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VOLATILITY MODELLING OF FINANCIAL MARKETS

By

PANAGIOTIS DELIS

A dissertation submitted in partial fulfillment of
the requirements for the degree of

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VOLATILITY MODELLING OF FINANCIAL MARKETS

Abstract

by Panagiotis Delis, Ph.D.
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Forecasting oil price volatility is considered of major importance for numerous stakeholders, including, policy makers, industries and investors. The first study (Chapter 2) examines and evaluates the main factors that oil price volatility forecasters should consider before constructing their forecasting models. Such factors are related to: i) direct vs iterated forecasts, ii) the incorporation of continuous and jump components, iii) the importance of semi variance volatility measures, and iv) OLS vs time-varying parameter (TVP) estimation procedures. We evaluate the performance of these factors for both the realized and implied volatility measures of the WTI crude oil price, based on statistical loss functions, as well as, their economic use. The results show that depending on whether end-users are interested in forecasting the realized or the implied volatility, the factors influencing the accuracy of forecasts are different.

In the third Chapter, we enhance the modelling framework by incorporating exogenous information in the proposed models. Nowadays, it is noteworthy the fact that the global uncertainty plays a major role to the economic outlook and more specifically the financial and energy markets. In this study (Chapter 3), we focus on the impact of the various uncertainty

factors on the oil price volatility, which is considered crucial not only for the global economy but also for the financial markets because of its financialization. However, uncertainty can be captured by different factors, which provide dissimilar information to oil price volatility. We categorize those factors to the following classes: Implied volatility indices, financial stress indices and other indicators related to the uncertainty environment, such as economic policy uncertainty, geopolitical risk and business conditions. Our main findings provide strong evidence that taking uncertainty indicators into account enhances the predictive accuracy of oil price volatility at all forecasting horizons. Moreover, the results indicate that the Dynamic Model Averaging (DMA) is considered significant for forecasting oil price volatility by combining the different indicators of the three classes and giving the corresponding weight to the model.

However, crude oil investors would be interested not only in maximizing their profits but also in minimizing the risk of their portfolios, which could be managed by hedging the crude oil portfolio in an efficient way. In this regard, the existing literature has studied the interrelations between crude oil and other asset classes, including stock, foreign exchange markets and market reflecting macroeconomic conditions focusing mainly on their returns. In this study (Chapter 4), we concentrate on the time-varying correlation of the volatility measures of crude oil and three asset classes using a dynamic conditional correlation (DCC) model. The main objective of this study is to examine the optimal portfolio weights, constructed by the variance-covariance matrix, for portfolios comprised of the aforementioned volatility measures and to identify whether the investors could benefit from the interactions of the WTI crude oil and the three assets. The aim of this study is to focus on volatility and not on returns, since investors and academics concentrate their attention to the volatility of crude oil recent years. The results of the correlations indicate a time-varying behavior, which gives a signal to investors that they have to re-balance their portfolios regularly in order to minimize their portfolios' risk. Finally, the findings show that the asset that offers higher opportunities for hedging the WTI crude oil volatility is that of the U.S. T-bills, which represents the market related to macroeconomic conditions.

Finally, Chapter 5 aims to investigate the predictive information of the daily crude oil realized volatility on the U.S. economy. More specifically, oil price volatility has attracted the attention of the academic community because of its crucial impact on the economic outlook. However, there is a gap in literature with regard to the impact of crude oil price volatility on special aggregates of industrial production. In this study (Chapter 5), we propose MIDAS models including different daily crude oil realized volatility measures in order to investigate their impact in an out-of-sample analysis. For comparison reasons, we study the effect not only of crude oil but also of other assets' realized volatility, such as S&P500 index, U.S. dollar index and U.S. T-bills. Moreover, we set a group of monthly macroeconomic, oil-related and uncertainty-related variables as predictors of the industrial production aggregates in order to evaluate whether the impact of the daily realized volatility measures is significant or the monthly predictors themselves offer adequate information. The results show that crude oil realized positive semivariance can definitely provide higher forecasting performance in the models used for producing energy-related industrial production measures, which is not the case for the non energy-related ones.

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Dedication

This dissertation/thesis is dedicated to my family (Kostas, Ioanna and Dimitra) who provided me with both emotional and financial support. Without their daily support, I would not have been in the position to complete my research studies. I really appreciate what they have done for me all these years! This dissertation is also dedicated to my supervisor, Stavros, who helped me dealing with several professional and personal issues (by spending infinite number of hours in calls).

Chapter One

Thesis introduction

At the beginning of this journey the main goal was to better understand the evolution of the realized volatility of a crucial for the economy market, namely the crude oil, over time. We, first, tried to identify some key characteristics of the aforementioned market in order to incorporate them in the modelling framework we implemented. More specifically, crude oil prices appear to be highly volatile and also subject to structural breaks. This holds true for the crude oil volatility as well. Therefore, we thought that by applying suitable time-varying parameter models and including uncertainty indicators as potential drivers of the crude oil volatility, the forecasting performance of simple models could be enhanced, which is a combination of elements that has not been investigated so far. A a later point in time, we aimed to investigate the existence of potential hedging opportunities between crude oil volatility and other assets' volatility, since it is considered of major importance for crude oil investors to minimize the risk of their portfolios. In this regard, we had in mind that crude oil market has unique characteristics and properties in comparison with the stock and foreign exchange markets and this fact could justify an efficient hedging between crude oil and those asset classes. Finally, we had as a main objective to show that crude oil realized volatility[1] is information-rich for the U.S. industrial production which is one of the main pillars for economic growth.

Summarizing the general aspects of the present dissertation, we mainly study the volatility

[1] We also investigated the performance of other assets' realized volatility.

of the crude oil market, which has attracted the attention of academics, investors and policy makers. This happens due to the fact that the oil market is considered a highly profitable investment for investors' and financial institutions' portfolios. Moreover, oil price volatility can drive uncertainty for energy companies or oil-intensive companies. Therefore, the entire economic outlook through industrial production levels could be affected by oil price volatility, which is one of the main features being investigated in this dissertation.

In the first chapter, the research question of what matters when developing oil price volatility forecasting frameworks is answered. According to this study, several static and dynamic models are developed with different forecasting approaches for the generation of multi-step ahead forecasts and implemented. The evaluation of the produced crude oil volatility forecasts is conducted by using not only statistical loss functions but also trading strategies. The results show that depending on whether end-users are interested in generating realized volatility (RV) or implied volatility (IV) forecasts, the factors having influence on the forecasting accuracy are different. More particularly, for RV, direct forecasting, which is based on time-varying parameter (TVP) models could provide more accurate forecasts compared to the alternative modelling frameworks. The forecasting performance is also enhanced when incorporating realized semi variance measures in the right hand side of the regression's equation. With regard to IV forecasts, TVP procedures seem to improve the forecasting performance of the applied models. Finally, the results for the case of IV provide evidence that the continuous component and the semivariance measures offer better forecasting gains in the longer run horizons.

It is noted that no exogenous variables are incorporated in the Chapter 2. The impact of several factors of uncertainty on oil price volatility is investigated in the second study (Chapter 3). This is an out-of-sample analysis that categorizes factors of uncertainty in the following classes: IV indices, financial stress indices and other indicators related to the uncertainty environment and use them as potential drivers of oil price volatility. The main finding is that taking into consideration uncertainty factors helps to enhance the predictive accuracy of oil price volatility at all forecasting horizons. In more detail, the use of Dynamic Model Averaging (DMA) is considered significant for forecasting oil price volatility by combining the different indicators of the three classes and giving the corresponding weight to the model. Our evaluation framework includes the use of statistical loss functions and also one strategy of trading the United States Oil Fund (USO), which confirm the high impact of the uncertainty factors on oil price volatility.

Having focused on forecasting the crude oil volatility is considered useful for oil investors. However, they would be really interested in minimizing the risk of their portfolios. Therefore, in this dissertation, we study the existence of potential hedging opportunities between crude oil price volatility and volatility measures of other assets from stock, foreign exchange markets and the market that reflects macroeconomic conditions. In this specific study, which is maintained in Chapter 4, the focus is on the time-varying correlation of the volatility measures of crude oil and three asset classes using a dynamic conditional correlation (DCC) model. The main objective of this study is to examine the optimal portfolio weights, constructed from the variance-covariance matrix, for portfolios comprised of the aforementioned volatility measures and to identify whether the investors could benefit from the interactions of WTI crude oil and the three assets. The results of the correlations indicate a time-varying behavior, which gives a signal to investors that they have to re-balance their portfolios regularly in order to minimize their portfolios' risk. Finally, the findings show that the asset that offers higher opportunities for hedging WTI crude oil volatility portfolio is that of U.S. T-bills, which represents the market related to macroeconomic conditions.

As mentioned above, crude oil volatility is crucial for the economic outlook. In this study, we focus on the impact of oil price volatility measures on the U.S. economy and more specifically on special aggregates of industrial production. Due to the fact that oil price RV measures are sampled at daily frequency (constructed by using intra-day data) and industrial production at monthly frequency, a MIDAS modelling framework is used in order to investigate the impact of RV measures when generating out-of-sample industrial production forecasts. For comparison reasons, we study the effect of not only crude oil but also other assets' realized volatility, such as S&P500 index, U.S. dollar index and U.S. T-bills. Moreover, we set a group of monthly macroeconomic, oil-related and uncertainty-related variables as predictors of the industrial production aggregates in order to evaluate whether the impact of the daily RV measures is significant or the monthly predictors themselves offer adequate information. The results show that crude oil realized positive semivariance can definitely provide higher forecasting performance in the models used for producing energy-related industrial production measures, which is not the case for the non energy-related ones.

Finally, We hope that we managed to provide an integrated modelling framework that focuses on the realized volatility of crude oil, which is found to be information-rich for the economic outlook and therefore professional forecasters, investors and policy makers should take the findings of this dissertation into account.

Chapter Two

What matters when developing oil price volatility forecasting frameworks?

2.1 Introduction

According to Elder and Serletis, 2010, oil price uncertainty exerts a great impact on the global economy and a number of studies have shown that such uncertainty has important macroeconomic effects (Ferderer, 1996; Kilian & Park, 2009). Moreover, crude oil prices have been highly volatile in recent years, with prime examples the price swing observed during the Global Financial Crisis of 2007-2009 and the oil price collapse period of 2014-2016. The aforementioned claims render oil price volatility forecasts very important.

Accurate forecasts of crude oil price volatility are also important because of the financialisation of the oil market, as Pen and Sevi, 2017 note. In addition Silvennoinen and Thorp, 2013 observe that the oil market is considered as a highly profitable investment for financial institutions' portfolios. Furthermore, oil price volatility could drive uncertainty at higher level for oil companies or oil-intensive industries, placing it as one of the most important factors for oil risk management purposes. Such features place oil price volatility forecasts at the centre of the policy makers', oil-intensive industries' and investors' attention.

The advent of ultra-high frequency data (intra-day) offers the ability for the so-called realized volatility (RV) to be estimated. Thus, models based on lower frequency returns have lost their appeal due to the fact that they are not capable of exploiting all the available information in the dataset. Among the realized volatility modelling frameworks, one of the most popular and widely used is the heterogeneous autoregressive (HAR) model proposed by Corsi, 2009. The HAR model exploits the heterogeneity of market participants, which originates from the difference in investment horizons. Recent literature has shown that HAR model is considered the standard benchmark for forecasting volatility dynamics (Chen et al., 2010; Degiannakis & Filis, 2017; Sévi, 2014).

Furthermore, given the importance of forecasting realized volatility, the literature focuses on developing different approaches, which are, nevertheless, based on the HAR model. Regarding oil volatility forecasting, indicative recent studies are those of Klein and Walther, 2016, F. Ma et al., 2018, Zhang and Wang, 2019 and Lin et al., 2020. Many studies enhance the HAR model to capture the jump component in volatility, which is estimated by decomposing quadratic variation into continuous and jump terms[1]. Some papers investigate whether the jump components can provide higher forecasting accuracy in HAR-type models when referring to crude oil market. Sévi, 2014 and Prokopczuk et al., 2016 conclude that the decomposition of quadratic variation into continuous component and jumps does not lead to higher forecasting accuracy. In contrast, L. Liu et al., 2018 and F. Ma et al., 2018 find that incorporating an appropriate jump component can provide higher performance in forecasting realized volatility of crude oil futures market[2]. Moreover, according to Patton, 2011, in the context of stock market volatility, realized semi variance components provide useful information for forecasting realized volatility with one part coming from the high-frequency returns and a part

[1] Barndorff-Nielsen and Shephard, 2006, Tauchen and Zhou, 2011 and Andersen et al., 2012 have proposed a number of non-parametric tests to detect jumps.

[2] L. Liu et al., 2018 suggest that the decomposed jumps with a certain threshold can improve the forecasting performance. F. Ma et al., 2018 conclude that proposed new variables, such as signed jump variations, have a significant influence on the future volatility.

from the negative high-frequency returns.

Most existing studies, which use HAR-type models to forecast oil price volatility, use constant coefficients estimated by OLS method. However, time-variation in the parameters of the HAR modelling framework is also an important consideration. This study is motivated by Spiegel, 2008, who asks if academics could "produce an empirical model that allows for economic changes over time that is also capable of determining the 'right' parameter values in time to help investors?" (p. 1454). This question is considered really important for the researchers due to the fact that the issue of time-variation in coefficients might affect the results. In general, coefficients may vary over time due to changes in monetary policies, in the institutional framework and in market sentiments.

More specifically, a number of studies note that the statistical property of asset volatility (e.g., volatility persistence) undergoes frequent structural breaks or switches between different regimes due to extreme events and economic policies[3]. Therefore, the autoregressive coefficients that determine the impact of past volatility terms on current volatility can change over time and this is one reason why we implement TVP models in the HAR-type structure. We also follow a broad number of studies that find evidence of time-variation in the coefficients and show that models, which take into consideration this issue, present better forecasting performance than constant-parameter models[4].

According to Wang et al., 2016, the existing HAR-type models are predictive regressions with constant coefficients and they cannot capture changes in predictive relationships. The most implemented TVP methodology in the literature relies on the work of Koop and Korobilis, 2012, which was first proposed by Raftery et al., 2010. This methodology is simple and offers

[3]For further details see Calvet and Fisher, 2004, Banerjee and Urga, 2005, C. Liu and Maheu, 2008 and Rapach and Strauss, 2008.

[4]For further details see studies such as: Mcaleer and Medeiros, 2008 and Bollerslev et al., 2016.

the ability to avoid MCMC-based Bayesian methods.

Using the TVP HAR-type models, one target of this study is to assess if the forecasting performance of the models that will be estimated under the TVP modelling framework can outperform the simple HAR-RV estimated by OLS. However, it is noted that there is a large number of TVP approximations, which use the Kalman filter for updating the observation and state equations. The latter process constitutes the structure of a dynamic model as West and Harrison, 1997 have analytically explained. Most of the published studies rely on the work of Koop and Korobilis, 2012, since they use the dynamic model averaging (DMA) model, which is a model that allows not only the parameters but also the regressors to change over time. In this case, we allow coefficients to change over time in order to capture potential structural breaks without taking into account changes of regressors, since no exogenous information is considered in these forecasting models.

Furthermore, it is a challenging task to decide on the TVP approximation that provides better forecasting performance. The TVP model proposed by Raftery et al., 2010 includes the approximation of the forgetting factor. Nevertheless, Grassi et al., 2017 suggest that another approximation, namely the standardized self-perturbed Kalman filter, proves to be a valid alternative to online methods based on forgetting factors. The concept of this methodology is that the measurement error variance enters directly in the updating step in a way that the update of the state equation becomes endogenously determined by the amount of uncertainty in the data. In this chapter, both approaches are implemented in order to fill the gap on this concern. Moreover, in the case of the crude oil market, as far as we are aware, there is no published study that provides realized and implied volatility forecasts of crude oil market under a TVP modelling framework, which is a significant contribution of this study to the literature.

Another issue that we also investigate in this chapter is whether the forecasts of oil price volatility have to be obtained from the direct or the iterated approach when referring to multi-step ahead forecasts. According to Marcellino et al., 2006, the iterated forecast entails the estimation of an autoregression, then iterating it upon that autoregression to obtain the multiperiod forecast. In contrast, the forecast based on the direct approach entails regressing a multiperiod-ahead value of the dependent variable on current and past values of the variable. Most studies which focus on forecasting realized and implied volatility use the direct approach when forecasting multiple steps ahead. The only exception is the work of Degiannakis and Filis, 2017, who provide iterated forecasts in order to forecast crude oil realized volatility without, though, comparing them with forecasts provided by the direct approach which is investigated in this study. Moreover, to the best of our knowledge, there is no study that provides iterated forecasts through TVP-HAR models. In this study, the different explanatory components are separately modeled (e.g. jump component, realized semi variance components), which is in line with the study of Busch et al., 2011, in order to provide the iterated forecasts. It is important to note that the iterated method is prone to bias if the one-step ahead model is misspecified. In addition, there are contradictions among different studies for choosing the most efficient forecasting method[5]. This study provides a thorough assessment as to which is the most efficient way, when it comes to direct or iterated forecasts.

Interestingly enough, all the aforementioned studies are interested in forecasting the realized volatility measure and not the implied volatility. However, it is widely known that volatility, which is implied by an option's price, is considered as capturing the anticipated asset volatility over the remaining life of the relevant option[6]. Thus, in this paper, we implement this forecasting framework for both the realized and implied volatility measures of the WTI oil prices.

Overall, this study brings together all aforementioned factors related to the most appropri-

[5]Cox, 1961 suggests that direct multi-period forecasts can be more efficient than iterated forecasts.

[6]Early studies have shown that implied volatility is interpreted as an efficient volatility forecast (Harvey & Whaley, 1992).

ate modelling framework, thus, it investigates what really matters when obtaining multi-step ahead (1-day up to 66-days ahead) oil price realized and implied volatility forecasts. Hence, this chapter offers a comprehensive forecasting framework in this line of research.

The findings show that for the WTI realized volatility forecasts it is really important to include the realized semi variance measures in the HAR models by using the TVP estimation methodology, which improves the forecasting performance of the models. The choice of direct forecasting procedure also improves the WTI realized volatility predictions. By contrast, the decomposition of the realized volatility into its continuous and jump components does not enhance the forecasting performance of the models. In the case of OVX, we also find that the TVP outperforms the rest of the competing models, although this is evident only based on the economic evaluation of the forecasts. In addition, for the long-run horizons, we also show that the information obtained from the continuous component and the semi variance measures of the WTI realized volatility improves the OVX forecasting performance.

The remainder of the chapter is structured as follows. In Section 2.2 the calculation of the different realized volatility measures, which have been included in the modelling specifications is described. In Section 2.3 we describe the data used in the study. In Sections 2.4 and 2.5 we present the modelling and forecasting frameworks, respectively. Section 2.6 details the statistical and economic evaluation framework, whereas Section 2.7 discuss the out-of-sample forecasting results. Finally, Section 2.8 concludes the study.

