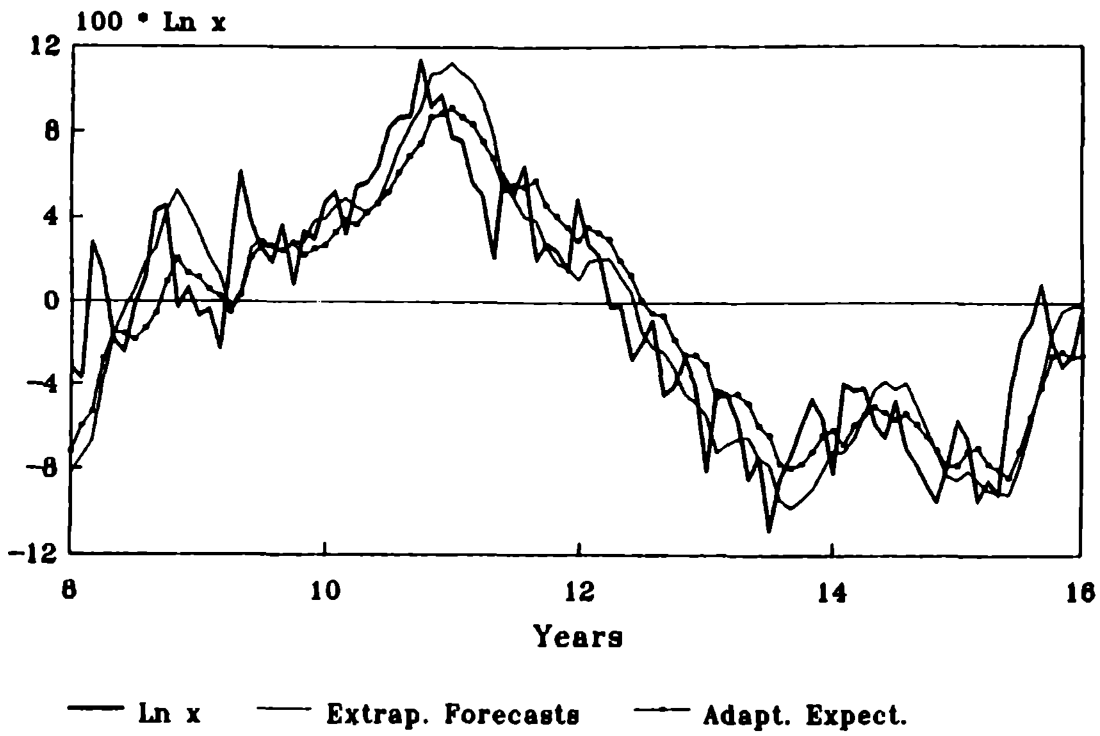

Table 1 presents some quantitative results for different degrees of the stochastic noise. The standard deviation of the error terms covers the range from $\sigma = 0$, the deterministic case, to $\sigma = 5/100$, where the noise begins to dominate the cyclical pattern of x. One realization of the stochastic disturbances is sufficient to gain the basic insights. (The seed of the sequences of pseudo-random numbers from which the ε^t were derived was the same for each σ.)

Naturally, the root mean square prediction error exceeds the standard deviation of the shocks, but the difference is not too large. Regression forecasts show smaller prediction errors than adaptive expectations when the series x is smooth, adaptive expectations yield comparatively better forecasts when the noise level of the series increases.

Table 1
Root mean square prediction error with respect to Ln x_t (times 100) and Durbin-Watson statistic of prediction errors (in parantheses)

	Standard Deviation σ of the ε_t-Shocks (times 100)				
	0.0	0.5	1.0	2.0	5.0
Regression Forecasts	0.26 (0.00)	0.76 (0.69)	1.44 (0.77)	2.83 (0.79)	7.04 (0.80)
Adaptive Expectations	0.72 (0.01)	0.97 (0.39)	1.42 (0.72)	2.50 (0.93)	5.93 (1.03)

The Durbin-Watson coefficients in Table 1 measure the extent of systematic prediction errors with respect to first-order serial correlation[10]. They indicate strong positive autocorrelation for low values of σ. At higher values of σ, adaptive expectations display somewhat less autocorrelation than the regression forecasts, though it is still significant. In many applications economic agents may nevertheless regard the forecast errors as tolerable, or expect only minor improvements from other forecast methods. More importantly, a model builder may be willing to accept these errors in formulating and investigating small and tractable macrodynamic models of the economy as a first approximation to the 'true' processes of expectations formation; and he/she may do this in a deterministic framework as an approximation to a stochastic setting, where the bias in the forecasts would be less annoying.

