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On the linearity of the relationship between the nominal wage rate and the rate of profit in linear production systems

by
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The aim of this assignment is to examine the implications of the linear relationship between the nominal wage rate and the rate of profit in linear production systems.

After giving a brief description of the linear production system of basic and non - basic commodities[1], we will examine which price normalizations[2] imply the existence of linearity between the nominal wage rate and the rate of profit. Finally we will show, based on the results of the previous analysis, that the comparison of available production techniques concerning their profitability is in the general case impossible[3].

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Let us consider a linear production system (of simple production) $[A, \ell, X]$, that produces a gross product - which is symbolised with the column vector X- using the productive[4] technique of production $[A, \ell]$. The matrix A, $A=[a_{ij}] \geq 0$, is the nxn matrix of technical coefficients, the element a_{ij} of which represents the amount of commodity i needed to produce one unit of commodity j, with i,j = 1,2,...n, while the vector ℓ, $\ell=[\ell_j] > 0$, is the strict positive row vector of inputs in direct homogenous labour, the component ℓ_j of which represents the necessary amount of direct labour for the production of one unit of commodity j.

1. For the meanings of "basic" and "non-basic" commodities, see P.Sraffa (1960), §6.
2. The normalization equation is the equation through which the price of a simple or composite commodity is placed equal to a positive constant. This commodity is called normalization commodity.
3. For the prevailing opinion see, among others, Pasinetti (1985), Ch. 6, Abraham-Frois (1991), pp. 396-400 , 465-477, Bidard (1991), Ch. 7.
4. In mathematical terms this assumption means that the maximum eigenvalue of matrix A is smaller than unity. See Solow (1952), pp. 29-33.

As it is known, the prices of n produced commodities of the given production system, if we introduce the usual hypotheses[5], are determined by the following equation system:

$$p = pA(1 + r) + w\ell \qquad (1)$$

where p stands for the 1xn row vector of the prices of n commodities, w - by assumption- stands for the uniform nominal wage rate and r- by assumption - for the uniform rate of profit. System (1) consisting of n equations has two degrees of freedom and finding the absolute prices requires the introduction of a normalization equation and the exogenous definition of one of the two variables of the distribution of income. Indeed, the introduction of a normalization equation of the form

$$pu = c \qquad (2)$$

where u, $u \geq 0$, is a nx1 column vector, which represents the normalization commodity and $c > 0$ is the normalization constant, permits the inference of a relationship between w and r, (known as w-r-relationship):

$$w = \frac{c}{\ell[I-(1+r)A]^{-1}u} \qquad (3a)$$

for each w in the $(0, w_{max}]$[6] and

$$r = R_v = \frac{1-\lambda_v^A}{\lambda_v^A}, \text{ for } w=0 \qquad (3b)$$

where λ_v^A the v eigenvalue of the matrix A, with v = 1, 2,..., n.

In the situation where the technique $[A, \ell]$ is reducible, the system $[A, \ell, X]$ produces apart from say k basic also n-k non - basic commodities. However, we will not take into consideration the existence of non - basics which do not enter the production of non - basic commodities. Therefore (1) is analysed in the subsystems:

$$p_1 = p_1A_{11}(1+r) + w\ell_1 \qquad (4a)$$

$$p_2 = (p_1A_{12} + p_2A_{22})(1+r) + w\ell_2 \qquad (4b)$$

where: $[A_{11}, \ell_1]$ is the production technique of the basic subsystem, $[A_{22}, \ell_2]$ the production technique of the non - basic subsystem, A_{12} a kx(n-k) semipositive matrix, representing the inputs that the non - basic subsystem receives from the

5. For a detailed presentation, see Stamatis (1988), pp. 1-12.

6. With w_{max} (=maximum nominal wage rate) we symbolize the price that results from the w-r-relation if we place r = 0.

basic subsystem and p_1, p_2, 1xk and 1x(n-k) row vectors of the prices of the basic and non-basic commodities, respectively.