2.2 Estimating realized volatility

To measure the daily quadratic variation using intra-day data, a widely known realized measure is used, the so-called realized volatility (RV). Thus, let the intra-day returns be denoted by $r_{t,j} = log(\frac{p_{t,j}}{p_{t,j-1}})$, where $p_{t,j}$ is the oil price, for $j = 1, 2, \ldots, M$, denoting the time of observation within a particular day, and $t = 1, 2, \ldots, T$ represents the number of trading days. According to Andersen and Bollerslev, 1998, the daily realized volatility, DRV_t, is defined as the sum of squared intra-day returns:

$$DRV_t = \sqrt{\sum_{j=1}^{M} r_{t,j}^2} \quad . \tag{2.1}$$

The realized volatility converges to the integrated volatility as the sampling frequency and the number of intra-day intervals (M) approach infinity. In this chapter, we rely on Hansen and Lunde, 2005 and we work with annualized realized volatility series, which are calculated as:

$$RV_t = \sqrt{252}\, DRV_t. \tag{2.2}$$

Barndorff-Nielsen and Shephard, 2006 prove that the integrated volatility can be estimated by the realized bipower variation, RBV_t, as:

$$RBV_t = \sqrt{252}\, \mu_1^{-2} \frac{M}{M-2} \sum_{j-3}^{M} |r_{t,j-2}||r_{t,j}|, \tag{2.3}$$

where $\mu_1 = \sqrt{2/\pi}$ and $M/(M-2)$ denotes an adjustment for the sample size. Barndorff-Nielsen and Shephard, 2006 and Huang and Tauchen, 2005 apply the z-statistic in order to identify the discontinuous jump variation:

$$Z_t = \sqrt{M} \frac{(RV_t - RBV_t)RV_t^{-1}}{(\mu_1^{-4} + 2\mu_1^{-2} - 5) max(1, \frac{RTQ_t}{RBV_t^2})}, \tag{2.4}$$

where RTQ_t is the realized tri-power quarticity:

$$RTQ_t = \sqrt{252}\ M\mu_{4/3}^{-3}(\frac{M}{M-4})\sum_{j=4}^{M}|r_{t,j-4}^{4/3}||r_{t,j-2}^{4/3}||r_{t,j}^{4/3}|, \tag{2.5}$$

where $\mu_{4/3} = 2^{2/3}\Gamma(7/6)\Gamma(1/2)^{-1}$. The daily discontinuous jump variation $J_t^{(d)}$ can be defined by

$$J_t^{(d)} = I(Z_t > \phi_\alpha)(RV_t - RBV_t), \tag{2.6}$$

where $I(.)$ is the indicator function, which identifies the significance of the Z_t statistic in excess of a given critical value of the Gaussian distribution ϕ_α.

Additionally, the continuous sample path variation $C_t^{(d)}$ can be calculated by

$$C_t^{(d)} = I(Z_t \leqslant \phi_\alpha)RV_t + I(Z_t > \phi_\alpha)RBV_t, \tag{2.7}$$

where I(*) is an indicator function and α equals 0.99.

Barndorff-Nielsen et al., 2010 propose the daily realized semi variance, which can capture the variation solely from negative or positive returns.The daily positive realized semi variance estimator, $RSV_t^{(d+)}$, is calculated as:

$$RSV_t^{(d+)} = \sqrt{252}\sum_{j=1}^{M} I(r_{t,j} \geqslant 0)r_{t,j}^2. \tag{2.8}$$

Similarly, the daily negative realized semi variance estimator, $RSV_t^{(d-)}$, is defined as:

$$RSV_t^{(d-)} = \sqrt{252}\sum_{j=1}^{M} I(r_{t,j} < 0)r_{t,j}^2. \tag{2.9}$$

2.3 Data

The dataset of this study includes both daily and tick-by-tick transaction data for the implementation of the forecasting procedure. More specifically, the WTI realized volatility estimator is calculated using tick-by-tick transaction data of the front-month futures contracts for the WTI crude oil. The tick-by-tick transaction data are used to construct the time series at the different sampling frequencies from 1 minute up to 120 minutes. The 10 minutes sampling frequency has been chosen, since minimizes the autocovariance bias induced by microstructure noise issue.

In this study, we also use OVX as another oil volatility measure since it has become an important instrument for trading oil price volatility and has attracted the attention of investors. More specifically, OVX is introduced by the Chicago board of options exchange (CBOE) in order to measure the market's expectation of 30-day volatility. In more detail, the CBOE Volatility Index methodology that has been applied for the OVX computation uses options on the United States Oil Fund with a wide range of strike prices. It is also important to note that OVX values are annualized to cover the upcoming 12-month period, as reported by the CBOE, by annualizing the interpolated value[7], taking its square root and expressing the result in percentage points, which is consistent with the calculation of the annualized realized volatility measure used in this chapter. Regarding OVX, the data are readily available at a daily frequency. Moreover, the common sample for both WTI realized volatility and OVX is from January 4, 2010 to October 30, 2017 and the number of observations is 1971 (trading days). The source of the obtained tick-by-tick data is TickData, whereas the data for the OVX are obtained from CBOE[8].

[7] The CBOE Volatility Index methodology is applied for the calculation of the OVX by interpolating two weighted sums of options midquote values, which represent the expected variance of the Euro to Dollar exchange rate up to two option expiration dates that bracket a 30-day time horizon.

[8] Please find the data using the following link: https://www.cboe.com/us/indices/dashboard/ovx/.

Figure 2.1 portrays the annualized realized volatility and OVX series. It is apparent that high values of volatility are observed in the late 2014 - early 2015 period due to the sudden decline of the oil prices. Moreover, from the descriptive statistics of those two variables, which are reported in Table 2.1, it is observed that the mean of OVX is higher than that of the realized volatility. Furthermore, we observe that realized volatility is more volatile with higher coefficient of variation than that of OVX. Similar results can be also observed for the variables $\log(RV)$ and $\log(OVX)$.

Descriptive statistics

Series	RV	log(RV)	OVX	log(OVX)
Mean	28.3834	3.2557	33.7542	3.4693
Median	25.6305	3.2438	32.4500	3.4797
Maximum	99.0252	4.5954	78.9700	4.3691
Minimum	7.09270	1.9591	14.5000	2.6742
Std. Dev.	13.0343	0.4173	10.6991	0.3185
CV	0.4592	0.1282	0.3170	0.0918
Skewness	1.63346	0.2498	0.70935	-0.1407
Kurtosis	6.79648	3.0568	3.58926	2.7073
Daily Obs.	1971	1971	1971	1971

Table 2.1 This table presents the descriptive statistics of the volatility series. CV = coefficient of variation. All series, RV, log(RV), OVX and log(OVX), are stationary according to the ADF unit root test.

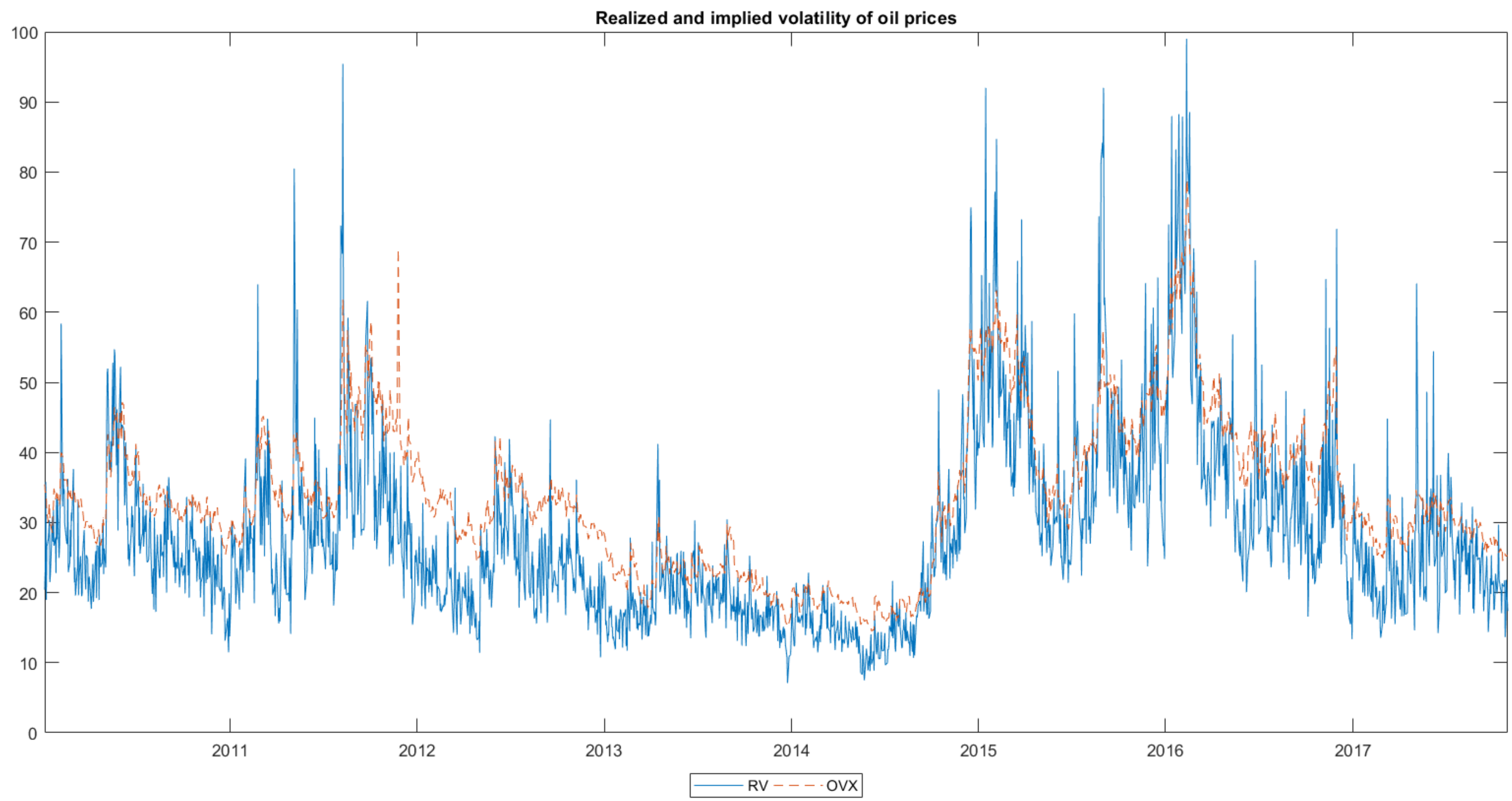

Figure 2.1 Realized and implied volatility of oil prices.

2.4 Modelling framework

2.4.1 Naive model specification

A simple Random Walk (RW) without a drift is considered as this naive model and it is written as:

$$log(RV_t) = log(RV_{t-1}) + \varepsilon_t, \tag{2.10}$$

where RV_t is the annualised realized volatility of the WTI crude oil at day t and ε_t is a white noise.

2.4.2 Simple HAR-type model specification

In this study, we employ the HAR modelling framework by Corsi, 2009, who proposes an additive cascade model of realized volatility aggregated at different time horizons. It could be also considered as a simple AR-type model in the realized volatility that includes volatilities realized over different time horizons and is thus called heterogeneous autoregressive (HAR) model. One of its advantages is its simplicity. The HAR model provides a flexible method to fit the partial autocorrelation function of the empirical data with a step function and it can be easily estimated by simple OLS. Furthermore, the HAR model captures the persistence properties of financial data, as well as, long-memory models, such as fractionally integrated one, even if it does not belong to the class of long-memory processes. The basic idea is that market participants have a different perspective of their investment horizon. Typically, three components are used with daily, weekly and monthly length.

Although the HAR model introduced by Corsi, 2009 is specified in terms of levels of realized volatility, we follow the literature, which considers models of the log of realized volatility [9]. Eq. (2.11) presents the HAR-RV model:

$$\begin{aligned} log(RV_t) = \hat{a}_0^{(t)} + \hat{a}_1^{(t)} log(RV_{t-1}^{(d)}) + \hat{a}_2^{(t)} log(RV_{t-1}^{(w)}) \\ + \hat{a}_3^{(t)} log(RV_{t-1}^{(m)}) + e_t, \end{aligned} \tag{2.11}$$

where e_t is the residual term and $\hat{a}_0^{(t)}, \hat{a}_1^{(t)}, \hat{a}_2^{(t)}, \hat{a}_3^{(t)}$ are the estimated parameters. Furthermore, the components are calculated as: $log(RV_{t-1}^{(d)}) = log(RV_{t-1})$; $log(RV_{t-1}^{(w)}) = \left(5^{-1} \sum_{k=1}^{5} log(RV_{t-k})\right)$; $log(RV_{t-1}^{(m)}) = \left(22^{-1} \sum_{k=1}^{22} log(RV_{t-k})\right)$, which is in line with Corsi and Renò, 2012.

We also investigate the impact of the decomposition of quadratic variation to continuous and jump components in forecasting realized volatility. Thus, we replace in the structure of the simple HAR model the RV components with the continuous components[10]. The HAR-C model is written as:

$$\begin{aligned} log(RV_t) = \hat{a}_0^{(t)} + \hat{a}_1^{(t)} log(C_{t-1}^{(d)}) + \hat{a}_2^{(t)} log(C_{t-1}^{(w)}) \\ + \hat{a}_3^{(t)} log(C_{t-1}^{(m)}) + e_t. \end{aligned} \tag{2.12}$$

Moreover, according to Patton and Sheppard, 2015, the model HAR-RSV uses the structure of the HAR model over negative and positive RSV. This model (HAR-RSV) is described as follows:

$$\begin{aligned} log(RV_t) = {} & \hat{a}_0^{(t)} + \hat{a}_1^{(t)} log(RSV_{t-1}^{(d+)}) + \hat{a}_2^{(t)} log(RSV_{t-1}^{(w+)}) \\ & + \hat{a}_3^{(t)} log(RSV_{t-1}^{(m+)}) + \hat{a}_4^{(t)} log(RSV_{t-1}^{(d-)}) + \hat{a}_5^{(t)} log(RSV_{t-1}^{(w-)}) \\ & + \hat{a}_6^{(t)} log(RSV_{t-1}^{(m-)}) + e_t. \end{aligned} \tag{2.13}$$

[9] Using log transformed RV data is closer to being normally distributed, and there is also no need to impose any non-negativity restrictions on the fitted and forecasted volatility.

[10] It should be noted here that in the HAR-C model we had initially included the jumps components, as well, i.e. we developed the HAR-CJ model. However, it was found that the HAR-CJ model does not provide additional predictive information compared to the HAR-C model. This is suggestive of the fact that the jump components do not provide incremental forecasting gains, as also noted by the recent literature (Prokopczuk et al., 2016; Sévi, 2014).

Before completing this part, it would also be convenient to condense the representation of the aforementioned models somewhat. Thus, $\mathbf{x_t} = \left[1,\ log(RV_{t-1}^{(d)}),\ log(RV_{t-1}^{(w)}),\ log(RV_{t-1}^{(m)})\right]$ is defined as the (1×4) vector of HAR-RV components. Also define $y_t = log(RV_t)$. Then, the following equation replaces Eq. (2.11)[11]:

$$y_t = \boldsymbol{x_t} \boldsymbol{\alpha_t} + \varepsilon_t, \tag{2.14}$$

where $\boldsymbol{\alpha_t} = \left[a_0^{(t)},\ a_1^{(t)},\ a_2^{(t)},\ a_3^{(t)}\right]'$ is the corresponding parameter vector and ε_t is the error term.

2.4.3 Estimation of time-varying parameter HAR-type models

It is observed that the autoregressive coefficients that determine the impact of past volatility terms on current volatility can change over time, mainly because of frequent structural breaks. Hence, the detection of potential structural breaks is an important consideration in the forecasting exercise. Thus, we apply the iterated cumulative sums of squares (ICSS) algorithm for testing multiple breaks in the unconditional variance of crude oil prices. As shown in Table 2.2, seven breaks were detected[12]. From Figure 2.2, which portrays the WTI returns and the detected structural breaks, we observe that most of the breaks appear after 2014, which is a highly volatile period for the crude oil market.

[11]The HAR-RV model is presented for simplicity reasons. The similar framework can be applied to each of the aforementioned models.

[12]Inclán and Tiao, 1994 propose a cumulative sum of squares statistic in order to test the null hypothesis of a constant unconditional variance against the alternative hypothesis of a break in the unconditional variance.

Structural breaks

Break points	Time period	Standard deviation
7	January 5, 2010 - October 28, 2011	2.0111
	October 29, 2011 - November 21, 2012	1.6970
	November 22, 2012 - August 14, 2014	1.0919
	August 15, 2014 - November 28, 2014	1.9547
	November 29, 2014 - January 6, 2016	2.7559
	January 7, 2016 - March 18, 2016	4.2456
	March 19, 2016 - December 19, 2016	2.2992
	December 20, 2016 - October 30, 2017	1.5689

Table 2.2 The structural breaks in the volatility of crude oil prices as detected by the ICSS algorithm.

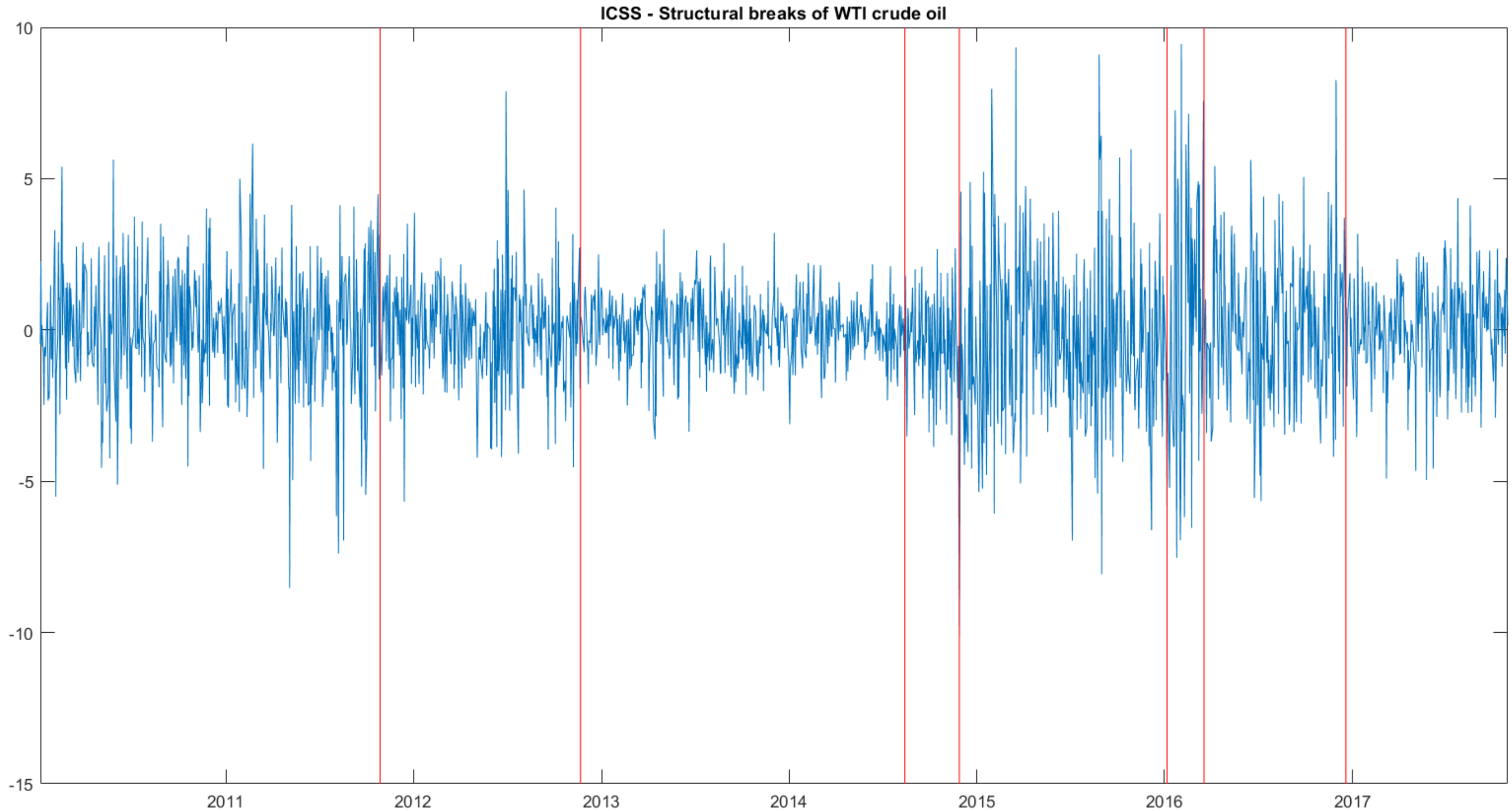

Figure 2.2 WTI daily returns and the structural breaks that have been detected by ICSS algorithm.

