Ibn-Sina's Anticipation of Burdian and Barcan Formulas

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Abstract

In Quantified Modal Logic, there are four fundamental theses with significant ontological consequences. Of these theses, which have been the center of much discussions since 1950's, two are attributed to the French philosopher Jean Buridan (1300-1358) and the other two to the contemporary American philosopher Ruth Barcan. But as history has it, it can be shown that all these theses were anticipated by Ibn-Sina (980-1037). The paper is both historical and expositional.

I. Contemporary attributions of some theses in QML

In 1947 Carnap wrote: (p. 196)[1]

> Any system of modal logic without quantification is of interest
> only as a basis for a wider system including quantification. If such
> a wider system were found to be impossible, logicans would probably
> abandon modal logic entirely.

Surprisingly Carnap's anticipation turned out to be right, but exactly in the opposite direction! What captured the attention of modal logicians most, was propositional modal logic, and quantified modal logic (QML) has remained under-developed. Of course there are many systems of, and many approaches to QML[2]. But apart from provability interpretation of it, other treatments of the subject have been of interest to mostly philosophical logicians, and QMLs have become a battle ground for heated controversies over philosophy problems such as essentialism, transworld identity, existence, possible worlds and the very notion of necessity.

But among many versions of QML, the one I am interested in here is a system in which all the following formulas are derivable:

$$BF: \quad \forall x \Box Fx \quad \rightarrow \quad \Box \forall x Fx$$
$$CBF: \quad \Box \forall x Fx \quad \rightarrow \quad \forall x \Box Fx$$
$$BF: \quad \Diamond \forall x Fx \quad \rightarrow \quad \Box \forall x Fx$$

BF and CBF stand for the Barcan formula and the Coverse Barcan formula. These names drive from Ruth C. Barcan (later Mrs. J. A. Marcus), who called attention to the formulas in her paper of 1946[3]. Also BUF stands for the Buridan formula, attributed to Jean Buridan, French philosopher of the 14th century, by Alvin plantinga[4] (p. 58). BF and CBF have important semantical properties. To see how, I begin with the simplest model validating all the three formulas.

As to the language of QML, it contains individual constants and variables, and nplace predicates ($n \geq 0$). Well formed formulas of QML are inductively constructed out of $\neg, \rightarrow, \forall$, and $\Box$. Furthermore, $\Diamond$ and $\exists$ are defined in the standard way. The simplest model of QML is the sequence $M = < W, R, D, I >$, where W is a non-empty set of possible worlds, R a binary equivalence relation over the members of W, D a non-empty set of individuals and I an interpretation function which assigns to every term (variable or constant) a member of D : $I(t) \epsilon D$, and to each n-place predicate letter A a set of n-tuples of D : $I(A_{t1}, \ldots, A_{tn})$ is $< 0_I, \ldots, 0_n >$ where $0_i \epsilon D$. The truth-value of each wff at a world $w \epsilon W$ is given by the following rules:

$I(A): I(A_{t1}, \ldots, A_{tn})$ is true in ω iff $< I(t_I), \ldots, I(t_n) > \epsilon I(A)$ in ω;

$I(\neg A): I(\neg A)$ is true in ω iff $I(A)$ is false in ω;

$I(A \rightarrow B): I(A \rightarrow B)$ is true in ω iff $I(A)$ is true ω,
then I (B) is true in ω;

$I(\forall x \forall x): I(\forall x Ax)$ is true in ω iff for every I' which is just like
I except possibly that $I(x) \neq I'(x)$ $I'(Ax)$ is true in ω;

$I(\Box A): I(\Box A)$ is true in ω iff for every ω' such that $\omega R \omega'$, $I(A)$
is true in ω'.

It is easy to verify that BF, CBF and BUF are all valid in M. Other logically and philosophically interesting aspects of this simple semantics are as follow:

1. BF imposes the condition that whenever $\omega R \omega'$, then $D(\omega') \subseteq D(\omega)$ and *CBF* the condition that whenever $\omega R \omega'$, then $D(\omega) \subseteq D(\omega')$. So if we have both *BF* and *CBF*, then $\omega R \omega'$ implies $D(\omega) = D(\omega')$. Therefore to have both formulas, the domains of all worlds should have the same objects. M has this property, so it is called a constant model of QML:

2. According to BF and CBF, to move from one world to another nothing comes into existence and nothing passes out of existence.

3. Since R is an equivalence binary relation, all S5 modal axioms are valid in M, in fact in the frame $< W,R >$ of M.

> Finally to obtain the simplest QML respecting our semantics, it suffices to add the laws and rules of classical quantification theory to the propositional modal logic S5 (pp.245-247)'. This is a kind of systems that philosophical logician like R.Barcan, E.N. Zalta, B. LinsKY[6] and T. Williamson[7] argue that it best represents our intuitive understanding of metaphysical nature of alethic modality.

2. Ibn-Sina's anticipations

According to the received texts of the ancient Greek and the early Islamic period Ibn-Sina (Avicenna, 980-1037) was the first logician who realized that the meanings of modalquantified propositions are sensitive to the order of their quantifiers and modal operators and extensively discussed what are now called Buridan and Barcan formulas. So in Kitab al-Ibara, the third volume of his major work Al-Shifa (in Latin known as Sufficientia) Ibn-Sina writes: (pp.115-116)[8]

> If we say that (*) every human being possibly is a writer,...., no one would doubt it,..., but if we say that (**) Possibly every human being is a writer, and possibility comes before quantifier, then that would be really doubtful. (My translation from Arabic.)

Here, Ibn-Sina, after some illuminating remarks, concludes that (**) implies (*) but not vice versa. So he accepts BUF and rejects its converse CBUF as a

modal principle. As to BF and CBF he writes: (pp.115-116 again)[9] But to say that some people possibly are not writers is modally the same as saying that possibly some people are not writers.

In modern symbolism this is formulized as:

$$\exists x \Diamond \neg W(x) \leftrightarrow \Diamond \exists x \neg W(x)$$

and by taking negations of both sides, we have

$$\forall x \Box \neg W(x) \leftrightarrow \forall x W(x)$$

which is the conjunction of BF and CBF in one biconditional.

Finally, and this is what I regard as Ibn-Sina's most important innovation in the subject, he discovered that there are two kinds of modality and coined the names "mode of predication" (Jahat-e Haml) and "mode of quantifier" (Jahat-e sour) for what, more than one hundred years later, medeival logicians called respectively de re and de dicto modality. This is a distinction which has become the subject of many discussions in modern philosophy of logic and language.

In conclusion I think, to do justice to the pioneer work of Ibn-Sina, it would be fair to re-name Buridan and Barcan formulas as Ibn-Sina-Buridan and Ibn-Sina-Barcan formulas.

Acknowledgment

References

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