

A Simple Linear Static Model of International Trade

by
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We shall present below a simple linear static model of foreign trade, which holds for two or more countries, and we shall briefly set out its analytical usefulness.

So, assuming an 'open' system of production, i.e. a system having trade exchanges with other systems, which in a given period operates at x levels of activity,

$$x \geq 0, \quad (1)$$

where x is a $n \times 1$ vector, and produces the gross production Φ ,

$$\Phi \geq 0, \quad (2)$$

where Φ is a $n \times 1$ vector. Both x and Φ are given.

Assuming also

- Γ , $\Gamma \geq 0$, the $n \times n$ matrix of outputs,
 - A , $A \geq 0$, the $n \times n$ matrix of inputs in already existing domestic means of production,
 - C , $C \geq 0$, the $n \times n$ matrix of intermediate inputs,
 - L , $L \geq 0$, the $k \times n$ matrix of inputs in each of the k different kinds of labour, $1 \leq k \leq n$,
 - DL , $DL \geq 0$, the $n \times n$ matrix of real wages,
 - (Im) , $(Im) \geq 0$, the $n \times n$ matrix of imports,
 - K , $K \geq 0$, the $n \times n$ matrix of consumption of capitalists,
 - S , the $n \times n$ matrix of net investments,
 - (Ex) , $(Ex) \geq 0$, the $n \times n$ matrix of exports,
- when the system operates at unitary activity levels, where*
- D , $D \geq 0$, the $n \times k$ matrix of k different wage rates.

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All the columns of the matrixes Γ , A , L , D and therefore of the matrix DL are positive or semi-positive. The total inputs of the system are equal to $Ax + Cx + DLx + (Im)x$.

We shall include in the total inputs of the system also the consumption of capitalists Kx . Consequently, the following holds for the total inputs of the system $\bar{A}x$:

$$\bar{A}x = [A + C + DL + (Im) + K]x,$$

where

$$\bar{A} = A + C + DL + (Im) + K. \quad (3)$$

The following holds for the total output (= gross production) Φ of the system

$$\Phi = \Gamma x. \quad (4)$$

How is the total output Φ used?

Regarding the use of the total output Φ , the following holds

$$\Phi = C^*x + A^*x + (DL)^*x + K^*x + (Ex)x, \quad (5)$$

where C^*x the intermediate *outputs*, A^*x the means of production produced during the period, $(DL)^*x$ the wage commodities produced during the period, K^*x the luxury commodities produced during the period and $(Ex)x$ the exports of the period.

Because, by definition, the intermediate inputs Cx of the system are always equal to its intermediate outputs C^*x ,

$$Cx = C^*x,$$

the following holds for the net output Ψ of the system

$$\begin{aligned} \Psi &= \Phi - \bar{A}x = \Gamma x - \bar{A}x = (\Gamma - \bar{A})x = \\ &= [A^* + C^* + (DL)^* + K^* + (Ex)]x - [A + C + DL + K + (Im)]x = \\ &= \{(A^* - A) + [(DL)^* - DL] + (K^* - K) + [(Ex) - (Im)]\}x = \\ &= S + E, \end{aligned}$$

The vector S ,

$$S = \{(A^* - A) + [(DL)^* - DL] + (K^* - K)\}x,$$

evidently represents the net investment of the system and the vector E ,

$$E = [(Ex) - (Im)]x,$$

represents the net exports of the system (see Stamatis 1992). Consequently, the net output Ψ of the system is identical to the saving of the system. Thus, the system uses the net outputs of Ψ for net investments and for net exports.

Both the vector S and the vector E may clearly have any composition. That is, they may contain (a) only positive quantities of commodities, (b) positive and zero quantities of commodities, (c) positive, zero and negative quantities of commodities, (d) positive and zero quantities of commodities, (e) zero and negative quantities of commodities or, lastly, (f) only negative quantities of commodities. The same evidently holds also for the vector of net outputs of the system Ψ ,

$$\Psi = S + E.$$

Assuming F , $F \geq 0$, the $n \times 1$ vector of the system's stocks of means of production and consumption commodities (wage commodities and luxury commodities that enter only the consumption of capitalists).