The detection of these structural breaks motivates the use of the TVP methodology, so as to investigate its capability to improve the forecasting ability of the HAR models. In this section, the specification of TVP parameter HAR models are presented[13]. We present the procedure for the case of the HAR-RV model only, since the same approach is followed for all remaining models. The specification of the HAR-RV model under the TVP specification of Grassi et al., 2017 can be written as:

$$y_t = \boldsymbol{x_t}\boldsymbol{\alpha_t} + \varepsilon_t \quad \text{for} \quad \varepsilon_t \sim N(0, H_t), \tag{2.15}$$

$$\boldsymbol{\alpha_t} = \boldsymbol{\alpha_{t-1}} + \boldsymbol{u_t} \quad \text{for} \quad \boldsymbol{u_t} \sim N(\boldsymbol{0_{4\times 1}}, \boldsymbol{\Sigma_{u_t}}). \tag{2.16}$$

The vector $\boldsymbol{\alpha_t}$ represents the time-varying regression coefficients. It is also assumed that the error sequences ε_t and $\boldsymbol{u_t}$ are internally and mutually independent at all of the leads and lags. The model given by Eq. (2.15) and Eq. (2.16) is considered an attractive one that provides for time-varying parameters in contrast to traditional, constant coefficient models (even when the aforementioned models provided by Eq. (2.11), Eq. (2.12) and Eq. (2.13) are estimated recursively).

There are studies proposing models that capture potential structural breaks. For example, Luo et al., 2020 construct the Infinite Hidden Markov (IHM) models, which are capable of capturing potential structural breaks in oil price volatility. Those models have been also implemented by Luo et al., 2019 in order to forecast the realized volatility of agricultural commodity futures. It is important to mention that the proposed IHM process is estimated by using the Markov chain Monte Carlo (MCMC) posterior sampler, which makes the whole forecasting framework more complicated because of the large number of parameters.

[13]More details for the whole procedure analyzing each step can be found in the Appendix

Moreover, according to Koop and Korobilis, 2012, the estimation of the dynamic model averaging that uses combinations of TVP models is considered computationally infeasible using MCMC-based Bayesian methods. Thus, one simple approximation that they implement is the forgetting factor λ in order to avoid estimating $\boldsymbol{\Sigma}_{\boldsymbol{u}_t}$. However, Grassi et al., 2017 suggest the standardized self-perturbed Kalman filter, which avoids the calibration of a design parameter as the perturbation term is scaled by the amount of the uncertainty in the realized volatility data in this study. The difference between the aforementioned studies is that they propose alternative ways to process the new information at each point in time, where the process of the updating equation of the state covariance matrix is modified by using some approximations. In detail, the perturbation term that is included in the updating equation of the state covariance matrix is weighted by the measurement error variance estimate. Thus, we rely on the latter approximation, namely the standardized self-perturbed Kalman filter, which was initially proposed by Park and Jun, 1992[14].

2.5 Forecasting realized volatility and oil price implied volatility index

It is important to note here that the same forecasting methodology is implemented for both WTI realized volatility and OVX. Therefore, OVX forecasts are generated by replacing RV with OVX in the corresponding equations of Section 2.4. Moreover, most studies implement a direct forecasting approach mainly for practical reasons. According to Nonejad, 2017, obtaining iterated forecasts is not feasible given the computational time it takes to generate iterated fore-

[14] In this study, the TVP methodology proposed by Raftery et al., 2010, which relies on the approximation of the forgetting factor has been also implemented but it is outperformed by the TVP HAR model with the standardized self-perturbed Kalman filter.

casts. Since we concentrate on the forecasting performance of the models, both direct and iterated forecasting procedures are implemented in order to assess which method provides more accurate out-of-sample forecasts.

Moreover, one important note is that the multi-step ahead forecasting procedure of this study is based on obtaining point forecasts since they are more useful for traders. Thus, y_{t+h} is defined as $y_{t+h} = log(RV_{t+h})$.

2.5.1 Direct forecasting procedure

The direct forecasting procedure for obtaining out-of-sample forecasts of realized volatility is based on Buncic and Gisler, 2016. In particular, distinct regressions are applied for each horizon. More specifically, the regression is written as follows:

$$y_{t+h} = \boldsymbol{x_t \alpha^{(h)}} + \varepsilon_{t+h}. \tag{2.17}$$

The direct forecast is $\boldsymbol{x_t \hat{\alpha}_t^{(h)}}$, where $\boldsymbol{\hat{\alpha}_t^{(h)}}$ is an estimate of $\boldsymbol{\alpha^{(h)}}$ that only relies on data up to period t.

2.5.2 Iterated forecasting procedure

The HAR-RV 1-day-ahead iterated forecast is written as:

$$\begin{aligned} RV_{t+1|t} = exp(&\hat{a}_0^{(t)} + \hat{a}_1^{(t)} log(RV_t^{(d)}) + \hat{a}_2^{(t)} log(RV_t^{(w)}) \\ &+ \hat{a}_3^{(t)} log(RV_t^{(m)})), \end{aligned} \tag{2.18}$$

where $log(RV_t^{(d)}) = log(RV_t)$, $log(RV_t^{(w)}) = \left(5^{-1} \sum_{k=1}^{5} log(RV_{t-k+1})\right)$ and $log(RV_t^{(m)}) = \left(22^{-1} \sum_{k=1}^{22} log(RV_{t-k+1})\right)$.

Due to the fact that the logarithmic transformation is used in the models, the error variance term $0.5\sigma_\epsilon^2$ should be incorporated, but it has marginal effect on the results.

The s-days-ahead iterated forecasts of the HAR-RV model, for $s \geqslant 2$ are computed as follows:

$$\begin{aligned} RV_{t+s|t} = exp(\hat{a}_0^{(t)} + \hat{a}_1^{(t)} log(RV_{t,s}^{(d)}) + \hat{a}_2^{(t)} log(RV_{t,s}^{(w)}) \\ + \hat{a}_3^{(t)} log(RV_{t,s}^{(m)})), \end{aligned} \tag{2.19}$$

where $log(RV_{t,s}^{(d)}) = log(RV_{t+s-1|t})$, $log(RV_{t,s}^{(w)}) = \Big(s^{-1} \sum_{k=1}^{s-1} log(RV_{t-k+s|t})$ $+ (5-s)^{-1} \sum_{k=s}^{5} log(RV_{t-k+s})\Big)$ and $log(RV_{t,s}^{(m)}) = \Big(s^{-1} \sum_{k=1}^{s-1} log(RV_{t-k+s|t})$ $+ (22-s)^{-1} \sum_{k=s}^{22} log(RV_{t-k+s})\Big)$, which are used for s-days-ahead forecasts.

It is important to note that for the 1-day ahead forecast of the RV, the models use data that belong to the information set at time t. However, for the 2-days ahead horizons onwards, the use of future data that do not belong to the information set at time t would have been required. Regarding the HAR-RV model, the future value of the RV that the model uses is the 1-day ahead forecast i.e. at $t+1$, but the future $log(C_t)$ or $log(RSV_t)$ (where RSV denotes the generic term for both the positive and negative semi variance measures) values are unknown to the forecaster. Thus, inspired by the published work of Degiannakis and Filis, 2017, we forecast those components through the simple HAR model in order to use them as the required values for producing RV and OVX for s-days ahead forecasts.

Thus, the equations that have been estimated in order to forecast $log(C_t)$ and $log(RSV_t)$ are the following:

$$\begin{aligned} log(C_t) = \hat{a}_0^{(t)} + \hat{a}_1^{(t)} log(C_{t-1}^{(d)}) + \hat{a}_2^{(t)} log(C_{t-1}^{(w)}) \\ + \hat{a}_3^{(t)} log(C_{t-1}^{(m)}) + e_t, \end{aligned} \tag{2.20}$$

$$\begin{aligned} log(RSV_t) = \hat{a}_0^{(t)} + \hat{a}_1^{(t)} log(RSV_{t-1}^{(d)}) + \hat{a}_2^{(t)} log(RSV_{t-1}^{(w)}) \\ + \hat{a}_3^{(t)} log(RSV_{t-1}^{(m)}) + e_t, \end{aligned} \tag{2.21}$$

which follow the structure of a HAR-RV model as mentioned above.

2.6 Forecast evaluation

2.6.1 Prediction settings

The initial sample period is T=1065 days and we use the remaining T_1 days for out-of-sample forecasting. When obtaining direct h-step ahead forecasts under the HAR-type modelling, a sample of extra $h-1$ days is required at the beginning of the sample[15]. Furthermore, a rolling window approach with fixed length of 1000 days is used. In other words, the data from the 1^{st} to the 1000^{th} is chosen, the parameters are re-estimated, and we, finally, fulfill the forecasting procedure. The choice of T=1000 days is justified by the fact that a large sample is required for estimating the proposed models[16].

2.6.2 Evaluation functions

The forecasting accuracy of the models is evaluated using two evaluation functions, namely the Mean Squared Predicted Error (MSPE) and the Mean Absolute Error (MAE), which are defined as:

$$MSPE^{(s)} = \frac{1}{T}\sum_{t=1}^{T}(RV_{t+s|t} - RV_{t+s})^2, \tag{2.22}$$

and

$$MAE^{(s)} = \frac{1}{T}\sum_{t=1}^{T} \mid RV_{t+s|t} - RV_{t+s} \mid, \tag{2.23}$$

where $RV_{t+s|t}$ is the s-days-ahead realized volatility forecast, whereas RV_{t+s} is the realized volatility at time $t+s$.

[15] This is a reason why we keep the maximum of the forecasting horizons minus 1 (when 1-day ahead forecasts are generated we don't need this adjustment), which is 65 days ahead, before the starting point of the initial sample.

[16] Most of the aforementioned literature on forecasting crude oil price volatility uses similar sample sizes.

2.6.3 Model Confidence Set

To assess the significant differences among the predicitions of the different forecasting models, we utilize the established Model Confidence Set (MCS) procedure developed by Hansen et al., 2011, which identifies the set of the best models, according to their forecasting accuracy in terms of a loss function, without an a priori choice of a benchmark model because we would like to evaluate the forecasting accuracy of these models simultaneously and not against a benchmark model. In this study, the loss function, which is used is the MSPE.

Starting with the goal of the MCS test, given a set of candidate forecast models, M_0, it investigates at a predefined level of significance a, which set of models survive an elimination algorithm. The MCS procedure starts with the full set of models $M = M_0 = \{1, \ldots, m_0\}$, and repeatedly tests the following null hypothesis of equal predictive ability:

$$H_{0,M} : E(d_{i,i*,t}) = 0, \;\; \forall \; i, i^* \in M, \tag{2.24}$$

where $d_{i,i*,t} = \Psi_{i,t} - \Psi_{i*,t}$ is defined as the evaluation differential for $i, \; i^* \in M_0$ and $\Psi_{i,t} = (RV_{t+s|t} - RV_{t+s})^2$, where $RV_{t+s|t}$ is the s-days-ahead oil realized volatility forecast obtained from model i. This procedure is repeated until the null is not rejected any longer. The MCS is computed for $a = 0.1$ using a block bootstrap with 10,000 bootstrap replications[17].

2.6.4 Directional accuracy

We also consider an additional evaluation technique, the Direction-of-Change (DoC), in order to examine the proportion of forecasts, which predict the direction of the actual volatility movement correctly. This kind of forecasting evaluation technique is of major significance for trading strategies and asset allocation. Let us denote as $P_{i,t}$ a dummy variable that takes the

[17] For technical details of the MCS procedure see Hansen et al., 2011.

value of 1 for each trading day t that model i forecasts the direction of the actual volatility movement s trading days ahead correctly, whereas it takes the value 0 otherwise:

$$P_{i,t}^{(s)} = \begin{cases} 1, & \text{if } RV_{t+s} > RV_t \text{ and } RV_{t+s|t} > RV_t, \\ 1, & \text{if } RV_{t+s} < RV_t \text{ and } RV_{t+s|t} < RV_t, \\ 0, & \text{otherwise.} \end{cases} \tag{2.25}$$

Then, the proportion of forecasted values that have predicted the direction of the actual volatility movement ($DoC^{(s)}$) correctly is calculated as:

$$DoC^{(s)} = \frac{\sum_{t=1}^{T_1} P_{i,t}^{(s)}}{T_1} \times 100, \tag{2.26}$$

where T_1 is the number of out-of-sample forecasted values.

2.6.5 Trading strategy

Apart from the statistical loss functions and the directional accuracy, we further gauge the forecasting performance of the different models using an economic loss function. This kind of forecasting evaluation technique is based on a quasi trading strategy. It works as follows: if the forecasted oil price volatility of model i at time $t+s$ is higher than that of the actual volatility at time t, the trader assumes a long position in realized volatility. If the forecasted volatility of model i at time $t+s$ is lower than that of the actual volatility at time t, then the trader assumes a short position. Thus, the model's i cumulative returns over the out-of-sample forecasting period is calculated as:

$$r^{(i)} = \sum_{t=1}^{T_1} \left(\frac{(RV_{t+s} - RV_t) d_t^{(i)}}{RV_t} \right), \tag{2.27}$$

where $d_t^{(i)} = 1$ if $RV_{t+s|t} > RV_t$ and $d_t^{(i)} = -1$ if $RV_{t+s|t} \leqslant RV_t$.

We reiterate that the same equations (2.22)-(2.27) are used for the evaluation of OVX forecasts.

2.7 Out-of-sample results

2.7.1 Evaluation functions results

We start the analysis of the results based on the MSPE statistical loss function, whereas the results based on MAE are qualitatively similar and can be found in the Appendix.

The results for the WTI oil price realized volatility are shown in Table 2.3, whereas the OVX results are presented in Table 2.4, respectively. In these tables, the values of the MSPE are reported for the Random Walk (RW) model, and in the next lines the results for the remaining models are presented for six different horizons, namely 1, 5, 10, 15, 22, 44 and 66 days ahead [18]. A value below 1 denotes that the corresponding model outperforms the RW.

[18] In these lines, we report the loss functions' ratios, relative to the RW model.

Realized volatility - MSPE ratios of HAR-type models to the RW model

Days ahead	1	5	10	15	22	44	66
RW	84.57	140.09	176.85	198.97	235.00	296.78	362.17
OLS - DIRECT							
HAR-RV	0.81	0.80	0.79	0.81	0.80	0.82	0.80
HAR-C	0.83	0.81	0.81	0.83	0.82	0.83	0.80
HAR-RSV	0.81	0.78	0.78	0.81	0.78	0.81	0.80
OLS - ITERATED							
HAR-RV	0.81	0.79	0.79	0.82	0.81	0.82	0.80
HAR-C	0.83	0.80	0.80	0.83	0.83	0.83	0.81
HAR-RSV	0.81	0.77	0.78	0.81	0.80	0.81	0.81
TVP - DIRECT							
HAR-RV	0.81	0.77	0.76	0.78	0.77	0.79	0.79
HAR-C	0.83	0.78	0.77	0.79	0.79	0.80	0.79
HAR-RSV	0.81	0.74	0.73	0.75	0.74	0.76	0.78
TVP - ITERATED							
HAR-RV	0.81	0.78	0.77	0.79	0.77	0.77	0.75
HAR-C	0.83	0.79	0.79	0.81	0.79	0.78	0.75
HAR-RSV	0.81	0.76	0.76	0.78	0.76	0.77	0.76

Table 2.3 The results of the MSPE loss function for different forecasting horizons regarding RV forecasting errors. Values represent ratios of HAR-type models to the RW model. A ratio below 1 suggests that MSPE of the corresponding HAR-type model outperforms that of the RW model. The actual MSPE values are presented only for the RW model.

Oil price implied volatility index - MSPE ratios of HAR-type models to the RW model

Days ahead	1	5	10	15	22	44	66
RW	3.52	16.59	29.69	45.04	64.88	121.36	192.21
OLS - DIRECT							
HAR-OVX	1.01	1.02	1.05	1.04	1.04	1.00	0.92
HAR-OVX-C	6.91	2.21	1.79	1.54	1.37	1.10	0.86
HAR-OVX-RSV	6.25	2.08	1.71	1.50	1.33	1.05	0.87
OLS - ITERATED							
HAR-OVX	1.01	1.03	1.06	1.05	1.03	1.02	0.96
HAR-OVX-C	6.91	2.19	1.72	1.47	1.31	1.05	0.87
HAR-OVX-RSV	6.25	2.03	1.59	1.37	1.22	1.00	0.87
TVP - DIRECT							
HAR-OVX	1.01	0.99	0.99	0.99	1.00	0.99	0.99
HAR-OVX-C	7.19	2.31	1.97	1.63	1.43	1.26	1.03
HAR-OVX-RSV	6.20	2.27	1.88	1.66	1.42	1.22	1.06
TVP - ITERATED							
HAR-OVX	1.01	1.02	1.04	1.01	0.97	0.93	0.85
HAR-OVX-C	7.19	2.12	1.63	1.37	1.20	0.94	0.77
HAR-OVX-RSV	6.20	2.00	1.55	1.32	1.15	0.92	0.79

Table 2.4 The results of the MSPE loss function for different forecasting horizons regarding OVX forecasting errors. Values represent ratios of HAR-type models to the RW model. A ratio below 1 suggests that MSPE of the corresponding HAR-type model outperforms that of the RW model. The actual MSPE values are presented only for the RW model.

Starting with the RV results, it is observed that each HAR-type model is able to significantly outperform the RW forecasts for each forecasting horizon. Moreover, the TVP model presents better results compared to the forecasts that have been produced by using OLS. The HAR-RSV (TVP/DIRECT) model seems to generate superior forecasts for all horizons, which means that the inclusion of the realized semi variance components under a TVP estimation framework into a HAR model improves the forecasting performance. This latter model is capable of reducing the forecasting error by more than 20% (compared to the RW) in short and mid-term horizons, and more than 25% in long-term horizons. When the HAR-RSV (TVP/DIRECT) model is compared with the simple HAR-RV (OLS/DIRECT) model, which is widely used in recent studies, it reduces the forecasting errors by almost 8% for all horizons.

Regarding the statistical evaluation of the OVX forecasts, the results of the models compared to RW do not show better forecasting performance, especially for some horizons. More specifically, the RW outperforms all the remaining models when forecasting OVX 1-day ahead. In case of of 5 and 10 days ahead, only the HAR-RV (TVP/DIRECT) model presents a ratio under 1, which means that it produces more accurate forecasts than the RW. Moreover, for long-term horizons, HAR-type (TVP/ITERATED) models outperform all the remaining models including the RW. It is interesting to note that the best models for the longer horizons, namely 44 and 66 days ahead, include the continuous and realized semi variance components. Hence, the inclusion of the realized semi variance measures helps to reduce forecasting errors in longer forecasting horizons.