Finally, we assume that the maximum eigenvalue of the matrix A_{11} $\left(\lambda_m^{A_{11}}\right)$ can be greater than, less than or equal to the maximum eigenvalue of the matrix A_{22} $\left(\lambda_m^{A_{22}}\right)$ [7]. This implies that the maximum rate of profit of the basic subsystem R_1 $\left[=\frac{1-\lambda_m^{A_{11}}}{\lambda_m^{A_{11}}}\right]$ can be, respectively less than, greater than or equal to the maximum rate of profit of the non- basic subsystem $R_2\left[=\frac{1-\lambda_m^{A_{22}}}{\lambda_m^{A_{22}}}\right]$.[8]

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As it is known, T.Miyao[9] showed, for the first time, that in the general case, there are generally more commodities other than the standard commodity of P.Sraffa[10], which, if they are used as measure of the prices of commodities, they imply a linear w-r relationship. However Miyao's analysis is restricted to irreducible production techniques. In what follows, we develop an alternative approach to this case, and then we will examine all possible cases of reducible production techniques $\left(\lambda_m^{A_{11}}\right) \gtreqless \left(\lambda_m^{A_{22}}\right)$ and we will show that, on the one hand, there are normalization commodities which include non-basics as well and imply linear w-r relationships , and on the other hand, in the case where $\left(\lambda_m^{A_{11}}\right) < \left(\lambda_m^{A_{22}}\right)$, there are commodities which are connected to the standard commodity of S.Vasilakis[11], in exactly the same way as the commodities of T. Miyao are connected to the standard commodity of P.Sraffa, that is, they are a generalization of Miyao's standard commodities.

7. We note that the reducible production systems aren't generally considered in the relative bibliography because of the "paradox" situations that appear in them. For the study of such systems and the explanation of the "paradox", see Stamatis (1984), pp. 293-333 and Vouyiouklakis/Mariolis (1992),(1993).
8. For the meaning of the maximum profit rate of the subsystems, see Stamatis (1988), pp. 13-17.
9. Miyao (1977). See also Abraham-Frois/Berrebi (1978), Stamatis/Dimakis (1981), Bidard (1991), pp. 54-61.
10. P. Sraffa (1960) Chapter IV and V. See also Stamatis (1988), pp. 62-64.
11. Vassilakis (1982). See also Stamatis (1988), pp. 56-62, Abraham-Frois / Berrebi (1989), pp. 129-132.

I. Let the matrix A be irreducible. We will examine under which circumstances it is possible for two of commodities to exist, say γ_I, γ_{II}, the relative price of which is not affected by changes in the distribution of income. That is, when, - for every r such that $p > 0$ [12] - it holds:

$$\frac{p\gamma_I}{p\gamma_{II}} = e \tag{5}$$

where e is a real constant.

From (1) to (5) it follows[13]:

$$\frac{w\ell\left[I-(1+r)A\right]^{-1}\gamma_I}{w\ell\left[I-(1+r)A\right]^{-1}\gamma_{II}} = e \Leftrightarrow$$

$$\frac{\ell\gamma_I + (1+r)\ell A\gamma_I + (1+r)^2\ell A^2\gamma_I + \dots}{\ell\gamma_{II} + (1+r)\ell A\gamma_{II} + (1+r)^2\ell A^2\gamma_{II} + \dots} = e \Leftrightarrow$$

$$\ell(\gamma_I - e\gamma_{II}) + (1+r)\ell A(\gamma_I - e\gamma_{II}) + (1+r)^2\ell A^2(\gamma_I - e\gamma_{II}) + \dots\dots = 0 \tag{6}$$

where I is the identity nxn matrix.