The source of the most severe prediction errors in the example is that the (major) peaks and troughs of x are recognized too late. Now, if agents are aware that they live in a cyclical environment and the variable x, say, has increased for some time above 'normal', then it will seem more likely to them that the series is about to peak. If agents stuck to the rigid rule of adaptive expectations, they would miss this turning point and overestimate the series in the first stage of the downturn then setting in. In a great deal of decision problems, especially those with irreversibilities, overshooting will be more

10. Values of the DW statistic in the neighbourhood of 2 signify the absence of first-order autocorrelation. Smaller (larger) values reveal positive (negative) autocorrelation, the polar cases being DW = 0 and DW = 4, respectively.

costly in such a situation than a possible underestimation. Hence, agents may hesitate near the (suspected) turning points and adjust their expectations more slowly. This subject is taken up in Section 7 below.

6. Delayed Adjustments under a Longer Time Horizon

One issue that is hardly ever discussed in the macroeconomic literature on simple expectation mechanisms is the question of the time horizon. That is, what period is it that expectations about a dynamic variable $x = x(t)$ do refer to? In the previous sections we joined the usual interpretation that expectations are formed at the beginning of the (possibly infinitesimally) short period and x^e is the realization of x that is expected to prevail over the rest of this 'period'. Alternatively, however, $x^e(t)$ may be conceived of as a point estimation of some average value of x over a medium range of time from t to $t + T$, say (where the near future might be weighted more heavily). There is no general answer to the above question for the length of the time horizon of expectations, but its discussion must not neglect the kind of the decision problems which have recourse to the variable x^e.

As a prominent example, take the rate of inflation in macrodynamic models of a Keynesian type. There are two building blocks where expected inflation plays an important role: in an expectations-augmented (wage or price) Phillips curve, and in a function representing investment expenditures on fixed capital which, besides other variables, are influenced by the variations of a real rate of interest (cf., for example, the macroeconomic textbook by Sargent 1979, Ch. V). In the latter case, because of the long life-time of capital goods and the (mostly) irreversibility of investment, the horizon T should be quite long[11]. Since, on the other hand, $\beta \to \infty$ is to mean that expected inflation tends to come close to realized inflation in the next short period, there is no particular reason why large values of the adjustment speed β should claim priority in rationality; a good prediction of price changes over the next quarter or month will generally be of rather limited use for the evaluation of an investment project that reaches far into the future. For concreteness, the

11. A short horizon would correspond to a neoclassical portfolio theory when there is no need to anticipate the rate of price increase over a longer term because the portfolio can be reshuffled at any time in the future. In fact, in neoclassical theory capital has been merely understood as material factors of production, with little attention paid to its fixity.

relevant time horizon may be thought of as extending over a period between two and five years[12].

The time horizon underlying the Phillips curve, on the other hand, derives from the assumption on the timing of wage settlements. If wages of the total labour force are renegotiated every short period (of length h), we have T = h. A more realistic device is to hypothesize that wage settlements only refer to a segment of the labour force, for which it remains fixed for the next T years. The wage bargains that are taking place every adjustment period then refer to different segments, and w is an index of present and past wage settlements. This can be easily made more precise by employing certain uniformity assumptions so that, denoting by $w^s(t)$ the wage settlements at time t, the wage index w(t) in continuous time is defined as

$$w(t) = (1/T) \int_{t-T}^{t} w^s(\tau)\, d\tau \, .$$

Differentiating with respect to time leads to

$$\dot{w}(t) = (1/T)\,[w^s(t) - w^s(t-T)] \, .$$

The time horizon T that is made explicit by this procedure may be one or two years[13].

Macroeconomic models where inflationary expectations enter an investment function as well as a Phillipis curve mechanism usually employ the same rate of expected inflation in these schedules. An implication of our reasoning is that, under closer conceptual scrutiny, two different notions of expected inflation are involved. Certainly, working with uniform expectations will be required for analytical tractability of these models, but future and more ambitious versions should also consider a differentiation in the time horizon of *the* expected rate of inflation.

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12. This time span roughly corresponds to usual values of a payback time limit, within which an investment project is required to pay back the money advanced; see Blatt (1983, pp. 279, 288). It is in itself a psychological variable.
13. The approach of staggered wage contracts is known from its application in a certain branch of rational equilibrium business cycles. These theorists like to work with it (in a simple discrete-time setting) because, with some suitable random shocks superimposed, the lags provide an easy means to generate serial correlation in output and prices. A seminal paper in this respect is Taylor (1980).