2.7.2 Model Confidence Set procedure results

Even though the aforementioned results suggest the combination of TVP specification with the inclusion of the leverage effect by decomposing realized variance into positive and negative returns in the HAR model, it is vital to find which model specifications can be included among the best models. The results for the MCS test are presented in Tables 2.5 and 2.6 for the RV and OVX, respectively.

Realized volatility - MCS test

Days ahead	1	5	10	15	22	44	66
RW	0.002	0.001	0.000	0.000	0.000	0.000	0.000
OLS - DIRECT							
HAR-RV	**0.904**	0.028	0.018	0.001	0.013	0.000	0.000
HAR-C	**0.568**	0.026	0.000	0.000	0.001	0.000	0.000
HAR-RSV	**0.987**	0.046	0.025	0.010	0.037	0.001	0.000
OLS - ITERATED							
HAR-RV	**0.904**	0.046	0.018	0.000	0.007	0.000	0.000
HAR-C	**0.568**	0.028	0.000	0.000	0.000	0.000	0.000
HAR-RSV	**1.000**	0.046	0.025	0.010	0.013	0.001	0.000
TVP - DIRECT							
HAR-RV	**0.987**	0.046	0.025	0.020	0.037	0.006	0.002
HAR-C	**0.554**	0.046	0.025	0.010	0.016	0.001	0.002
HAR-RSV	**0.904**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	0.002
TVP - ITERATED							
HAR-RV	**0.987**	0.046	0.025	0.020	**0.239**	**0.898**	**0.671**
HAR-C	**0.554**	0.046	0.025	0.010	0.037	0.013	**1.000**
HAR-RSV	**0.904**	**0.172**	**0.184**	0.020	**0.293**	**0.898**	0.002

Table 2.5 The results of the MCS test for different forecasting horizons related to RV forecasts. Figures in bold denote the model that belongs to the confidence set of the best performing models.

Oil price implied volatility index - MCS test

Days ahead	1	5	10	15	22	44	66
RW	**1.000**	**0.298**	**0.366**	**0.269**	**0.693**	**0.374**	0.000
OLS - DIRECT							
HAR-OVX	**0.574**	**0.217**	0.008	0.053	0.009	0.002	0.000
HAR-OVX-C	0.000	0.000	0.000	0.000	0.000	0.000	0.000
HAR-OVX-RSV	0.000	0.000	0.000	0.000	0.002	0.002	0.000
OLS - ITERATED							
HAR-OVX	**0.574**	0.032	0.008	0.053	0.044	0.001	0.000
HAR-OVX-C	0.000	0.000	0.000	0.000	0.000	0.000	0.000
HAR-OVX-RSV	0.000	0.000	0.000	0.001	0.009	0.065	0.000
TVP - DIRECT							
HAR-OVX	**0.574**	**1.000**	**1.000**	**1.000**	**0.693**	**0.375**	0.000
HAR-OVX-C	0.000	0.000	0.000	0.000	0.000	0.000	0.000
HAR-OVX-RSV	0.000	0.000	0.000	0.000	0.004	0.000	0.000
TVP - ITERATED							
HAR-OVX	**0.574**	**0.217**	**0.158**	**0.269**	**1.000**	**0.803**	0.000
HAR-OVX-C	0.000	0.000	0.000	0.000	0.004	**0.375**	**1.000**
HAR-OVX-RSV	0.000	0.000	0.000	0.001	0.009	**1.000**	0.000

Table 2.6 The results of the MCS test for different forecasting horizons related to OVX forecasts. Figures in bold denote the model that belongs to the confidence set of the best performing models.

From Table 2.5, we make the following observations related to RV forecasts. First of all, the RW, HAR-type (OLS/DIRECT) and HAR-type (OLS/ITERATED) models are never among the best performing models at all forecasting horizons. However, the HAR-RV (TVP/ITERATED) belongs to the confidence set of the best performing models at the longer forecasting horizons. The best model is the HAR-RSV (TVP/DIRECT) for almost all forecasting horizons, which means that the inclusion of the semi variance measures in the HAR model is considered of major importance for forecasting oil RV.

In the case of OVX (see Table 2.6), it is remarkable that RW is among the best models. In more detail, for short and mid term horizons, the HAR-RV (TVP/DIRECT) outperforms the remaining models except for the case of 1-day ahead for which the RW presents better results. This has to be taken into account since there are studies that use the HAR-OVX model in order to obtain 1-day ahead OVX forecasts. For longer forecasting horizons, the incorporation of the $C_t^{(d)}$, as well as, $RSV_t^{(d+)}$ and $RSV_t^{(d-)}$ components, help the TVP-HAR models to produce significantly better results than the remaining models. It is also noticed that for both RV and OVX, when creating forecasts 44 and 66-days ahead, the iterated approach outperforms the direct, which asserts that forecasters should predict separately the different components in order to provide better results.

2.7.3 Directional accuracy results

We further this analysis turning attention to the directional accuracy of these forecasting models. The DoC results are presented in Tables 2.7 and 2.8, which report the proportion of forecasted values that have correctly predicted the direction of realized volatility and oil price implied volatility index movement, respectively. In the case of RV, it is shown that the HAR-type models that have been estimated by the TVP specification exhibit higher directional accuracy of the oil volatility movement, compared to the HAR-RV (OLS/DIRECT) model, which is the most widely used HAR model specification in the literature. From Table 2.7, we draw the conclusion that the HAR-RSV (TVP/DIRECT) model is capable of predicting the direction of change at a much higher rate compared to the other models in short and mid-term horizons. The same model estimated with the TVP method presents better directional accuracy in long-term horizons. In particular, the HAR-RSV (TVP/DIRECT) model is able to increase the directional accuracy by approximately 5% in short and mid-term horizons and by more than 3% for long term horizons compared to the HAR-RV (OLS/DIRECT). Interestingly enough, the HAR-C (TVP/ITERATED) and the HAR-RSV (TVP/ITERATED) models present directional accuracy more than 67% at 66-days ahead.

Realized volatility - directional accuracy

Days ahead	1	5	10	15	22	44	66
OLS - DIRECT							
HAR-RV	68.01	67.16	65.32	64.35	66.91	61.66	61.78
HAR-C	65.93	68.01	66.06	65.20	67.89	61.17	62.52
HAR-RSV	64.71	68.74	63.86	64.22	67.28	61.78	60.32
OLS - ITERATED							
HAR-RV	68.01	67.40	64.35	64.71	66.54	61.90	64.84
HAR-C	65.93	67.89	64.84	64.84	66.42	62.52	65.45
HAR-RSV	64.71	68.62	65.20	65.08	68.01	62.88	64.59
TVP - DIRECT							
HAR-RV	67.89	68.25	66.42	66.30	68.86	63.49	62.88
HAR-C	66.54	67.89	66.42	66.42	68.13	61.17	63.25
HAR-RSV	66.30	70.57	67.52	67.40	69.35	64.35	63.25
TVP - ITERATED							
HAR-RV	67.89	67.03	66.54	67.03	68.25	64.71	68.01
HAR-C	66.54	68.25	65.69	67.28	67.77	65.45	67.52
HAR-RSV	66.30	69.35	65.57	66.91	68.99	64.71	67.16

Table 2.7 The numbers on the table show the proportion of forecasts that have predicted the direction of realized volatility correctly. Their significance is gauged by employing the test of Pesaran and Timmermann (2009), under the null hypothesis of no directional accuracy. We find that for all models and all forecasting horizons the null hypothesis is rejected at 1% level of significance.

Oil price implied volatility index - directional accuracy

Days ahead	1	5	10	15	22	44	66
OLS - DIRECT							
HAR-OVX	51.77	51.89	50.55	50.92	53.72	54.21	60.81
HAR-OVX-C	52.63	53.72	49.94	50.55	46.76	51.40	65.69
HAR-OVX-RSV	52.14	52.75	50.06	52.50	50.06	49.82	63.49
OLS - ITERATED							
HAR-OVX	51.77	52.50	52.26	54.58	53.72	51.04	61.90
HAR-OVX-C	52.63	52.50	50.55	50.06	49.21	52.99	63.61
HAR-OVX-RSV	52.14	51.04	50.18	52.50	50.43	53.48	63.37
TVP - DIRECT							
HAR-OVX	51.89	53.48	52.01	53.85	50.06	49.82	55.56
HAR-OVX-C	52.26	51.89	49.33	51.77	48.72	47.74	55.56
HAR-OVX-RSV	52.50	52.38	51.65	54.21	50.55	50.06	54.82
TVP - ITERATED							
HAR-OVX	51.89	53.85	53.72	57.02	58.24	54.46	61.66
HAR-OVX-C	52.26	52.01	47.86	50.67	51.16	57.75	67.16
HAR-OVX-RSV	52.50	50.18	49.69	49.94	50.31	57.51	66.67

Table 2.8 The numbers on the table show the proportion of forecasts that have predicted the direction of implied volatility correctly. Their significance is gauged by employing the test of Pesaran and Timmermann (2009), under the null hypothesis of no directional accuracy. We find that for all models and all forecasting horizons the null hypothesis is rejected at 1% level of significance.

Notably, as it is shown in Table 2.8, when forecasting OVX at short and mid-term horizons, the HAR-OVX (TVP/DIRECT) model presents directional accuracy of more than 53% compared to the values of HAR-OVX (OLS/DIRECT), which are close to 51%. However, the contribution of the realized semi variance components under the TVP methodology at longer horizons is material. In particular, for 66-days ahead, the increase of the directional accuracy that HAR-C (TVP/ITERATED) and HAR-RSV (TVP/ITERATED) provide compared to HAR-RV (TVP/ITERATED) is approximately 10%.

2.7.4 Trading strategy results

As an additional evaluation method, we assess the forecasting performance of the competing models based on their economic use, as gauged by a simple trading strategy. It should be noted that for brevity we do not present the results for all forecasting horizons but only the results for 22-, 44- and 66-days ahead. The results are qualitatively similar for the shorter run horizons.

Focusing first on the RV forecasts, Figures 2.3, 2.4 and 2.5 show the cumulative trading returns over the out-of-sample period. For 22 and 44 days ahead, it is obvious that the HAR-RSV (TVP/DIRECT) model obtains the highest trading gains and the RW is the only one that exhibits negative returns. In the case of 66 days ahead, what matters more is the use of the iterated approach under the TVP method in the forecasting strategy.

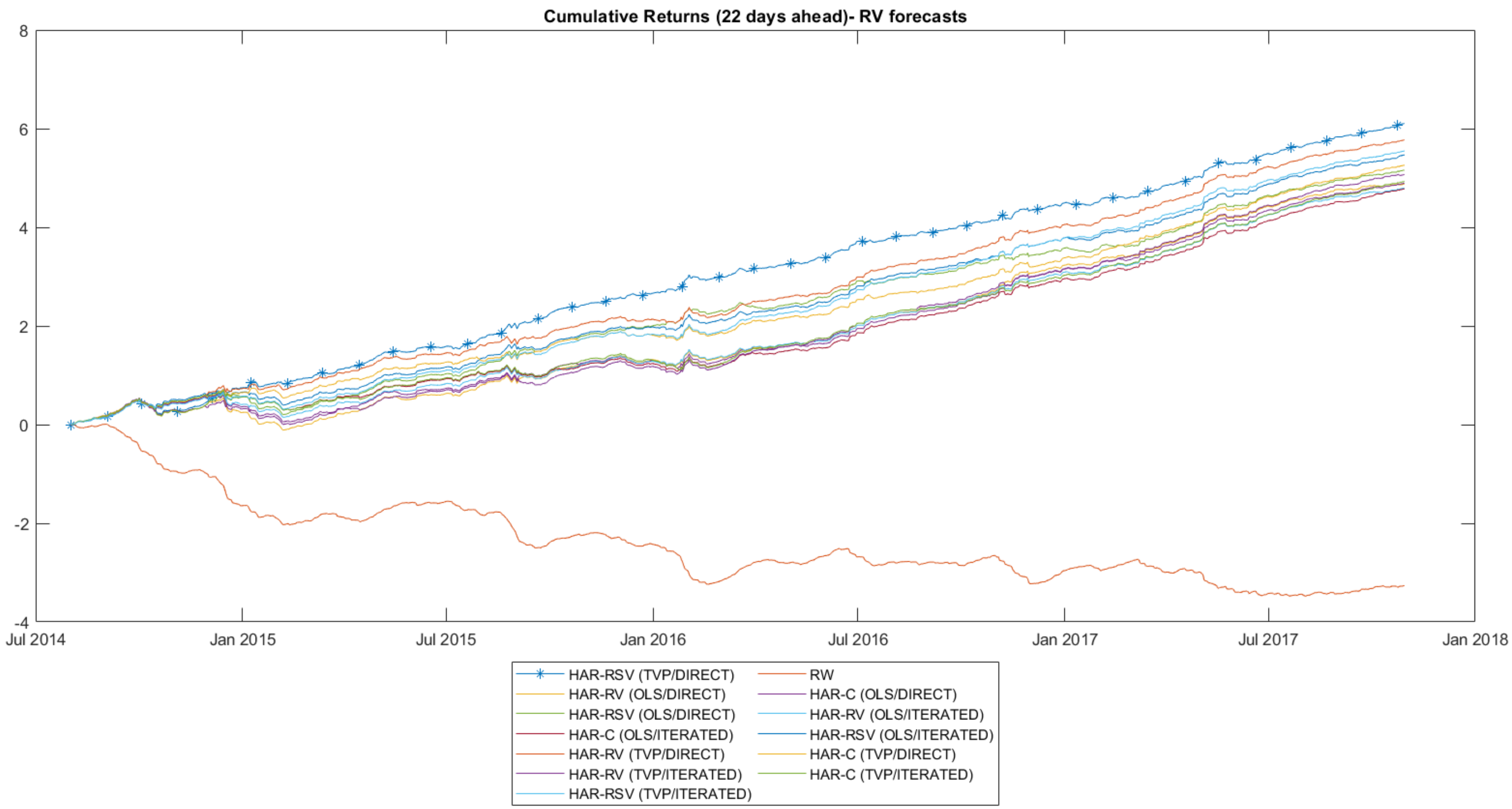

Figure 2.3 Cumulative trading returns when forecasting RV 22-days ahead.

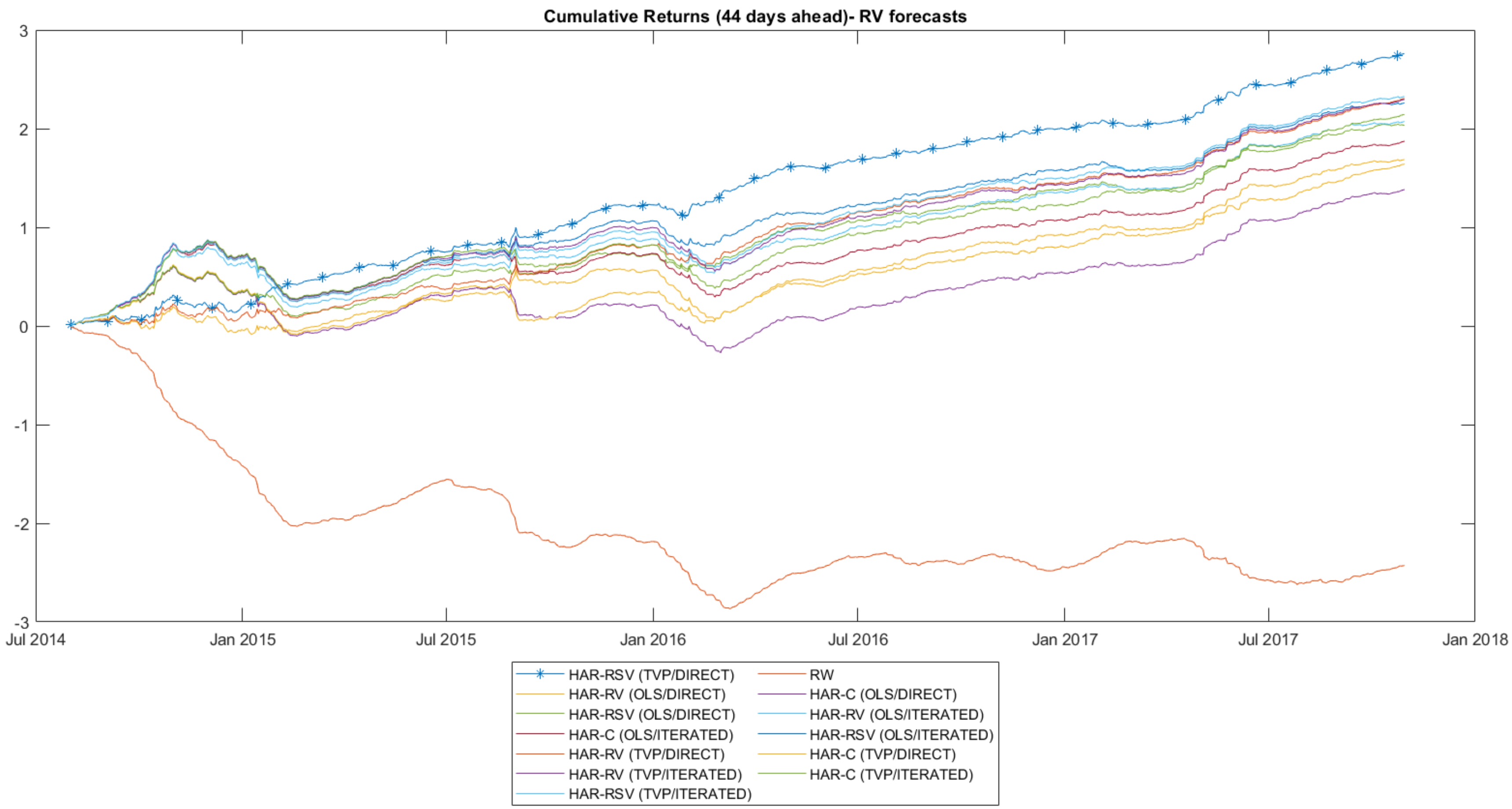

Figure 2.4 Cumulative trading returns when forecasting RV 44-days ahead.