Therefore, our first postulate (relation (5)) is satisfied if and only if, the following holds:

$$\ell(\gamma_I - e\gamma_{II}) = \ell A(\gamma_I - e\gamma_{II}) = \ell A^2(\gamma_I - e\gamma_{II}) = \dots\dots = 0 \Leftrightarrow$$

$$\ell A^i f = 0, \quad i = 0,1,2,\dots \tag{7}$$

where: $f \mathrel{\widehat{=}} \gamma_I - e\gamma_{II}$

However, due to Cayley - Hamilton's theorem (each matrix satisfies its own characteristic polynomial) equation (7) is reduced to:

$$\ell A^i f = 0 \Leftrightarrow$$

$$L^* f = 0 \tag{8}$$

12. Because for each $r \in [0, \frac{1-\lambda_m^A}{\lambda_m^A} \mathrel{\widehat{=}} R)$ we have $[I-(1+r)A]^{-1} > 0$, in that interval it holds, $w>0$, $p > 0$. See Pasinetti (1985), pp. 290-291.

13. For each $r \in [0,R)$ it holds : $[I-(1+r)A]^{-1} = I+(1+r)A+(1+r)^2A^2+\dots\dots$ See Pasinetti (1985), pp. 280-282.

where: t = 0,1,2,.....n-1, (n: the number of the produced commodities) and

$$L^* \widehat{=} \begin{bmatrix} \ell \\ \ell A \\ \vdots \\ \ell A^{n-1} \end{bmatrix} \qquad (9)$$

Let q^* be the right eigenvector of the A matrix which corresponds to its maximum eigenvalue, in other words, the standard commodity of P. Sraffa. If the rank of the nxn matrix L^* is n, there is no f such that $L^* f = 0$ holds. On the contrary, if the rank is n-d, with $1 \leq d \leq n-1$, the system (8) has d - parametric infinite solutions. In the second case, when we normalize the price vector with

$$p\,(q^* + f) = c \qquad (10)$$

the w-r -relation that follows is linear, because - due to (7) - it hold

$$\ell\,[I - (1 + r)\,A]^{-1} f = 0.$$

Indeed from (1) and (10) it follows:

$$p = w\ell[I-(1+r)A]^{-1} \Rightarrow$$

$$p(q^*+f) = w\ell[I-(1+r)A]^{-1}(q^*+f) \Rightarrow$$

$$c = w\left[\ell q^*\left(\frac{1}{1-(1+r)\lambda_m^A}\right) + \ell[I-(1+r)A]^{-1}f\right] \Rightarrow$$

$$c = w\ell q^*\left(\frac{1+R}{R-r}\right) \Rightarrow$$

$$w = \frac{c}{\ell q^*}\left(\frac{R-r}{1+R}\right). \qquad (11)$$

We summarize the conclusions of the above analysis:

a) When the vectors ℓ, ℓA, ... ℓA^{n-1}, are linearly dependent, there exist commodities, other than q^*, which if they function as normalization commodities, they imply a linear w-r-relation.

b) These commodities result from q^* if we add to it a row vector f that has the following characteristics: its price is, for every value of the rate of profit, equal to zero, because the direct labour which is required for its production (ℓ f), the direct labour which is required for the production of its means of productions (ℓ A f) etc., is equal to zero.

c) The relative price of q^* and of these commodities does not change with changes of the income distribution irrespective of the kind of normalization of the prices.

d) When the rank of L^* is n-1, vector f is a right eigenvector of A[14].

e) The normalizations with the vector q^{**} or with the vector $q^{**} + f$ (where q^{**} the right eigenvector of A, which corresponds to an eigenvalue other than the maximum), under the condition that $\ell q^{**} \neq 0$, result in a linear w-r- relation, as well. However, even if $q^{**} + f \geq 0$, these normalizations do not have an economic meaning[15].