The following evidently holds

$$F_\tau = \sum_{t=0}^{\tau-1} S_t,$$

where t is the period of production and τ the present period of production.

Therefore

$$\Delta F_t = S_t, \forall t,$$

that is

$$\Delta F = S,$$

where ΔF is the absolute change of F .

The following holds for the net output of the system Ψ_i in commodity i

$$\Psi_i = S_i + E_i,$$

where $S_i \geq 0$ and $E_i \geq 0$. When $S_i < 0$, then of course the investment of the system in commodity i is deinvestment. And when $E_i < 0$, E_i represents the net *imports* of the system.

Assuming that $\Psi_i > 0$. Then we can discern five cases:

Case 1: $S_i > 0$ and $E_i = 0$. In this case, $\Psi_i = S_i$, that is, the saving Ψ_i of the system in commodity i is equal to investment S_i of the system in commodity i . The saving Ψ_i of the system in commodity i is used solely and exclusively for investment and not for net exports also.

Case 2: $S_i = 0$ and $E_i > 0$. In this case, $\Psi_i = E_i$, that is, the saving Ψ_i of the system in commodity i is equal to the net exports of the system in terms of commodity i . The saving Ψ_i of the system in commodity i is used solely and exclusively for net exports of commodity i . With the net exports of E_i , the system meets consumption and/or investment of the other systems in commodity i .

Case 3: $S_i < 0$ and $E_i > 0$. In this case the net exports E_i of the system in commodity i are

$$E_i = \Psi_i - S_i = \Psi_i - \Delta F_i,$$

where $S_i (= \Delta F_i) < 0$. Consequently, the net exports of the system in commodity i originate not only from saving Ψ_i of the system in commodity i but also from the reduction ΔF_i of stocks F_i of the system in commodity i . Thus, the system exports as net export also part of its stocks of commodity i in order to meet consumption and/or investment of the other systems in commodity i .

It is worth noting that to the extent the system meets investment of other systems in commodity i with this net export of commodity i , it increases the stocks of those systems in commodity i , thus reducing its own stocks of commodity i .

Case 4: $S_i > 0$ and $E_i > 0$. In this case, the system uses the saving Ψ_i in commodity i partly for investment S_i , that is to increase stocks F_i of commodity i , and partly for net exports E_i , with which it meets consumption and/or the investment of other systems in commodity i .

Case 5: $S_i > 0$ and $E_i < 0$. In this case, the system invests in commodity i more than it saved in commodity i , $S_i > \Psi_i$. The system covers the difference $\Psi_i - S_i (= E_i)$ with net imports of commodity i . Thus, in this case, the other systems cover with their saving and/or with their stocks of commodity i part of the investment of the system in commodity i .

Assuming now that $\Psi_i < 0$. We discern here also five cases:

Case 1: $S_i < 0$ and $E_i = 0$. In this case, the system covers part of its consumption or all of its consumption in commodity i from its stocks F_i or commodity i .

Case 2: $S_i = 0$ and $E_i < 0$. In this case, the system covers part of its consumption or all of its consumption in commodity i with net imports.

Case 3: $S_i > 0$ and $E_i < 0$. In this case, the system covers all of its investment and part of its consumption or all of its consumption in commodity i with net imports of commodity i .

Case 4: $S_i < 0$ and $E_i < 0$. In this case, the system covers part or all of its consumption in commodity i from its stocks of commodity i and from net imports of commodity i .

Case 5: $S_i < 0$ and $E_i > 0$. In this case, the system covers from its stocks of commodity i not only part of its consumption or all its consumption of commodity i but also, in the form of net exports of commodity i , the consumption and/or investment of the other systems in terms of commodity i . It is worth noting that to the extent the system covers the investment of the other systems in commodity i , it increases the stocks of commodity i of those systems, reducing its own stocks of commodity i .