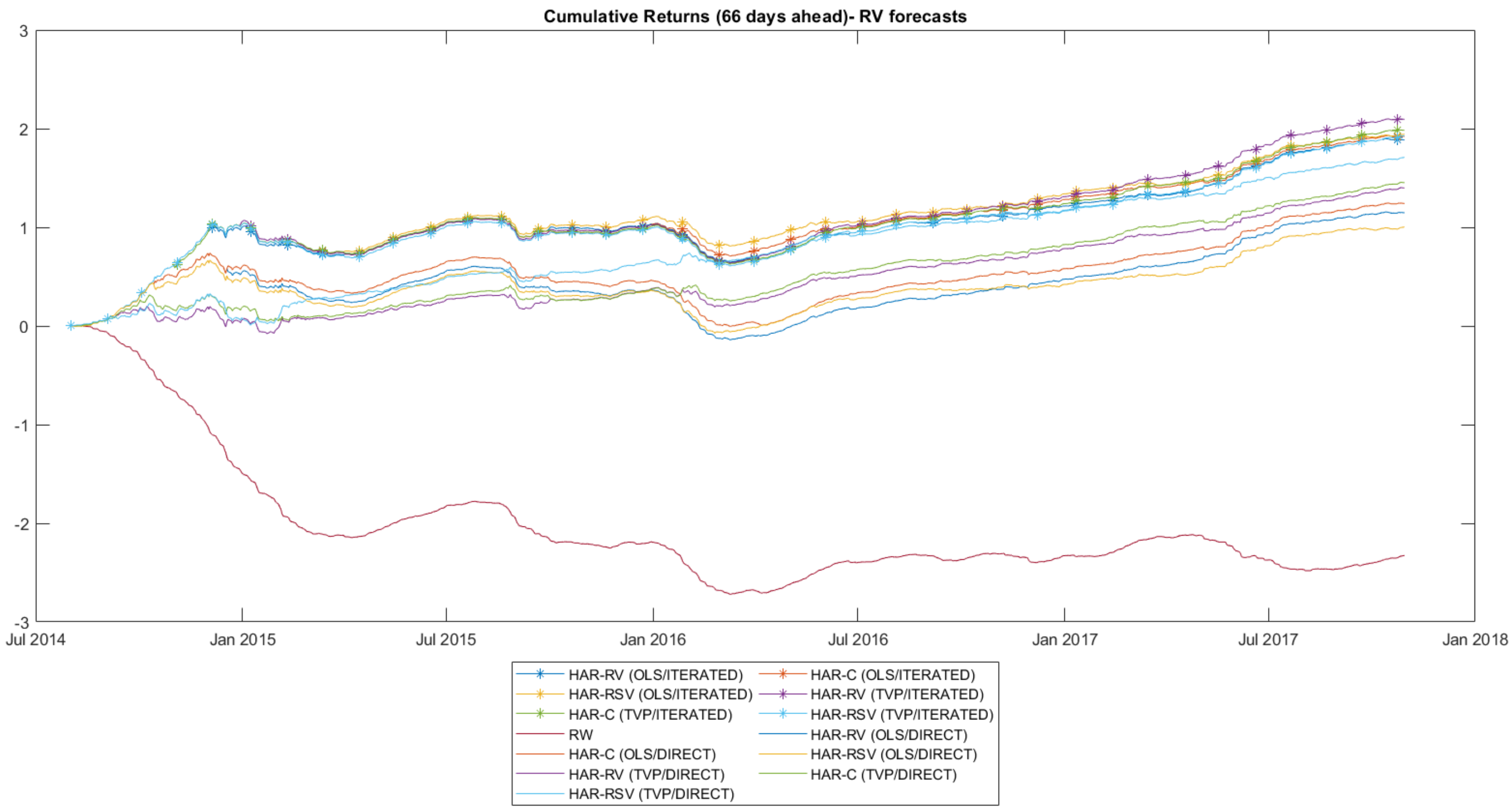

Figure 2.5 Cumulative trading returns when forecasting RV 66-days ahead.

It is considered of major importance to note that for 66 days ahead, among the best models are those that include the continuous components by using the iterated approach. Thus, the inclusion of the aforementioned components provide superior results for the case of OVX, which is a tradeable asset and the investors could benefit from such forecasts.

Finally, from Table 2.9, which summarises the annualized cumulative returns, it is observed that the returns from the RW are negative for both RV and OVX. However, by including the realized semi variance components in the simple HAR model, the cumulative trading returns increase substantially in the case of RV. More specifically, using the forecasts from the HAR-RSV (TVP/DIRECT) the annualized cumulative returns reach the level of almost 190%, 85% and 53% at forecasting horizons 22, 44 and 66 days ahead, respectively. For the case of OVX the annualized cumulative returns are approximately 39%, 36% and 50% at forecasting horizons 22, 44 and 66 days ahead, respectively. We can also conclude that the TVP models significantly outperform the models estimated by OLS, in terms of trading gains. Finally, regarding the iterated and direct approach, it is observed that for the forecasting horizons of 22 and 44 days ahead the gains are higher when implementing the direct methodology. Nevertheless, the iterated forecasts provide higher gains for both RV and OVX at 66 days ahead.

Annualized cumulative trading returns

Days ahead	RV			OVX		
	22	44	66	22	44	66
RW	-100.40	-74.70	-71.62	-35.46	-37.12	-42.39
OLS - DIRECT						
HAR-RV/OVX	150.02	51.91	35.27	-16.56	-6.42	12.36
HAR-RV/OVX-C	155.96	42.50	38.18	1.98	19.45	56.25
HAR-RV/OVX-RSV	158.78	62.54	30.86	22.18	27.05	54.17
OLS - ITERATED						
HAR-RV/OVX	147.61	63.73	57.99	1.82	5.24	28.67
HAR-RV/OVX-C	147.05	57.59	59.11	4.40	24.84	49.27
HAR-RV/OVX-RSV	168.22	69.55	59.81	12.27	27.54	47.64
TVP - DIRECT						
HAR-RV/OVX	177.54	70.92	43.01	-21.03	-16.55	-15.64
HAR-RV/OVX-C	161.82	50.50	44.67	18.72	20.97	46.73
HAR-RV/OVX-RSV	187.84	84.96	52.58	38.96	35.92	49.65
TVP - ITERATED						
HAR-RV/OVX	150.59	70.63	64.42	23.80	17.39	35.75
HAR-RV/OVX-C	151.67	65.97	60.99	15.50	38.77	53.71
HAR-RV/OVX-RSV	170.66	71.54	59.33	13.29	38.36	52.21

Table 2.9 The annualized cumulative trading returns when implementing a trading strategy on RV and OVX at forecasting horizons 22, 44 and 66 days ahead. The numbers refer to percentages (%).

2.8 Conclusion

Since the literature on forecasting oil price volatility has shown great interest, this chapter brings together all the different elements in this line of research and provides an answer as to what matters more for multi-step forecasts. Previous studies provide evidence that in the case of WTI crude oil the HAR-RV model outperforms all other competing forecasting models[19]. This study aims to improve the forecasting performance of the HAR-RV model in order to capture the changes of the coefficients by considering time-variation in HAR-type models' parameters and by implementing not only the direct approach but also the iterated methodology for obtaining multi-period forecasts for RV and OVX.

This study constructs HAR-type models by incorporating information only from crude oil volatility without the inclusion of exogenous variables. The HAR-type models that are estimated by the TVP framework with the standardized self-perturbed Kalman filter outperform the state of the art models under the OLS estimation for all forecasting horizons for both RV and OVX. Evaluating the aforementioned models, it is obvious for the case of RV that the impact of realized semi variance components is highly significant and TVP models, which include these components outperform the remaining models at all forecasting horizons. Furthermore, we should reiterate that the decomposition of quadratic variation to continuous and jumps components does not provide incremental predictive gains. These results are robust under the trading strategy, as well.

Regarding the generated OVX forecasts, HAR-OVX with OLS estimation does not present better results than RW, which is really notable since a lot of studies use HAR models estimated by OLS in order to produce forecasts for OVX. However, by estimating the HAR model under the TVP methodology, the forecasting performance is improved, especially for short- and mid-

[19] For further details see Sévi, 2014, Haugom et al., 2014 and Prokopczuk et al., 2016.

term horizons. One of the most interesting findings is that iterated forecasts including continuous components from the decomposition of quadratic variation provide much better performance in longer horizons, relative to other models. This is considered significant information especially for traders and investors. Finally, we suggest that forecasters should not use the same models in order to forecast oil RV and OVX, since the factors influencing the accuracy of their forecasts are different.

The results indicate some interesting directions for future research. First, something that could be really important for both investors and academics is extending this methodological framework with the inclusion of exogenous information. Finally, investigating whether the results of this study would remain qualitatively similar when focusing on forecasting the realized volatility of other crude oil futures (e.g. Brent) would be an important avenue of future research, since the interconnectedness of different crude oil markets price movements (e.g. between WTI and Brent) has recently attracted the attention of researchers and policy makers (Klein, 2018).

Chapter Three

The impact of the uncertainty environment on oil price volatility: An out-of-sample investigation

3.1 Introduction

Crude oil plays a crucial role in the international economy since it is regarded as a key commodity for all the international economies. According to Vo, 2011, a rise in oil price impacts production costs, which affect inflation. Furthermore, due to the financialization of oil markets, oil price shocks can affect financial markets immediately, which is one of the reasons why researchers focus on oil price volatility. According to Silvennoinen and Thorp, 2013, the importance of forecasting oil price volatility can also be explained by the fact that financial institutions regard the oil market as a profitable investment. Moreover, forecasting oil price volatility has received significant attention from researchers and policy makers due to the fact that it significantly affects the global economy and financial stability (Baumeister & Kilian, 2016; Charles & Darne, 2017; Ferderer, 1996; Wang et al., 2016).

Most of the existing papers on forecasting oil price volatility use high frequency data in order to estimate the daily realized volatility of crude oil. Moreover, as far as we are concerned, many papers use the oil volatility index (OVX) from the Chicago Board Options Exchange (CBOE) in order to improve the realized oil price volatility forecasts (Dutta, 2017; Haugom et al., 2014). It is widely known that implied volatility indices have been used in order to explain and predict the future uncertainty. In the case of the oil market, OVX is the market's expectation of future oil volatility. In the present study, the major implied volatility indices reflecting U.S.'s stock market including OVX are categorized to one class as a significant source of information explaining the future oil price uncertainty.

In addition, there are also other variables affecting the future uncertainty, which have not been extensively investigated. According to Nazlioglu et al., 2015, oil market movements are affected by financial stress through their impact on both economic activity and investor behavior. On the one hand, increased financial stress causes economic activity to slow down, which leads to lower energy demand and declining oil prices. On the other hand, the oil market is considered as an alternative profitable investment by investors. Therefore, when investors are adjusting their portfolios with respect to oil price changes, this will have repercussions on financial asset prices. Simultaneously, increased financial stress seems to cause investors to adjust their portfolios and, therefore, is likely to have an impact on the oil market. In this chapter, financial stress indices across regions are incorporated in the current methodological framework as a class of factors that are likely to have influence on the future oil price uncertainty.

Another class of factors reflecting uncertainty includes economic policy uncertainty and business conditions indices, which can be a representation of the economic and business conditions on a daily basis. Moreover, one key factor for capturing the geopolitical uncertainty through a text-search process is the geopolitical risk index (GPR) calculated by Caldara and Iacoviello, 2018[1], which is also included in this class of uncertainty factors. Finally, in this study, the dynamic model averaging (DMA) methodology is used for forecasting oil price volatility and for examining the different uncertainty factors in order to assess their predictive ability.

[1] For further details see https://matteoiacoviello.com/gpr.htm.

DMA is a combination method and it can help us recognize the impact of each class on oil price volatility not only in the short-run but also in the mid- and long-run horizons.

This chapter contributes to the literature in the following ways. First of all, its main contribution is the investigation of whether the inclusion of exogenous variables that reflect the uncertainty environment provides predictive power regarding future oil price volatility in an out-of-sample analysis. Second, this study aims to provide an answer to the question of which class of the above mentioned factors, and specifically which predictor, is most information-rich for the future oil price volatility. Finally, we examine whether the individual models or the combination methods provide higher predictive ability for the oil price volatility in terms of both statistical and economic evaluation techniques.

The findings of this chapter indicate that the predictive accuracy of oil price realized volatility is improved by incorporating different uncertainty factors depending on the forecasting horizon. More specifically, when referring to short-run horizons, the contribution of the inclusion of all indicators with greater emphasis on implied volatility indices in the DMA approach is highly significant under both the statistical and economic evaluation frameworks. At mid- and long-run forecasting horizons, the impact of economic policy uncertainty and geopolitical risk indices on future oil price volatility is considered high enough to outperform the competing models. Additional useful information is that VXN improves the forecasting ability of the naive models especially at mid- and long-run horizons, which can be also viewed when trading United States Oil Fund, LP (USO) under the trading strategy is assumed. Finally, It is shown that the DMA combination method generates higher trading returns compared to the remaining models at 22-days ahead, which means that DMA including all indicators is also information-rich at long-run horizons that cannot be confirmed from the statistical evaluation techniques.

The rest of the chapter is structured as follows. Section 3.2 provides the literature review.

Section 3.3 reports the estimation of the realized oil price volatility. Section 3.4 presents the data that has been used in this chapter. Section 3.5 introduces the methodology, which can be separated to the individual and combination modelling frameworks. Section 3.6 describes the evaluation framework, while Section 3.7 analyses the findings of the chapter. Finally, Section 3.8 presents the conclusions of the chapter.

3.2 Review of the literature

In recent decades, many studies have focused on the oil price volatility, which as mentioned above is considered of major importance for the global economy. The earliest studies concentrate on forecasting oil price volatility by using squared daily returns of oil futures prices as proxy of volatility (Sadorsky, 2006; Sadorsky & Mckenzie, 2008). Subsequently, a lot of papers turned their attention to oil price conditional volatility forecasting by implementing GARCH-family models. For example, Kang et al., 2009 indicate the usefulness of the CGARCH and FI-GARCH models for modelling and forecasting persistence in the volatility of crude oil prices. In a similar fashion, Nomikos and Pouliasis, 2011 find that Mix-GARCH and MRS-GARCH models improve the forecasting performance of oil price conditional volatility compared to the simple GARCH model.

Nevertheless, Andersen and Bollerslev, 1998 estimate daily volatility by considering intra-day data, which has been suggested as more information-rich. This alternative volatility measure is calculated by summing the squared intra-day returns. More specifically, Andersen et al., 2001, Andersen, Bollerslev, Diebold, and Labys, 2003 and Mcaleer and Medeiros, 2008 consider the realized volatility measure as a proxy of daily volatility. Many studies show that using intra-day data for volatility forecasting, the forecasting performance of the models is improved (Engle & Sun, 2007; Hansen & Lunde, 2005). In this context, Corsi, 2009 introduces the Heterogeneous Autoregressive (HAR) model, which produces realized volatility forecasts by capturing "stylized facts" in financial market volatility, such as long memory.

Regarding oil price realized volatility, many studies investigate which variables are able to increase the forecasting performance of the HAR model. As noted in Chapter 2, Sévi, 2014 finds that considering independently the squared jump component, the continuous component, signed jumps and realized semivariances of both signs do not provide any further information compared to the simple HAR model when forecasting oil price volatility under a out-of-sample framework. Prokopczuk et al., 2016 also suggest that modelling jumps does not significantly improve the accuracy of volatility forecasts in energy markets. However, F. Ma et al., 2018 conclude that adding the jump component and its intensity can substantially increase the forecasting accuracy.

Recently, studies such as Haugom et al., 2014 incorporate OVX in order to investigate if the content of the implied volatility indices contains predictive power for the realized volatility of crude oil. Their results indicate that including implied volatility significantly improves daily and weekly volatility forecasts. An additional study that presents similar results is that of Dutta, 2017, which reveals that the information content of OVX helps to provide more accurate volatility forecasts in comparison with the simple HAR model. Furthermore, Lv, 2018 proposes that OVX and its decomposition based on a certain threshold provide more accurate oil price volatility forecasts. Interestingly, he finds that large OVX values have slightly larger impacts than smaller values of OVX on future volatility, which can be used in a future research study.

Some studies include other factors, besides OVX, in order to capture the future uncertainty. For example, R. Ma et al., 2019 investigate through a GARCH-MIDAS model the impact of economic policy uncertainty (EPU) on the crude oil price volatility and specifically which EPU index provides the most forecasting power in the crude oil market. It is found that EPU has a positive and significant short-term effect on the crude oil price volatility. Moreover, a study by Mei et al., 2019 provides information that not only EPU, but also monetary policy uncertainty (MPU) significantly helps in forecasting oil volatility by considering both of them together rather than separating those factors. The methodology that is implemented in the latter study relies on the GARCH-MIDAS modelling framework. Finally, the impact of the geopolitical risk factor on the oil price volatility is analyzed in recent studies. J. Liu et al., 2019 examine the role of geopolitical risk in the future oil price volatility and find that it contains useful information and can provide higher economic gains compared to the forecasts obtained from the simple GARCH-MIDAS model. A more recent study of Mei et al., 2020 provides qualitatively similar results by implementing a MIDAS model for forecasting realized oil price volatility. This specific MIDAS-RV-GPR model is extended by replacing the RV with the continuous and jump components, which can outperform the benchmark and other competing models. It is also noteworthy that the frequency of the above mentioned uncertainty factors that are used, namely the EPU, MPU and GPR is monthly. However, EPU and GPR indicators are also available in daily frequency, which is what we aim to use in this chapter in order to extract further information.

Another class of uncertainty indices that are recently used in an out-sample analysis is the financial stress environment, which is included by Gkillas et al., 2020. In that study, the impact of global and regional measures of financial stress on forecasting oil price realized volatility is investigated. They conclude that extending the simple HAR model by including indices of financial stress, which differentiates among regional sources, helps to improve forecasting performance. A first study that examines the relationship of financial stress index and oil prices is done by Nazlioglu et al., 2015, who reveal a significant spillover between the energy (via WTI crude oil) and financial (via financial stress index) markets,both in terms of volatility and mean estimates.

3.3 Estimating realized volatility

Realized variance has been widely estimated as a proxy for the volatility by using intra-day data. In our case, we follow most of the studies and our analysis is also based on the realized variance estimator. We are then based on the calculation of the realized volatility as noted in Chapter 2 and we use the annualized realized volatility, which is defined as ARV_t.

In this study, the quadratic variation is not decomposed into continuous and jump components since the main objective is not the investigation of the role of jumps in forecasting oil price volatility.

3.4 Data

The dataset of this chapter includes daily and tick-by-tick transaction data, which is used for the estimation of the realized volatility. More specifically, the WTI realized volatility estimator has been calculated by using tick-by-tick transaction data of the front-month futures contracts for the WTI crude oil. It is also important to mention that the 10 minutes sampling frequency has been found to minimize the autocovariance bias induced by microstructure noise issue and this is why this data frequency is used. The source of the retrieved tick-by-tick data is TickData. Figure 3.1 portrays the annualized realized volatility and the OVX series. It is apparent that high values of volatility are observed in the late 2014 - early 2015 period due to the sudden decline of the oil prices.