II. Let the matrix A be reducible. Analogously to the situation of an irreducible technique, apart from the right eigenvector q_1^* of A_{11} which is connected to its maximum eigenvalue, there are other commodity bundles - consisting only of basic commodities - which give rise to a linear w-r- relation, if the rank of the matrix L_1^* is smaller than k[16], where:

$$L^* \equiv \begin{bmatrix} \ell_1 \\ \ell_1 A_{11} \\ \vdots \\ \ell_1 A_{11}^{k-1} \end{bmatrix} \tag{12}$$

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14. Because it holds : $\ell f = \ell A f = \ldots\ldots = \ell A^n f = 0$ (See relation (7)), it follows that: $L^*[f, Af, \ldots\ldots A^{n-1}f] = 0$. Therefore the rank of the nxn matrix $[f, Af, \ldots. A^{n-1}f]$ is one, since that of L^* is n-1. (According to the relative theorem, if for the matrices B, C of order μxp and pxv respectively, it holds BC = 0 and one of them has rank λ, then the rank of the other is not higher than the number p-λ. See Daskalopoulos, pp. 162-163).
15. As known (see for example Pasinetti (1985), p.290) q^{**} has at least one negative component. Also, because for each r > R and w > 0 the price vector p has at least one negative component (for the proof, see Mariolis (1992)), the normalization with the vector $q^{**} + f$ leads to an interval of values of r in which w is positive and the price of at least one commodity is negative.
16. Based on (4a) and following the same procedure as the one followed in the derivation of (11) we get: $w = \dfrac{c}{\ell_1 q_1^*}\left(\dfrac{R_1 - r}{1 + R_1}\right)$.

and k the number of basic commodities.

Also, if the rank of L^* is smaller than n, there are, apart from the right eigenvector of $A(q^*)$ which is connected with its maximum eigenvalue, other commodity bundles- consisting of basic and non basic commodities, which lead to a linear w-r - relationship.

a) Suppose that $\left(\lambda_m^{A_{11}}\right)=\left(\lambda_m^{A_{22}}\right)$ If the rank of the matrix L^* is smaller than n, there are commodities of the form: $q^*+f = (q_1^*,0)^T + (f_1,f_2)^T$, which lead to a linear w-r-relation[17]. However: i) The rank of L^* cannot be equal to 1, because the left eigenvector of A which corresponds to its maximum eigenvalue has its k first components zero (and the rest positive)[18] and therefore, for the production of basic commodities, direct labour should not be needed.

ii) Even though from a mathematical point of view, such a commodity could exist when the rank of L^* is smaller than n, it is not always economically significant. For if $\ell A^t f = 0$, $t = 0,1,2,...$ holds in some cases - when for example $f_1 \geq 0$ - it is required that f_2 has at least one negative component; thus $q^* + f$ will consist of at least one commodity in a negative quantity[19].

b) Suppose that $\left(\lambda_m^{A_{11}}\right)<\left(\lambda_m^{A_{22}}\right)$ If the rank of the matrix L^* (L_1^*) is smaller than n(k), there are, apart from the standard commodity q^* (q_1^*) of Vassilakis (P. Sraffa), other commodities as well which give rise to a linear w-r- relation.

Let us give, a relevant numerical example. Let a production system that uses the reducible technique $[A, \ell]$, where:

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17. For the determination of P. Sraffa's standard commodity in this case, see Egidi (1975) and Vouyiouklakis/Mariolis (1993). When $\lambda_m^{A_{11}} > \lambda_m^{A_{22}}$, for the right eigenvector of A which corresponds to $\lambda_m^{A_{11}}$ it holds exactly the same. Thus, it is not necessary to treat this case.
18. See Egidi (1975), pp. 11-12.
19. Let us give a numerical example. Let:

$$A=\begin{bmatrix} 0{,}3 & 1 & 1 \\ 0 & 0{,}2 & 0{,}1 \\ 0 & 0{,}1 & 0{,}2 \end{bmatrix}, \quad \ell=[1,1,1], \text{ with } \ell_m^{A_{11}}=\ell_m^{A_{22}}=0{,}3$$