If it is acknowledged that the time horizon of expectations is definitely longer than the short period and, so, myopic perfect foresight loses its significance as a benchmark case, delayed adjustments like adaptive expectations may appear in a new light. A general analysis of this matter is conducted by Heiner (1988). He starts out from the notion of a gap between an agent's competence in the decision making process and the difficulty in rightly interpreting information and selecting potential actions (even if information were complete). The stage of his investigation is set by the assumption that the environment, which is exogenous to the individual agent and can be characterized by a single decision parameter, is initially in a period of relative stability. At some point in time the decision parameter starts shifting toward a new value that remains constant for another uncertain period before shifting again. This framework allows Heiner to be compatible with the analytical tools that are traditionally used to study optimal behaviour. In particular, an optimal response exists that maximizes expected utility.

By contrast, an imperfect agent is a decision maker for whom there is a positive probability of missing the optimal response and adjusting initially either too soon or in the wrong direction[14]. Heiner demonstrates that, in comparison to a perfectly optimizing agent, an imperfect agent needs to respond at a relatively slow rate or with a noticeable delay in order to control decision errors. In other words, the very attempt to respond soon after a parameter starts shifting without severely constraining the expected rate of response will actually reduce expected utility, as compared to not doing so (p. 268). It follows that, given the explicit limitations imperfect agents are subjected to (rather than ruling them out by hypothesis), "behavioural rules (and the mechanisms needed to enforce them) can be viewed as fully rational even though they may prevent them from acting in the same manner as perfect agents would behave" (p. 269).

7. A Proposal of Flexible Adaptive Expectations

Heiner's arguments provide a theoretical basis for inertial behaviour in forecasting, which, especially if a longer time horizon is involved, may also take

14. The precise definition of an imperfect agent (Heiner 1988, pp. 263f) is more technical and in fact somewhat weaker. The following comparisons between perfect and imperfect agents rest on the supposition that similar adjustment costs apply to both of them (p. 272).

the form of adaptive expectations. On the other hand, a specification of a constant speed of adjustment as in equations (1) – (4) above is certainly not warranted. In this respect we may mention some empirical support that indeed agents will not always revise their expectations about inflation in the same relation to past observations. This means they additionally have a conception of a rate of inflation which they regard as 'normal' and, if the current rates of inflation deviate too much from it, to which they expect inflation to return. Incidentally, the value of normal inflation might shift over time. Kane and Malkiel (1976) establish that such return-to-normality elements play an important part in inflation forecasts[15].

If one wishes to avoid introducing a completely new mechanism, return-to-normality may also be viewed as affecting the speed of adjustment of adaptive expectations in different stages of the inflationary process. To tell a simple story, let π and π^e be actual and expected inflation, respectively, and consider a situation with $\pi > \pi^e$ that is characterized by an increasing π^e and accelerating prices, $d\pi / dt > 0$. In addition, suppose that in the past agents have learned that the upwards motion of π tends to be reversed; the higher π, the more likely this event seems to occur in the near future. If π^e were still increased at the same speed as before, it would later be found to be above current inflation, at a time when π is already on the downturn. Regarding the decisions about fixed investment, for example, such an overprediction would probably be more costly than a possible underestimation. It seems therefore reasonable to assume that, in order to reduce the risk of overprediction, the adjustments of π^e are more sluggish in the (suspected) late phase of the upswing of $\hat{p}$, or in the early phase of its downturn. Later, with π and π^e coming down again to medium values, adjustments in π^e might gain momentum.

Schmalensee (1976) is an experimental study which makes a very similar point. He presented a total of twenty-three subjects with price observations from a nine\-teenth-century British wheat market and had them submit both point and interval forecasts of five-year averages of the price series. A central issue was the role of turning points in this series. The survey results obtained show that peaks and troughs were indeed 'special' to the expectations formation process. Most important for our purpose, an extrapolative ex-

15. Specifically, a crude return-to-normality model in which all other mechanisms were absent outperformed all versions of the error-learning models investigated.

pectations model was found to be outperformed by an adaptive expectations model in which the speed of adjustment tended to *fall* during turning points.