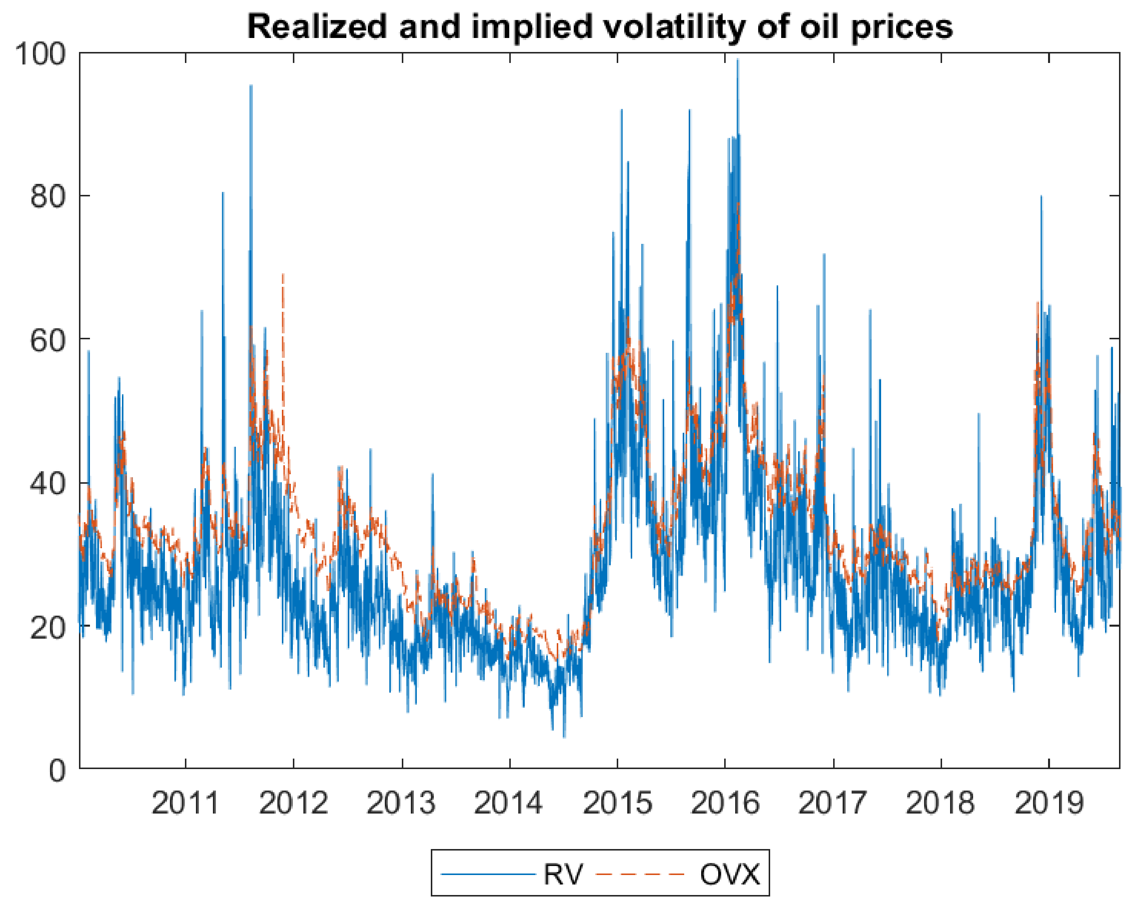

Figure 3.1 Realized and implied volatility of oil prices.

Besides the intra-day data, some implied volatility indices, namely the OVX, VIX, VXN and VXD are used in this study as predictors of the oil price volatility. This dataset is readily available at a daily frequency and is obtained from CBOE for the corresponding implied volatility indices.

Another class of uncertainty that is used in this chapter includes the EPU of U.S.[2] developed by Baker et al., 2016. We rely on U.S.'s EPU motivated by the study of Wei et al., 2017, who conclude that U.S.'s EPU index has superior predictive power for WTI spot oil volatility. This index is based on newspaper archives from Access World New's NewsBank service and measures the number of articles that contain at least one term from each of 3 sets of terms namely, economic or economy, uncertain or uncertainty, and legislation or deficit or regulation or Congress or Federal Reserve or White House. Moreover, GPR is added to the list of that reflect uncertainty. The GPR index reflects automated text-search results of the electronic archives of 11 national and international newspapers and it is calculated by counting, in each of the 11 newspapers, the number of articles that contain the search terms[3], which are related to geopolitical risks on a daily basis. Finally, the ADS business condition index is considered an uncertainty factor that is used in this chapter due to the fact that it is designed to track real business conditions at high observation frequency and its underlying economic indicators (such as weekly initial jobless claims, monthly industrial production, quarterly real GDP), blend high frequency and low frequency data[4].

Regarding the financial stress indices, the data is retrieved from the Office of Financial Research (OFR)[5], which provides a market-based snapshot of stress in financial markets on a

[2]For further details please visit the https://www.policyuncertainty.com/us_monthly.html.

[3]See https://www.matteoiacoviello.com/gpr.htm for further details.

[4]The ADS index on the web page https://philadelphiafed.org/research-and-data/real-time-center/business-conditions-index is updated in real time as new or revised data on the index's underlying components are released.

[5]The data is available for download from https://www.financialresearch.gov/financial-stress-index.

daily basis. This index is constructed using 33 financial market variables, such as yield spreads, valuation measures and interest rates. Furthermore, when the FSI index is positive, it means that the stress levels are above average. The index is negative when stress levels are below average. Figure 3.2 displays all the explanatory variables that are mentioned above categorized by class.

The sample for both WTI realized volatility and all the uncertainty indicators that are used is common from January 4, 2010 to August 30, 2019 and the number of observations is 2494 (trading days). Table 3.1 presents the descriptive statistics of those variables.

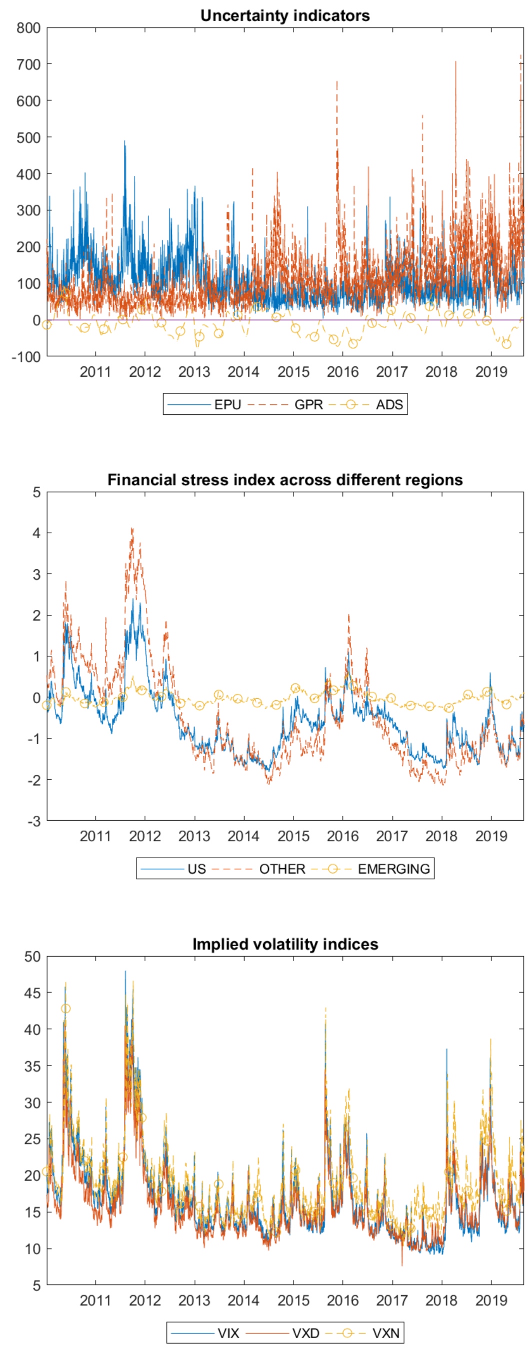

Figure 3.2 This figure depicts the indicators of the three uncertainty classes, namely the EPU, GPR, ADS from the first class, the FSI among regions from the second class and the VIX, VXD and VXN from the class including the major IV indices of the U.S. stock market.

Variable	USFSI	OTHERFSI	EMERGFSI	VIX	VXD	VXN
Mean	-0.5954	-0.4905	-0.0253	16.9430	16.0692	18.9949
Median	-0.6595	-0.9140	-0.0540	15.5750	14.8150	17.5500
Maximum	2.4080	4.1630	0.6300	48.0000	41.4500	46.6300
Minimum	-1.8090	-2.1620	-0.3420	9.1400	7.5800	10.3100
Std. Dev.	0.7943	1.2611	0.1645	5.6984	4.8556	5.4035
Skewness	1.0342	1.1817	0.9847	1.7100	1.7550	1.5575
Kurtosis	4.0671	4.0544	3.9872	6.6991	6.8793	5.9822
Jarque-Bera	562.9177	695.9212	504.3506	2637.4060	2844.1630	1932.4800
Observations	2494	2494	2494	2494	2494	2494

Table 3.1 Descriptive statistics of the variables that have been used in the empirical analysis for forecasting realized oil price volatility.

3.5 Modelling framework

3.5.1 Naive model specifications

A simple Random Walk (RW) without a drift is considered as our naive model and it is written as:

$$log(ARV_t) = log(ARV_{t-1}) + \varepsilon_t, \tag{3.1}$$

where ARV_t is the annualised realized volatility of the WTI crude oil at day t and ε_t is a white noise.

In addition to RW, the AR(1) is estimated, which is another naive model that has been widely used, since we would like to evaluate their forecasting performance compared to that of more sophisticated models. The AR(1) model specification is the following:

$$log(ARV_t) = \hat{a}_0^{(t)} + \hat{a}_1^{(t)} log(ARV_{t-1}) + e_t, \tag{3.2}$$

where e_t is the residual term.

3.5.2 Individual models - HAR model specification

In this chapter, the individual models are based on the HAR model specification proposed by Corsi, 2009. The HAR structure captures stylized facts of financial market volatility such as long memory and is motivated by the heterogeneous market hypothesis proposed by Muller et al., 1997. Thus, the main idea of the HAR specification is to use realized volatility aggregated over different time horizons in order to disentangle the information coming from the different market participants (e.g. short-term traders and long-term traders). The benchmark HAR-RV model is written as follows:

$$
\begin{aligned}
log(ARV_t) = \hat{a}_0^{(t)} + \hat{a}_1^{(t)} log(ARV_{t-1}^{(d)}) + \hat{a}_2^{(t)} log(ARV_{t-1}^{(w)}) \\
+ \hat{a}_3^{(t)} log(ARV_{t-1}^{(m)}) + e_t,
\end{aligned}
\tag{3.3}
$$

where e_t is the residual term and $\hat{a}_0^{(t)}, \hat{a}_1^{(t)}, \hat{a}_2^{(t)}, \hat{a}_3^{(t)}$ are the estimated parameters. Moreover, the components of the HAR structure, namely the realized volatilities aggregated over different time horizons, are calculated as: $log(ARV_{t-1}^{(d)}) = log(ARV_{t-1})$;
$log(ARV_{t-1}^{(w)}) = \left(5^{-1} \sum_{k=1}^{5} log(ARV_{t-k})\right)$;
$log(ARV_{t-1}^{(m)}) = \left(22^{-1} \sum_{k=1}^{22} log(ARV_{t-k})\right)$, which is in line with Corsi and Renò, 2012.

In this study, the HAR-RV model is extended by including OVX as an additional variable because of its large impact on oil price volatility according to many studies (Haugom et al., 2014; Lv, 2018). The HAR-OVX model specification is written as:

$$
\begin{aligned}
log(ARV_t) = \hat{a}_0^{(t)} + \hat{a}_1^{(t)} log(ARV_{t-1}^{(d)}) + \hat{a}_2^{(t)} log(ARV_{t-1}^{(w)}) \\
+ \hat{a}_3^{(t)} log(ARV_{t-1}^{(m)}) + \hat{\beta}_1^{(t)} OVX_{t-1} + e_t,
\end{aligned}
\tag{3.4}
$$

where $log(OVX_{t-1})$ is the one lagged OVX in logarithmic transformation. Since it is known that the HAR-OVX model provides predictive information on realized oil price volatility, we rely on this model and the additional factors reflecting uncertainty will be added in this specific model structure such as HAR-OVX-X, where X represents each uncertainty factor of the three different classes. First of all, denote by $X_t^{(UNC)}$ the uncertainty factor that represents each of the variables, which are included in the uncertainty class of geopolitical risk, economic policy uncertainty and ADS business conditions indicators. Each factor of the class of uncertainty, which maintains the financial stress indices, is written as $X_t^{(FSI)}$ and each factor of the third class (implied volatility indices), is represented as $X_t^{(IV)}$. Thus, three different categories of individual models are applied depending on the uncertainty class, written as follows:

$$\begin{aligned} log(ARV_t) &= \hat{a}_0^{(t)} + \hat{a}_1^{(t)} log(ARV_{t-1}^{(d)}) + \hat{a}_2^{(t)} log(ARV_{t-1}^{(w)}) \\ &+ \hat{a}_3^{(t)} log(ARV_{t-1}^{(m)}) + \hat{\beta}_1^{(t)} OVX_{t-1} + \hat{\beta}_2^{(t)} X_{t-1}^{(UNC)} + e_t, \end{aligned} \tag{3.5}$$

$$\begin{aligned} log(ARV_t) &= \hat{a}_0^{(t)} + \hat{a}_1^{(t)} log(ARV_{t-1}^{(d)}) + \hat{a}_2^{(t)} log(ARV_{t-1}^{(w)}) \\ &+ \hat{a}_3^{(t)} log(ARV_{t-1}^{(m)}) + \hat{\beta}_1^{(t)} OVX_{t-1} + \hat{\beta}_2^{(t)} X_{t-1}^{(FSI)} + e_t, \end{aligned} \tag{3.6}$$

$$\begin{aligned} log(ARV_t) &= \hat{a}_0^{(t)} + \hat{a}_1^{(t)} log(ARV_{t-1}^{(d)}) + \hat{a}_2^{(t)} log(ARV_{t-1}^{(w)}) \\ &+ \hat{a}_3^{(t)} log(ARV_{t-1}^{(m)}) + \hat{\beta}_1^{(t)} OVX_{t-1} + \hat{\beta}_2^{(t)} X_{t-1}^{(IV)} + e_t. \end{aligned} \tag{3.7}$$

Therefore, nine individual models are implemented, which are extensions of the HAR-OVX model. The forecasts obtained from these individual models will be evaluated in order to answer the question whether they provide additional information to the simple HAR-RV and the HAR-OVX models at different forecasting horizons or not. Before continuing to the next part, it is considered convenient for the current analysis to represent the aforementioned models in a more general way. Define $\mathbf{x_t}$=$\left[1 \;\; log(ARV_{t-1}^{(d)}) \;\; log(ARV_{t-1}^{(w)}) \;\; log(ARV_{t-1}^{(m)})\right]$ as the (1×4) vector of components of the simple HAR-RV. Moreover, $y_t = log(ARV_t)$ is defined, which is the logarithmic transformation of the realized oil price volatility. The vector of the uncertainty factors including at least OVX is represented as $\mathbf{x_{t,UNC}}$, $\mathbf{x_{t,FSI}}$, or $\mathbf{x_{t,IV}}$ depending on to which uncertainty class our exogenous variable belongs. One example of the latter vector could be the following: $\mathbf{x_{t,UNC}} = \left[OVX_{t-1} \;\; EPU_{t-1}\right]$, which refers to the HAR-OVX-EPU model. We can now replace Eq.(3.5), which corresponds to the simple HAR-RV, with the following one:

$$y_t = \boldsymbol{x_t \alpha_t} + \varepsilon_t, \tag{3.8}$$

where $\boldsymbol{\alpha_t} = \left[a_0^{(t)},\, a_1^{(t)},\, a_2^{(t)},\, a_3^{(t)}\right]'$ is the corresponding parameter vector. Regarding the different individual models, which correspond to Eq. (3.7-3.9), we replace them with a more general equation, as follows:

$$y_t = \boldsymbol{x_t \alpha_t} + \boldsymbol{x_{t,UNC|FSI|IV} \beta_t} + \varepsilon_t, \tag{3.9}$$

where $\boldsymbol{\beta_t}$ is a vector that includes at least OVX as an uncertainty indicator. For example, the HAR-OVX-EPU model can be defined under the Eq. (3.11) with the $\boldsymbol{\beta_t}$ vector denoted as $[\beta_{OVX}^{(t)}, \beta_{EPU}^{(t)}]'$.

After the estimation of the individual models, the forecasting framework is implemented, which is based on Buncic and Gisler, 2016. More specifically, the regression of the HAR-RV model can be defined as:

$$y_{t+h} = \boldsymbol{x_t \alpha^{(h)}} + \varepsilon_{t+h}. \tag{3.10}$$

The direct forecast is $\boldsymbol{x_t \hat{\alpha}_t^{(h)}}$, where $\boldsymbol{\hat{\alpha}_t^{(h)}}$ is an estimate of $\boldsymbol{\alpha^{(h)}}$ that only relies on data up to period t. Moreover, due to the fact that we focus on realized volatility forecasts in this chapter, we replace $y_{t+h} = log(ARV_{t+h})$ with $ARV_{t+h} = e^{y_{t+h}}$[6], where h denotes the days ahead that the forecasts are generated.

3.5.3 Dynamic model averaging (DMA) approach

Recent literature includes studies that capture structural breaks that are detected. Most of the models that are widely used so far assume constant coefficients, which is not the case in this

[6]It is noted here that the error variance term $0.5\sigma_\varepsilon^2$ should be incorporated in the forecasting, since logarithmic transformation is applied in the models, but it has negligible effect on the results.

study. In this methodological framework, time-varying parameters are assumed in order to capture potential structural breaks. Furthermore, in an individual modelling framework, the set of predictors included in the model remains constant over time. However, in this chapter, we allow for K models which utilize different sets of predictors to be applicable at different time periods. Therefore, it is allowed for both parameters of a model and the model itself to change over time by using the DMA approach proposed by Raftery et al., 2010. The state-space model[7] consists then of the two following equations:

$$y_t = \boldsymbol{x_t^{(k)}} \boldsymbol{\alpha_t^{(k)}} + \varepsilon_t^{(k)} \quad \text{for} \quad \varepsilon_t^{(k)} \sim N(0, H_t^{(k)}), \tag{3.11}$$

$$\boldsymbol{\alpha_t^{(k)}} = \boldsymbol{\alpha_{t-1}^{(k)}} + \boldsymbol{u_t^{(k)}} \quad \text{for} \quad \boldsymbol{u_t^{(k)}} \sim N(\boldsymbol{0_{4\times 1}}, \boldsymbol{\Sigma_{u_t}^{(k)}}). \tag{3.12}$$

where $k = 1, \ldots, K$, $\boldsymbol{\alpha_t^{(k)}}$ are the regression parameters of the HAR-RV model specification and the errors $\varepsilon_t^{(k)}$ and $\boldsymbol{u_t^{(k)}}$ are mutually independent at all of the leads and lags. Moreover, if there are m predictors in $\boldsymbol{x_t^{(k)}}$, the total number of possible combinations of these predictors is $K = 2^m$. According to the Eq. (3.13), which is based on the HAR-RV specification, the number of combinations is $K = 2^4 = 16$. This model can be estimated by using the Kalman filter method[8]. One key element that has to be mentioned is that the approximation of the forgetting factor λ, which is used in order to avoid estimating the state covariance matrix $\boldsymbol{\Sigma_{u_t}^{(k)}}$ is replaced by the standardized self-perturbed Kalman filter[9] proposed by Grassi et al., 2017. This specific

[7]This state space form is based on the simple HAR-RV model specification for simplicity reasons. The state space model is of similar fashion for the other models that include uncertainty indicators as predictors of the realized oil price volatility.

[8]More details for each step of the estimation of time-varying parameter (TVP) models and the combination of those models through the DMA approach can be found in the Appendix

[9]This methodological part is motivated by the first study (Chapter 2), which provides more details on this approximation.

approach avoids the calibration of a design parameter as the perturbation term is scaled by the amount of the uncertainty in the realized oil price volatility data.