The rank of L^* is two and thus there exist, other than P. Sraffa's standard commodity, a commodity which if it functions as a normalization commodity, leads to a linear w-r-relation. This is of the form: $\mu(1, 0, 0)^T+\nu(0, 1, -1)^T = (\mu, \nu, -\nu)^T$ where μ, ν are arbitrary positive constants.

$$A = \begin{bmatrix} 0 & 0,5 & 0,1 \\ 0,4 & 0 & 0,2 \\ 0 & 0 & 0,5 \end{bmatrix}, \quad \ell = \left[\frac{0,4}{\sqrt{0,2}}, 1,4 \right]$$

and

$$\lambda_m^{A_{11}} (= \sqrt{0,2}) < (\lambda_m^{A_{22}})(=0,5) \quad \Leftrightarrow \quad R_1 (\cong 1,236) > R_2 (=1).$$

Because the rank of the matrix L_1^* is equal to 1 (the $\ell_1 = \left[\frac{0,4}{\sqrt{0,2}}, 1 \right]$ is a left eigenvector of the matrix A_{11}, which is connected to the maximum eigenvalue $(=\sqrt{0,2})$, each normalization with only basic commodities leads to a linear w-r-relation. So if, for example, we introduce the normalization equation: $p_1 + p_2 = 1$, it follows:

$$w=0,2918 - 0,2361r \tag{13}$$

The standard commodity of Vassilakis is the right eigenvector of the A matrix, that corresponds to its maximum eigenvalue (=0,5). Thus, if we introduce the normalization equation : $3p_1 + 2,8p_2 + p_3 = 1$, it follows:

$$w=0,0527\,(1 - r) \tag{14a}$$

However, because the L^* matrix is of rank 2, there is another commodity which, if it functions as a measure of price, it leads to a linear w-r- relation. Indeed, with the introduction of the normalization equation : $4p_1 + 1,90557p_2 + p_3 = 1$, it follows

$$w=0,0527\,(1 - r) \tag{14b}$$

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According to the prevailing claim, the w-r- relation is linear only in the case we use the standard commodity of P.Sraffa, consisting of only basic commodities, as the normalization commodity. However, for each w-r-relation it always holds:

$$w = w_{max} - K_n r \tag{15}$$

where $w_{max}\left(\equiv \frac{pu}{\ell[I-A]^{-1}u} \right)$ is the constant average productivity of labour in price terms and therefore the maximum nominal wage rate of the normalization subsystem[20] and $k_n \left(\equiv \frac{pA[I-A]^{-1}u}{\ell[I-A]^{-1}u} \right)$ is the average capital intensity in price terms in the normalization subsystem, which for each w-r curve and a given rate of profit, say $\bar{r}$, is equal to the slope of the line which goes through the points $(0, w_{max})$ and $(\bar{r}, \bar{w})$[21].

Therefore a linear w-r-relation implies that the magnitude K_n does not change with changes in the rate of profit. Indeed, if the $q^* + f$ functions as a normalization commodity, where $Aq^* = \lambda_m^A q^*$ and $\ell A^i f = 0$,[22] $i = 0,1,2,...,$ then the average capital intensity in price terms in the normalization subsystem is constant and equal to that of the standard system:[23]

$$K_n = \frac{pA[I-A]^{-1}(q^* + f)}{\ell[I-A]^{-1}(q^* + f)} \Rightarrow$$

$$K_n = \frac{pAq^*\left(\frac{1}{1-\lambda_m^A}\right) + p(A + A^2 + ...)f}{\ell q^*\left(\frac{1}{1-\lambda_m^A}\right) + \ell(I + A + A^2 + ...)f} \Rightarrow$$