The slow down of the reactions may to some extent also be explained by Heiner's (1988) theoretical results. If in an upward motion of inflation we distinguish two regimes, a 'normal' regime and a regime of excessively high inflation rates, then the transition from the first to the second regime corresponds to a decision parameter that begins to shift. Consequently, the initial response to this event will be adjustments in expected inflation that are less rapid than in the previous normal regime. Realistically, however, the environment as it is perceived by the agents is less stylized than in the clear distinction between an old and a new regime. This means that the change in the speed of adjustment will be more gradual than the analysis by Heiner might suggest. Apart from that, gradual changes of the adjustment speed will be obtained if it is interpreted as an average across heterogeneous agents.

On the basis of the above reasoning it would now be fairly easy to endogenize the adjustment speed of inflationary expectations and make it a variable. We here propose an alternative with a slightly different specification. It is, however, conceptually equivalent. Recall that in order to determine the change in expected inflation, agents compare π^e to the current rate of inflation π. That is, the latter serves as a reference. Some flexibility may also be allowed for in such a reference rate of inflation. Introducing the notation f_{ref}, we conceive it as a *function* of current inflation, $f_{ref} = f_{ref}(\pi)$. In this way the rigid rule of adaptive expectations in (1), which here reads $\dot{\pi}^e = \beta(\pi - \pi^e)$, is generalized to

$$\dot{\pi}^e = \beta\,(f_{ref}(\pi) - \pi^e) \tag{8}$$

For simplicity, rising and falling inflation is treated symmetrically. Assume that over a medium range of inflation f_{ref} still coincides with π. Yet, when π has soared to higher levels the increase in the yardstick is faltering: $df_{ref}(\pi)/d\pi < 1$ then. Similarly when inflation rates are considered to be low. For numerical simulations, f_{ref} may specified as a piecewise linear function. Introducing two positive parameters d and α, define 'medium inflation' as a symmetrical interval $[\pi^* - d, \pi^* + d]$ around a (possibly subjectively perceived) steady state value of inflation π^* and suppose that the slope of f_{ref} outside this range is given by $\alpha < 1$. Then we have

$$f_{ref}(\pi) = \begin{cases} \alpha\pi + (1-\alpha)(\pi^* + d) & \text{if } \pi > \pi^* + d \\ \pi & \text{if } |\pi - \pi^*| \leq d \\ \alpha\pi + (1-\alpha)(\pi^* - d) & \text{if } \pi < \pi^* - d \end{cases}$$

Note that if π^e as well as π turn out to remain bounded in a macroeconomic model, these modified expectations might be described as partially (or qualitatively) self-fulfilling (otherwise, if the economy ran into hyperinflation, they would not be maintained). They differ from rational expectations, which are totally self-fulfilling, in a quantitative way. The significance of a flexibilization in the adaptive expectations mechanism as in equation (8) derives from the fact that in many dynamic models a high speed of adjustment β has a destabilizing effect, whereas slow adjustments tend to render a steady state position of the economy (locally) stable. Examples are Hadjimichalakis (1971), Tobin (1975), Hayakawa (1984), Franke (1992a) with respect to expectations of the rate of inflation in monetary (growth) models, and Franke and Lux (1993) with respect to sales expectations of firms in a Metzlerian model of the inventory cycle. If now the adjustment speed is large in a vicinity of the steady state and sufficiently low in the outer regions of the state space, and if in addition other nonlinearity effects in such models are limited or cancel out, then though the equilibrium will be unstable, the trajectories will nevertheless not explode. As a consequence, such systems will exhibit persistent and bounded fluctuations. This interaction of destabilizing and stabilizing forces can therefore constitute an important cycle-generating mechanism[16].

8. Conclusion

In the previous sections, a number of arguments have been collected which run counter the dominant expectational hypothesis in macroeconomic modelling, the assumption of rational expectations, and which make a plea in favour of the use of a simple rule such as adaptive expectations. The discussion took place on an empirical and theoretical level. In particular, it was pointed out that more attention should be paid to the time horizon underlying the

16. Within a macroeconomic model of inflation and distribution, a more detailed investigation of the resulting growth cycles can be found in Franke (1992b).

expectations. Advocating adaptive expectations was not meant to adopt a fixed speed of adjustment that equally applies in all stages of a dynamic process. It was rather suggested to combine adaptive expectations with elements of return-to-normal expectations, a procedure which would directly or indirectly give rise to a flexible speed of adjustment. This device has some empirical and theoretical underpinnings, at least as a first approximation to more sophisticated procedures of expectations formation, and their cycle-generating potential for the macrodynamics was shortly indicated.

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