In the current study, the main purpose is to explore the forecasting performance of the different uncertainty factors categorized into three classes, namely the uncertainty driven by economic policy uncertainty, geopolitical risk and business conditions, the uncertainty coming from the financial stress across regions and the uncertainty captured by the main implied volatility indices of the U.S. stock market. Thus, the DMA approach is implemented for each uncertainty class in order to generate forecasts of oil price volatility at multiple periods ahead. Therefore, Eq. (3.13) will be extended to the following:

$$y_t = \boldsymbol{x_t^{(k)}} \boldsymbol{\alpha_t^{(k)}} + \boldsymbol{x_{t,UNC|FSI|IV}^{(k)}} \boldsymbol{\beta_t^{(k)}} + \varepsilon_t^{(k)} \quad \text{for} \quad \varepsilon_t^{(k)} \sim N(0, H_t^{(k)}), \tag{3.13}$$

where there is one major difference. The vector of the uncertainty factors $\boldsymbol{x_{t,UNC|FSI|IV}^{(k)}}$ includes OVX and the whole indicators' list included in each category because we want to reveal the corresponding weight of each predictor to oil price volatility and not to rely on an individual uncertainty factor. In other words, one purpose of this chapter is the investigation of the predictive information of each factor for all uncertainty classes. Finally, since we need the incorporation of those uncertainty factors not to be restricted, one model is estimated including all variables of all uncertainty classes through the DMA approach in order to arrive at more general results for the contribution of the variables under investigation.

3.6 Forecast evaluation

3.6.1 Prediction settings

The settings regarding the modelling framework are as follows. First of all, the initial sample period is T=1043 days, since there is a need for $h-1$ days for implementing the direct forecasting approach and on additional 22 days (the component of the HAR structure with the maximum length). The 1000 days of the initial sample period T is the fixed length that is used for the rolling window estimation approach. More specifically, when referring to the rolling window estimation, we choose the data from the 1^{st} to the 1000^{th}, we then generate the forecasts by using the estimated parameters, we re-estimate the parameters for the data from the 2^{nd} to the 1001^{st} and we implement again the forecasting methodology. T=1000 days is justified by the fact that a large sample is required for estimating the proposed models. The remaining out-of-sample period is used for the evaluation of the oil price volatility forecasts and is defined as T_1.

3.6.2 Statistical loss functions

The first part of the evaluation of the models regarding their forecasting performance will be done using the widely used statistical loss functions, namely the Mean Squared Predicted Error (MSPE) and the Mean Absolute Error (MAE), which are defined as:

$$MSPE^{(h)} = \frac{1}{T_1}\sum_{t=1}^{T_1}(ARV_{t+h|t} - ARV_{t+h})^2, \tag{3.14}$$

and

$$MAE^{(h)} = \frac{1}{T_1}\sum_{t=1}^{T_1}\mid ARV_{t+h|t} - ARV_{t+h}\mid, \tag{3.15}$$

where $ARV_{t+h|t}$ is the h-days-ahead realized volatility forecast, ARV_{t+h} is the realized volatility at time $t+h$ and T_1 is the number of the out-of-sample data points.

3.6.3 Model Confidence Set

Additionally, the established Model Confidence Set (MCS) is used in order to further evaluate the predictions, since this specific procedure developed by Hansen et al., 2011 offers the advantage of identifying the set of the best models depending on a loss function, which is MSPE in our study. However, this procedure is also implemented by using MAE as a loss function, which provides qualitatively similar results.

The target of the MCS test is to investigate which set of models remains until the end, under an elimination algorithm, at a predefined level of significance a. At the beginning of the process, the full set of models $M = M_0 = \{1, \ldots, m_0\}$ is used and the following null hypothesis of equal predictive ability is repeatedly tested:

$$H_{0,M} : E(d_{i,i*,t}) = 0, \;\; \forall \; i, i^* \in M, \tag{3.16}$$

where $d_{i,i*,t} = \Psi_{i,t} - \Psi_{i*,t}$ is defined as the evaluation differential for i, $i^* \in M_0$ and $\Psi_{i,t} = (ARV_{t+h|t} - ARV_{t+h})^2$, where $ARV_{t+h|t}$ denotes the h-days-ahead oil price volatility forecast produced by using model i. This process is repeated until the null is not rejected any longer. The defined level of significance is $a = 0.1$ and a block bootstrap with 10,000 bootstrap replications is another predefined setting for fulfilling the procedure[10].

3.6.4 Trading strategy

Apart from the statistical evaluation of the obtained forecasts, which is based on the loss functions, we investigate the forecasting performance of the models by using an economic loss function. This kind of forecasting evaluation technique is based on trading USO. A simple trading strategy is applied, which works as follows: if the oil price volatility forecast of model

[10] For further details see Hansen et al., 2011.

i at time $t+h$ is higher than that of the actual oil price volatility at time t, the trader takes a short position in USO, which holds predominantly short-term NYMEX futures contracts on WTI crude oil. If the oil price volatility forecast of model i at time $t+h$ is lower than that of the actual oil price volatility at time t, then the position that the trader takes is long[11]. Thus, the cumulative return of model i, which is the metric for comparing the models, over the out-of-sample period is measured as[12]:

$$r^{(i)} = \sum_{t=1}^{T_1} \left(\frac{(USO_{t+h} - USO_t) d_t^{(i)}}{USO_t} \right), \tag{3.17}$$

where $d_t^{(i)} = 1$ if $ARV_{t+h|t} < ARV_t$ and $d_t^{(i)} = -1$ if $ARV_{t+h|t} > ARV_t$.

3.7 Out-of-sample results

3.7.1 Loss functions results

An this point, oil price volatility forecasts are evaluated by using the statistical loss functions, namely MSPE and MAE. Regarding both MSPE and MAE results, which can be found in Tables 3.2 and 3.3, respectively, the values of the two naive models and the simple HAR-RV are reported in the first rows, whereas the ratios of the corresponding model to the benchmark HAR-RV model are presented in the next rows of the table. These ratios can help us recognize which models perform better than others and mainly whether they provide better predictive accuracy in comparison with the simple HAR-RV model. For instance, a ratio that takes value below 1 denotes that the corresponding model outperforms the HAR-RV model. The results

[11] This idea is based on the sense that the trader is afraid of higher volatility and therefore (s)he assumes short position in USO.

[12] In this study, zero transaction costs are assumed because we are not focusing on the profits but in comparing the models which provide oil price volatility forecasts.

for both loss functions refer to five different forecasting horizons, namely 1, 5, 10, 15 and 22 days ahead.

MSPE results

Days-ahead	1	5	10	15	22
RW	74.92812	112.9533	141.7351	153.3292	187.9267
AR(1)	71.48478	103.8418	124.1968	134.5277	156.7742
HAR-RV	58.71964	87.50515	106.9928	121.4028	143.0997
HAR-OVX	0.913599	0.959522	0.980665	1.008551	1.007722
HAR-OVX-GPR	0.911765	0.948593	0.974812	0.99711	0.997739
HAR-OVX-EPU	0.907762	0.958769	0.968718	1.002888	1.002489
HAR-OVX-ADS	0.915976	0.965718	0.992219	1.022228	1.020747
HAR-OVX-USFSI	0.917274	0.968463	0.996901	1.03916	1.045858
HAR-OVX-OTHERFSI	0.909828	0.959308	0.9915	1.038028	1.05308
HAR-OVX-EMERGFSI	0.914774	0.968392	0.986006	1.014873	1.021001
HAR-OVX-VIX	0.910874	0.960551	0.980225	1.020985	1.030246
HAR-OVX-VXD	0.907329	0.946211	0.983489	1.018721	1.027408
HAR-OVX-VXN	0.916212	0.957868	0.971005	1.003369	1.00359
DMA-UNC	0.904325	0.978275	1.034682	1.044243	1.035657
DMA-FSI	0.900751	0.996188	1.026445	1.1068	1.122249
DMA-IV	0.897053	0.913063	0.97904	1.04845	1.02535
DMA-ALL	0.867821	0.90292	0.983021	1.100819	1.157492

Table 3.2 The results of the MSPE loss function for different forecasting horizons. Values represent ratios of the full set of the models to the HAR-RV, which is considered the benchmark model in this paper. A ratio below 1 suggests that MSPE of the respective model outperforms that of the HAR-RV model. The actual MSPE values are presented for RW, AR(1) and HAR-RV models.

MAE results

Days-ahead	1	5	10	15	22
RW	6.10246	7.434129	8.353564	8.353564	9.630948
AR(1)	5.858863	6.979731	7.682185	7.682185	8.690443
HAR-RV	5.190573	6.357207	6.998187	6.998187	8.209058
HAR-OVX	0.95553	0.96594	0.982697	0.982697	0.984607
HAR-OVX-GPR	0.956284	0.967176	0.985952	0.985952	0.991777
HAR-OVX-EPU	0.953191	0.968952	0.977608	0.977608	0.980887
HAR-OVX-ADS	0.956791	0.970215	0.99062	0.99062	0.995045
HAR-OVX-USFSI	0.959875	0.975151	0.992937	0.992937	1.003103
HAR-OVX-OTHERFSI	0.95568	0.973164	0.998129	0.998129	1.023486
HAR-OVX-EMERGFSI	0.955499	0.968833	0.987188	0.987188	0.996349
HAR-OVX-VIX	0.952626	0.968309	0.994183	0.994183	1.008376
HAR-OVX-VXD	0.951802	0.960009	0.984254	0.984254	1.003127
HAR-OVX-VXN	0.957124	0.972595	0.995466	0.995466	1.001912
DMA-UNC	0.953741	0.982148	1.018576	1.018576	1.018019
DMA-FSI	0.949533	0.988966	1.018683	1.018683	1.040901
DMA-IV	0.949333	0.953704	0.993267	0.993267	1.001921
DMA-ALL	0.942091	0.95835	1.009243	1.009243	1.064913

Table 3.3 The results of the MAE loss function for different forecasting horizons. Values represent ratios of the full set of the models to the HAR-RV, which is considered the benchmark model in this paper. A ratio below 1 suggests that MAE of the respective model outperforms that of the HAR-RV model. The actual MAE values are presented for RW, AR(1) and HAR-RV models.

From a first look, it is observed that the HAR-RV model always performs better than the two naive models, which is in line with the literature. It is also observed that the DMA-ALL model, which includes all uncertainty predictors, outperforms the remaining model in short-run horizons. This is also confirmed by viewing the MAE results, which are qualitatively similar. The fact that the DMA approach including all indicators reflecting uncertainty improves the forecasting ability, confirms our statement that combining different classes of uncertainty factors improves the predictive accuracy of realized oil price volatility. More specifically, according to MSPE results, the DMA-ALL model is capable of reducing the forecasting error by 13% in short-run horizons. It is also important to note that the DMA including IV indices performs well and seems to provide information on future oil price volatility.

Regarding mid-run horizons, it is observed that IV indices included as predictors in the DMA method, have a large impact on future oil price volatility. More specifically, regarding both MSPE and MAE results, it is noted that the forecasting error is reduced by almost 5% when implementing the DMA approach incorporating IV indices as predictors. In addition to this, the individual model HAR-OVX-VXN performs better than other models of its class. However, individual models representing the first uncertainty class, such as HAR-OVX-GPR and HAR-OVX-EPU, outperform the remaining models, which is a signal that economic policy uncertainty and geopolitical risk index are information-rich in mid-run horizons for oil price volatility forecasts. It is remarkable to mention here that these indicators when included in the DMA approach do not perform as well as they do when they are used in an individual modelling framework.

Regarding long-run horizons, it is obvious that the DMA approach does not beat even the simple HAR-RV model, which is the benchmark model in this study, in terms of MSPE and MAE loss functions. Moreover, the statement of the previous paragraph that individual models including economic policy uncertainty and geopolitical risk index outperform the rest of the models is confirmed, in terms of both MSPE and MAE loss functions. However, the forecasting error is slightly decreased even when we use the aforementioned models.

Thus, we can conclude that, under the loss functions of the evaluation framework, the DMA approach improves the predictive accuracy when referring to short-run horizons. The importance of IV indices is also significant in short- and mid-run horizons and more specifically the impact of VXN. Finally, at mid- and long-run horizons, indicators of the first uncertainty class, namely the EPU and GPR, outperform the remaining models when they are used in individual HAR model specifications. It is also crucial for our conclusion to mention that the aforementioned models include OVX by default, since it is considered of major importance for future oil price volatility[13]. Therefore, the above mentioned models that include uncertainty indicators enhance the forecasting ability of not only the HAR-RV model but also the HAR-OVX model.

3.7.2 Model Confidence Set procedure results

After having evaluated the models that have been used in the methodological part, it is considered efficient to find the set of the models that are included in the set of the best models. The first finding related to short-run horizons is that the two naive models and the HAR-RV do not belong to the confidence set of the best performing models. The results of the MCS test are presented in Table 3.4.

[13] The HAR-OVX model performs better than the HAR-RV model in short- and mid-run horizons.

MCS test

Days-ahead	1	5	10	15	22
RW	0.0000	0.0000	0.0002	0.0019	0.0002
AR(1)	0.0000	0.0005	0.0050	0.0133	0.0628
HAR-RV	0.0265	0.0976	**0.3459**	**0.9391**	**0.9674**
HAR-OVX	**0.1488**	**0.4944**	**0.6084**	**0.5255**	**0.6833**
HAR-OVX-GPR	**0.1488**	**0.5528**	**0.9131**	**1.0000**	**1.0000**
HAR-OVX-EPU	**0.1488**	**0.5528**	**1.0000**	**0.9391**	**0.9674**
HAR-OVX-ADS	**0.1488**	**0.1346**	0.0466	**0.1391**	**0.2793**
HAR-OVX-USFSI	**0.1223**	0.0976	0.0149	0.0133	0.0076
HAR-OVX-OTHERFSI	**0.1488**	**0.4944**	**0.3459**	**0.1077**	0.0076
HAR-OVX-EMERGFSI	**0.1488**	**0.1346**	**0.3459**	**0.5255**	**0.3427**
HAR-OVX-VIX	**0.1488**	**0.5528**	**0.7997**	**0.5039**	**0.2399**
HAR-OVX-VXD	**0.1488**	**0.5528**	**0.6084**	**0.5039**	**0.2399**
HAR-OVX-VXN	**0.1488**	**0.5528**	**0.9131**	**0.9391**	**0.9674**
DMA-UNC	**0.1488**	**0.4351**	0.0149	**0.5039**	**0.2793**
DMA-FSI	**0.1488**	0.0976	0.0149	0.0133	0.0076
DMA-IV	**0.1488**	**0.6636**	**0.9131**	**0.1391**	**0.3427**
DMA-ALL	**1.0000**	**1.0000**	**0.9131**	0.0133	0.0076

Table 3.4 The results of the MCS test for different forecasting horizons. Figures in bold denote the model that belongs to the confidence set of the best performing models.

The DMA including all uncertainty indicators is considered the best model at least for 1 and 5 days ahead. It is also observed that the combination of the DMA approach with all potential predictors do not belong in the set of the best models in mid- and long-run horizons, which confirms the results of the evaluation using the two loss functions. However, the models that incorporate variables of the first and the second class of uncertainty, except for the HAR-OVX-ADS model, belong always in the set of the best performing models.

Another finding is the fact that models which include FSI across regions do not belong in the best models, especially in mid- and long-run horizons. More particularly, the HAR-OVX-USFSI model that reflects the financial stress in the U.S. does not perform well and this is remarkable for the main findings of this chapter. Finally, we conclude from the results of the MCS test that IV indices, separately and combined in a DMA approach, always belong in the set of the best models and enhance the forecasting performance of the HAR-RV and HAR-OVX models, especially at short- and mid-run horizons. Regarding the first class of uncertainty and more specifically EPU and GPR indices, we conclude that their contribution to oil price volatility forecasting is crucial and enhances the predictive accuracy especially at mid- and long-run horizons, which is confirmed by the fact that they are the best models under the MCS test.

3.7.3 Trading strategy results

Another evaluation method, which is based on a simple trading strategy, uses 1-, 5-, 10-, 15- and 22-days ahead realized oil price volatility forecasts in order to take long or short position on USO. Figures 3.3 to 3.7 depict the cumulative trading returns over the out-of-sample period.

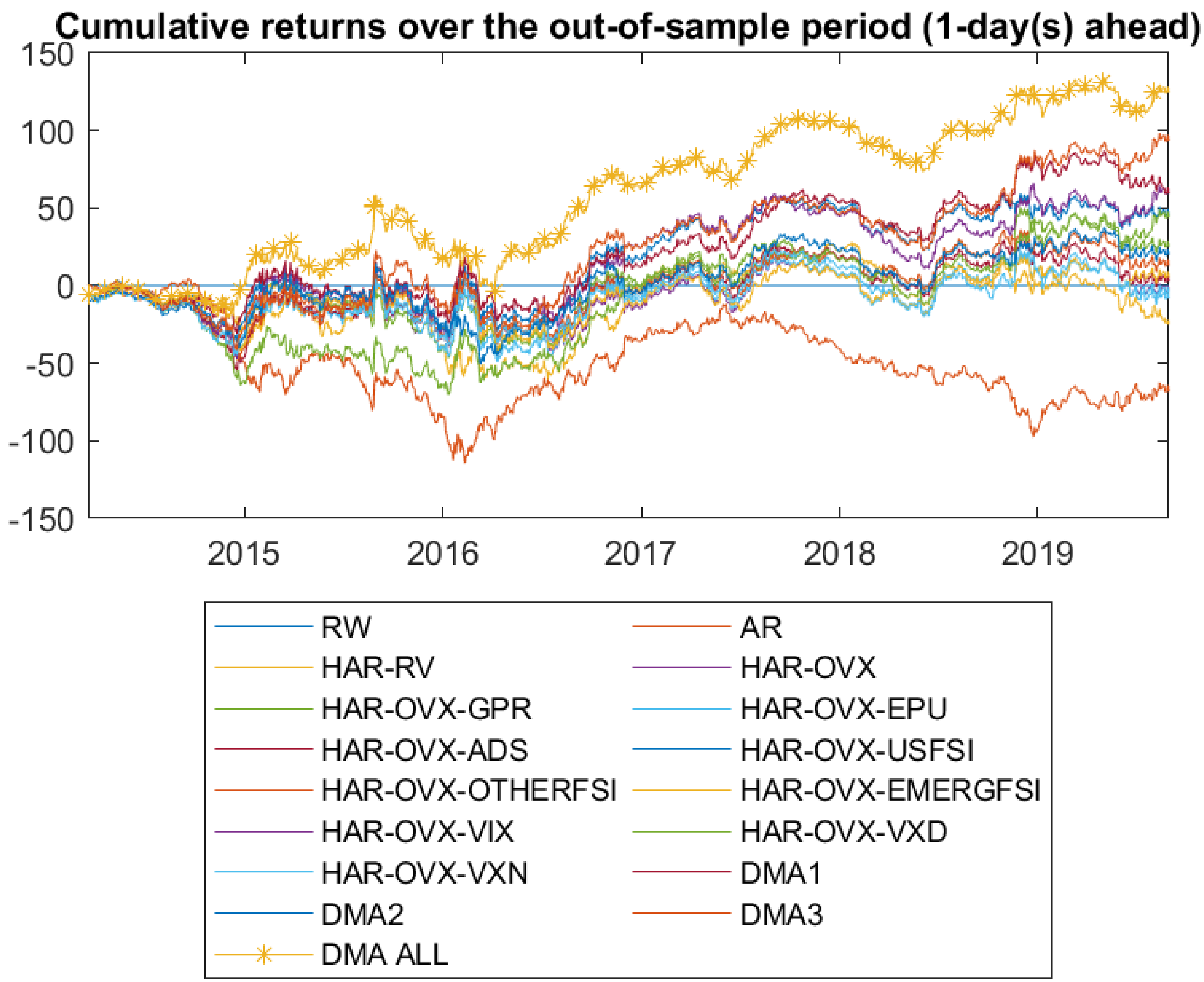

Figure 3.3 Cumulative trading returns when forecasting ARV 1-day ahead.