$$K_n = \frac{pAq^*}{\ell q^*} \Rightarrow$$

20. The normalization subsystem is the system which, using the linear technique $[A, \ell]$, produces as its net product the normalization commodity. G. Stamatis showed, for the first time, that the w-r- relation is a feature of the normalization subsystem and not of the production technique of the given production system. See, Stamatis (1984), pp. 275-293.
21. This matter is explained in detail in Vouyouklakis/Mariolis (1992), pp. 156-160.
22. Since $\ell A^i f = 0$, $i = 0,1,2,...,$ it follows:
    a) $\ell[I-A]^{-1}f = \ell(I+A+A^2+....)f = 0$
    b) $pA^i f = 0$, $i = 0,1,2,...,$ because: $pf = w\ell[I-(1+r)A]^{-1}f = 0$, $pAf(1+r) = pf - w\ell f = 0$ etc.
23. The standard system is the system that, using the linear technique $[A, \ell]$, produces as its gross product q^*. The average capital intensity in price terms of the standard system is equal to:

$$K_{q^*} = \frac{pAq^*}{\ell q^*}.$$

$$K_n = \frac{pq^*}{\ell q^*(1+R)} \quad \Rightarrow$$

$$K_n = \frac{c}{\ell q^*(1+R)} (= K_{q^*}) \qquad (16)$$

And so within the framework of this work we showed that apart from the known normalizations of P.Sraffa and T.Miyao and the less known normalization of S.Vassilakis there are other normalizations that lead to an invariable K_n and thus to a linear w-r- relation[24].

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We will show that, even if reducible production techniques are excluded, the comparison of the given production techniques concerning their profitability is in the general case impossible, because each judgment depends on the arbitrarily introduced price measure, that is, the arbitrarily introduced normalization equation[25].

Case 1. Suppose two irreducible production techniques are compared and for one it holds: rank $L^* = 1$, so that to each normalization corresponds a linear w-r-relation. Then it is possible, when we move from an arbitrary normalization to the normalization with the standard commodity of P.Sraffa (or with some of T.Miyao's

24. Alternatively we can treat the matter as follows: for each w-r-relation, irrespective of the price normalization, it holds $w = K_{q^*}(R-r)$ (that expression results from multiplying from the right of (1) with q^*) where K_{q^*}, for a given profit rate, say $\bar{r}$, is equal to the slope of the line that passes through the points (R, 0), $(\bar{r}, \bar{w})$. Therefore, a linear w-r-relation implies that the magnitude K_{q^*} and the price of the standard commodity (pq^*) do not change with changes in the profit rate. Thus, with the introduction of the price normalization equation $pq^* = c$ or the price normalization equation $p(q^*+f) = c$, one can secure the linearity of w-r-relation because they lead to a constant K_{q^*}. Indeed, they are the ones with that feature, since we have showed that the relative price of the commodities q^*, q^*+f are independent of the rate profit.

25. G. Stamatis, for the first time, showed that in the case, where the techniques are reducible, their comparison is impossible. See Stamatis (1993). *We emphasize that in the present paper we refer exclusively to irreducible and "non-adjacent" production techniques* (see the cases and the numerical examples that follow). Exactly for this reason, we consider that in the *general case* the comparison of the irreducible techniques (in terms of the envelope of the w-r curves) is impossible.

commodities, when they exist) of the other technique, so that both w-r- relation have at most one intersection point, that some switch and reswitching points "disappear".

Case 2. Suppose two irreducible production techniques to which the same standard commodity of P.Sraffa corresponds (or they have some of T.Miyao's commodities in common). Then it is possible, when moving from an arbitrary normalization to a normalization with the above commodity, to have the same results as in the previous case.

Case 3. Suppose a technique for which it holds: $1 < \text{rank } L^* < n$. Then, there exist at least two commodities - say γ_I, γ_{II} with $\gamma_I - e\gamma_{II} = f$ and $\ell A^t f = 0$, t =0,1,2,..., n-1, the relative price of which does not change with changes in the rate of profit. Based on the above, it can be easily shown, that, if γ_I, γ_{II} sequentially function as a normalization commodity and c, c/e function as constant of normalization respectively, the resulting w-r-relations are identical. In the case where we compare this technique with a technique in which its corresponding matrix L^* is of rank n, then moving from one normalization to another, the intersection points of the resulting w-r-relations change, because the w-r-relation corresponding to the second technique changes.