These commonplace but relatively complex relationships, which are established by virtue of the international division of labour, entail certain noteworthy things. As we saw, the following holds

$$\Psi = (\Gamma - \bar{A})x.$$

So, assuming that $(\Gamma - \bar{A})^{-1}$ exists. Then

$$x = (\Gamma - \bar{A})^{-1} \Psi \Rightarrow$$

$$\Gamma x = \Phi = \Gamma(\Gamma - \bar{A})^{-1} \Psi.$$

Because, as we saw, Ψ may have any composition, while $\Phi \geq 0$ holds for Φ , $(\Gamma - \bar{A})^{-1}$ may contain negative components. This means that in the framework of the international division of labour, the following may exist

- (a) systems, which consume and invest more than they produce ($pDLx + pKx + pS > \bar{Y}$) or *only* consume more than they produce ($pDLx + pKx > \bar{Y}$),

because for these, $pDLx + pKx + pS - pE = \bar{Y}$, $pE < 0$ holds, where $pS \geq 0$, and

- (b) systems, which consume less than they produce ($pDLx + pKx < \bar{Y}$) or consume *and* invest less than they produce ($pDLx + pKx + pS < \bar{Y}$), because for these, $pDLx + pKx + pS - pE = \bar{Y}$, $pE > 0$ holds, where $pS \geq 0$.¹

Because $pS \geq 0$ holds, both for the systems of category (a) and for those of category (b), it is possible for the systems of category (a) to grow ($pS > 0$), while for systems of category (b) to 'shrink' ($pS < 0$).

The existence of all these systems is due to the 'credits' which certain systems of category (a), through their net exports, give to systems of category (b).

Precisely because of these 'credits' however, it is also possible within the framework of the international division of labour for there to exist for a certain period of time –and this is what is noteworthy– not only systems that are not simply 'deficient' and for which $pE < 0$ therefore holds,² but also systems that are non-profitable, i.e. systems for which $\Pi \leq 0$, where Π is profit.

The following evidently holds for profit Π of the given system of production

$$\Pi = pKx + pS + pE.$$

It is clear that, because it is possible for $S \leq 0$, Π may be negative.

What is most important and noteworthy however is the following: Precisely because of the above-mentioned 'credits', the nominal net product $\bar{Y}$ of an 'open' system,

$$\bar{Y} = p(DL)x + pKx + pS + pE,$$

may be negative. Thus, the international division of labour allows for a certain period of time the existence not only of non-profitable but also of non-productive systems – even though 'open' systems in total, as a single system, constitute a profitable and consequently productive system.

1. $p, p > 0$, is the $1 \times n$ vector of prices and $\bar{Y}$ the nominal net product.

2. Because, as we noted previously, the vector E may contain positive and negative quantities of commodities, whether a system is or is not 'deficient' cannot in the general case be ascertained on the basis of the vector of net exports E of that system, but only on the basis of the nominal value pE of net exports of the system.

Whatever holds for each system of the 'closed' total of 'open' systems of production however also holds for each sector of a 'closed' system. Furthermore, because the model of the distinct sectors of all the distinct 'open' systems of production of a 'closed' total of such systems constitutes a 'holistic' reproduction of the model of the distinct 'open' systems of a 'closed' total of such systems, whatever holds for each system of the 'closed' total of 'open' systems of production holds not only for each sector of a 'closed' system of production, but also for each sector of each 'open' system of a 'closed' total of 'open' systems of production.

Therefore, what we set out above also holds for each sector of a 'closed' or 'open' system.

Lastly, we should like to make an observation regarding the multitude n of commodities which *may* be produced by the given 'open' system. The multitude n of commodities produced by the given system in all cases contain also the commodities which may be imported by the given 'open' system from the other 'open' systems, i.e. all the commodities which are produced by these latter systems. This is so because each commodity that is imported by the given 'open' system from other 'open' systems is, even if sold by the importer to the end consumer, a commodity produced by the given 'open' system which imported it (see Stamatis 1992a).

References

- Stamatis, G. (1992), *Reproduction, income circulation and national accounting*, 2nd edition, Kritiki Publications, Athens.
- Stamatis, G. (1992a), The role of 'circulation' in the reproduction of the economic system and in the production of surplus value, in: *Problems of Marxist economic theory*, 3rd edition, Kritiki Publications, Athens.