Finally, let us give two relevant numerical examples. In the first, with the change of the normalization equation, the value of the profit rate, where the two w-r curves of the compared techniques intersect, changes while in the second example, with the change of the normalization equation, the number of the intersection points of the two w-r curves changes. (See Case 1)

<u>**Example 1**</u>. Let the two production techniques be[26] :

$$A^I = \begin{bmatrix} 0{,}2 & 0{,}1 \\ 0{,}1 & 0{,}2 \end{bmatrix}, \quad \ell^I = \begin{bmatrix} 1, 1 \end{bmatrix}$$

$$A^{II} = \begin{bmatrix} 0{,}6 & 0{,}2 \\ 0{,}4 & 0{,}4 \end{bmatrix}, \quad \ell^{II} = \begin{bmatrix} 0{,}01, 0{,}01 \end{bmatrix}$$

If we normalize with $p_1 = 1$, then it follows:

$w^I = 0{,}7 - 0{,}3r$, $w^{II} = 16(1+r)^2 - 100\,(1+r) + 100$ and the two w-r curves intersect at $r^* = 0{,}2396$.

26. ℓ^I is a left eigenvector of A^I connected with its maximum eigenvalue ($\lambda_{\max}^{A^I} = 0{,}3$), but q* is common to A^I, A^{II}.

If we normalize with $p_1+p_2 = 1$, then it follows:

$w^I=0{,}35 - 0{,}15r$, $w^{II}=10 - 40r$ and the two w-r curves intersect at $r^* = 0{,}2422$

Example 2. Let the two production techniques be[27]:

$$A^I = \begin{bmatrix} 0{,}5 & 0{,}25 \\ 0{,}25 & 0{,}26 \end{bmatrix}, \quad \ell^I = \begin{bmatrix} 0{,}5492, & 0{,}3456 \end{bmatrix}$$

$$A^{II} = \begin{bmatrix} 0{,}5 & 0{,}25 \\ 0{,}25 & 0{,}25 \end{bmatrix}, \quad \ell^{II} = \begin{bmatrix} 0{,}5, & 0{,}5 \end{bmatrix}$$

If we normalize with $p_1 = 1$, then it follows:

$w^I=1{,}8208 - 1{,}1968(1+r)$, $w^{II}=0{,}125(1+r)^2 - 1{,}5(1+r)+2$ and the two w-r curves intersect at $r^* = 0{,}0197$, $r^{**} = 0{,}4059$.

If we normalize with $p_1+p_2\ 0{,}618 = 1$, in other words, if we take as normalization commodity the standard commodity of the second technique, then it follows:

$w^I=0{,}4493 - 0{,}8617r$, $w^{II}=0{,}4271 - 0{,}8090r$ and the two w-r curves intersect at $r^*=0{,}4213$.

Since the w-r-relation is in reality a characteristic feature of the normalization subsystem and not of the production technique of the given production system, the so-called comparison of techniques is in reality a comparison of normalization subsystems[28].

Changing the normalization equation, the corresponding normalization subsystems change and therefore the resulting conclusions as well. Finally, what is being compared with regard to their profitability are production systems producing the same net product (the normalization commodity) with different production techniques and not the production techniques themselves.

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27. ℓ^I is a left eigenvector of A^I connected with its maximum eigenvalue ($\lambda_{max}^{A^I} = 0{,}6573$).
28. This is clearly shown, in the case where, for a certain normalization, the two w-r-relations that result from techniques under comparison: a) have the same w_{max} and different Rs, b) intersect - within the significant economic price interval in only one point, c) one is strictly concave and the other strictly convex. In this case - as the direct result of the relation (15) - for each r, the chosen technique is the one in which the price of the means of production of the normalization subsystems is lower, while at the intersection point of the two w-r-relations the price of the means of production is equal in both normalization subsystems.

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