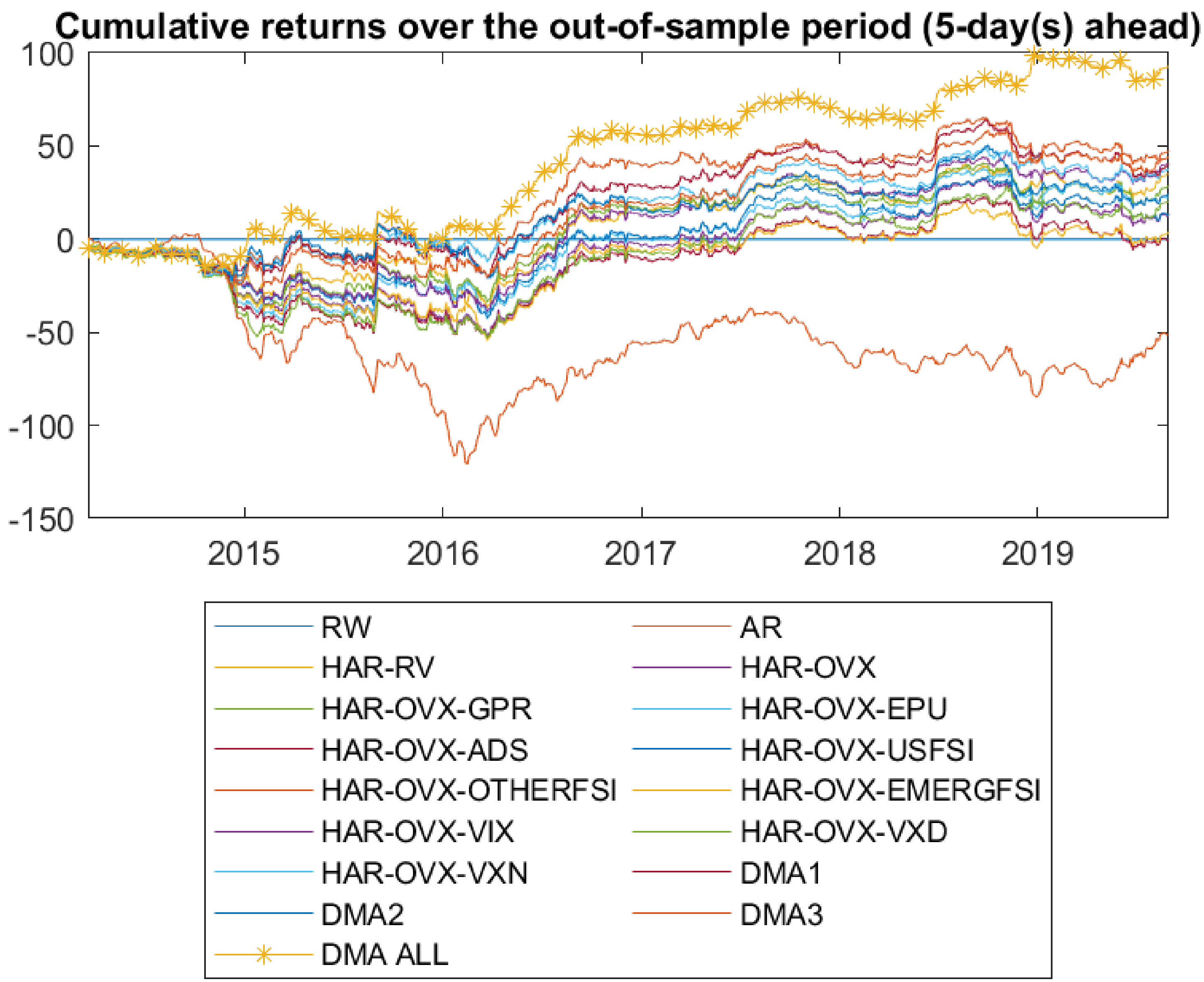

Figure 3.4 Cumulative trading returns when forecasting ARV 5-days ahead.

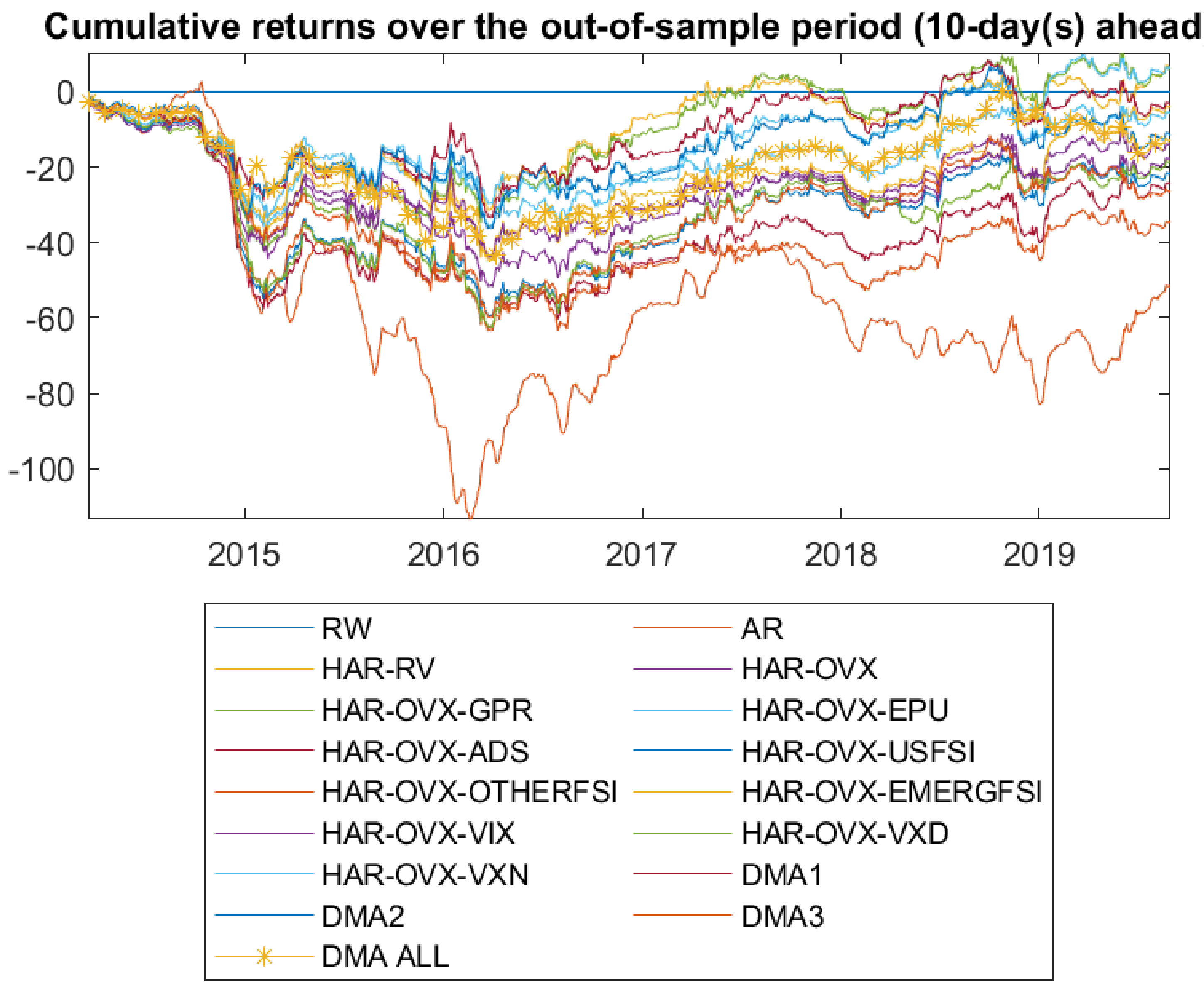

Figure 3.5 Cumulative trading returns when forecasting ARV 10-days ahead.

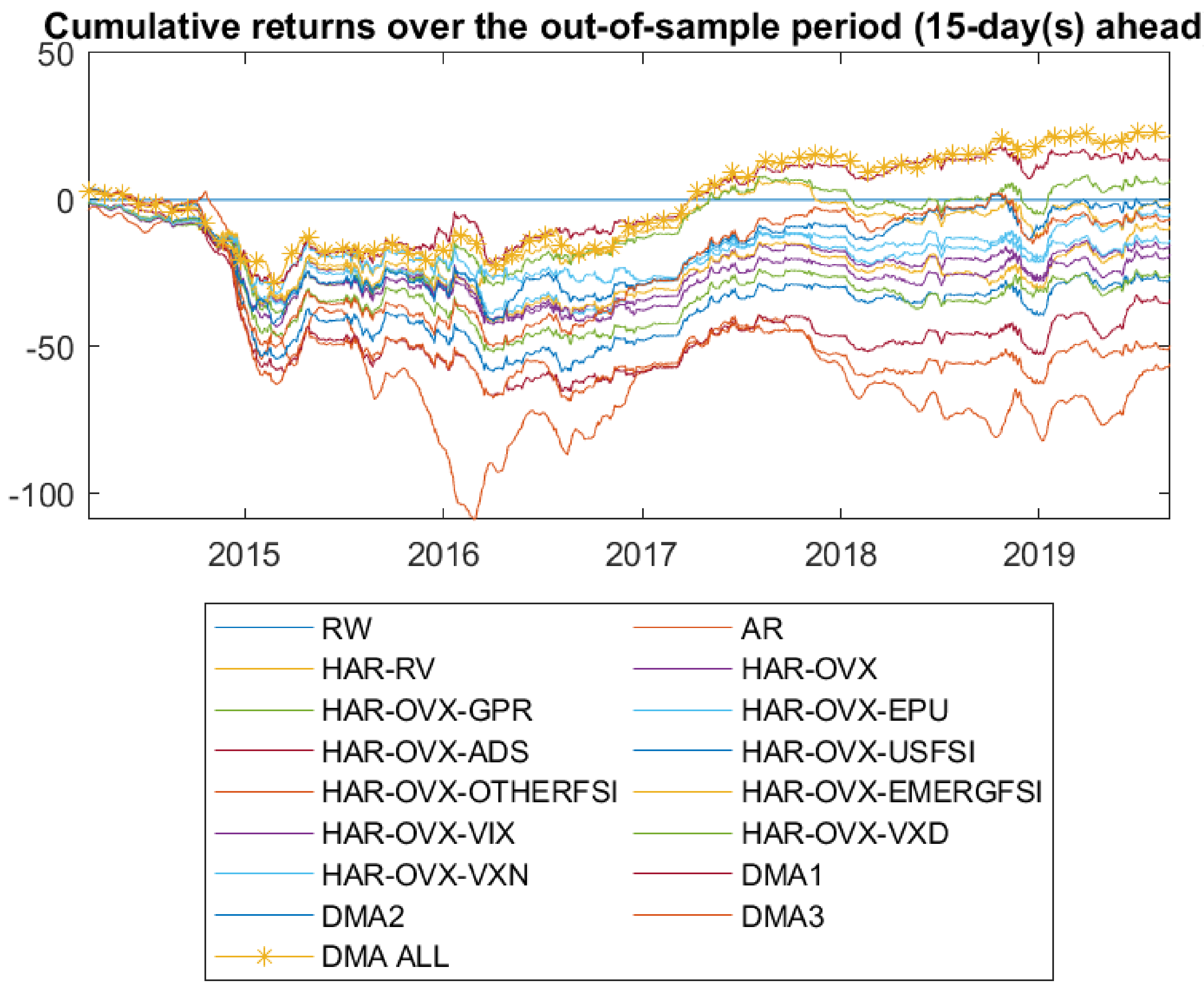

Figure 3.6 Cumulative trading returns when forecasting ARV 15-days ahead.

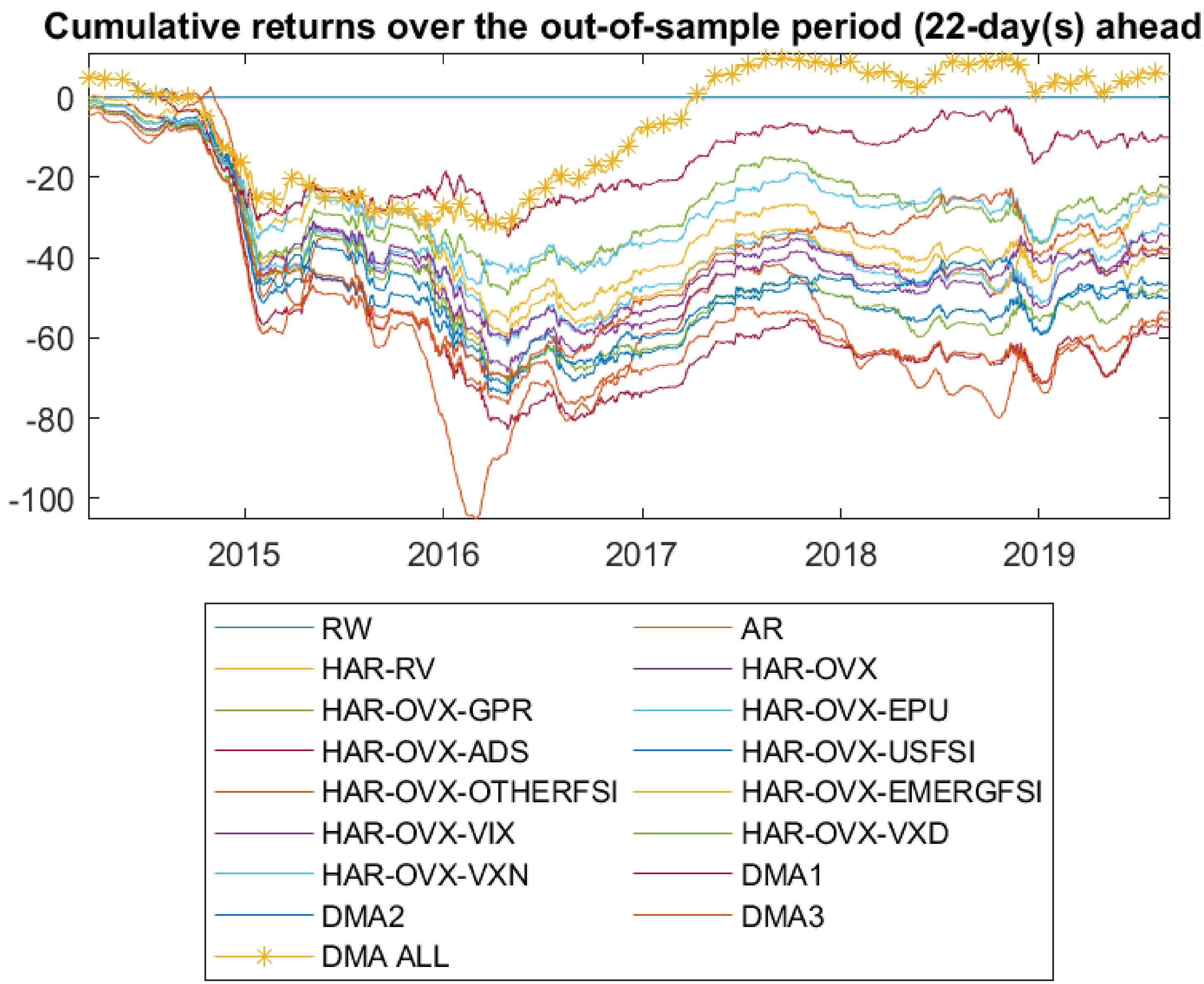

Figure 3.7 Cumulative trading returns when forecasting ARV 22-days ahead.

Focusing on the USO returns by using 1- and 5-days ahead oil price volatility forecasts, the conclusion that can be drawn is that the DMA approach including all predictors of the three classes generates the highest trading returns. Moreover, it is observed that the DMA approach, which includes indicators of the first and the third class obtains positive gains compared to most of the remaining models.

With regard to 10-days ahead forecasting, using individual models, namely the HAR-OVX-GPR and HAR-OVX-EPU models, and also the combination of the indicators of the first uncertainty class under the DMA approach provide the best results. This does not mean that they generate positive cumulative returns over the entire out-of-sample period, but even in those periods they obtain smaller losses compared to the remaining models.

Moreover, when assuming position in the USO based on 15- and 22-days ahead oil price volatility forecasts, is is observed that the DMA approach including all potential predictors and DMA which incorporates the indicators of the first uncertainty class, outperform the remaining models since they generate higher cumulative returns compared to the competing models. The fact that those factors are information-rich in long-run horizons is in line with the evaluation under the statistical loss functions and results of the MCS test.

Finally, from Tables 3.5 and 3.6, which show the annualized cumulative differences of the entire list of the implemented models compared to the models HAR-RV and HAR-OVX, it is observed that DMA approaches including all uncertainty factors and especially implied volatility indices generate higher trading returns referring to short-run horizons. More specifically, DMA-ALL generates 25% trading returns higher than HAR-RV and 22% higher than HAR-OVX, which can be considered of major importance for professional forecasters. The DMA-IV model which includes the main implied volatility indices of the U.S. stock market, generates trading returns 20% higher than HAR-RV and 17% higher than HAR-OVX. Regarding mid-run horizons, the impact of the first uncertainty class including the EPU, GPR and ADS indices is already obvious. More specifically, the inclusion of geopolitical risk in an individual model generates higher trading returns compared to both HAR-RV and HAR-OVX models. This difference of cumulative returns is almost 3%, which holds for long-run horizons, as well. One difference compared to the results of the previous evaluation techniques is the fact that DMA-UNC and DMA-ALL outperform the remaining models by generating higher trading returns at long-run horizons, which is not the case when the forecasts obtained by the models are evaluated using statistical loss functions and the MCS test. In this case, DMA-ALL generates more than 6% higher trading returns than HAR-RV and HAR-OVX models, when referring to the forecasting horizon of 22-days ahead. Finally, it is remarkable the fact that the HAR-OVX models generates higher trading returns than the HAR-RV model only at short-run horizons, which means that the predictive accuracy that OVX provides to the simple HAR-RV is restricted to short-run horizons. This statement is also confirmed by the previous evaluation techniques.

Annualized cumulative differences in trading returns (all vs HAR-RV)

Days ahead	1	5	10	15	22
RW	-11.212	-21.7416	-16.0331	-14.53	-8.29693
AR(1)	-9.22747	-16.2644	-10.1465	-9.20942	-2.27066
HAR-OVX	3.268394	-4.10577	-3.51336	-2.54182	-0.08099
HAR-OVX-GPR	8.330662	-1.10968	0.032817	1.351331	2.548685
HAR-OVX-EPU	3.097745	-2.37795	0.069991	-0.68605	0.891513
HAR-OVX-ADS	4.421342	-6.37209	-5.33188	-5.86268	-3.58658
HAR-OVX-USFSI	11.94351	-3.98232	-4.87983	-4.54381	-1.72999
HAR-OVX-OTHERFSI	6.209019	1.974884	-7.14553	-8.50208	-3.0255
HAR-OVX-EMERGFSI	5.542214	-5.88071	-2.03655	-1.45384	1.91698
HAR-OVX-VIX	14.37264	0.846483	-3.95023	-2.99978	0.665908
HAR-OVX-VXD	11.82085	-2.93133	-4.32969	-4.12689	-1.7715
HAR-OVX-VXN	2.662846	0.274656	-2.02537	-2.27084	2.089364
DMA-UNC	14.42257	0.827353	-2.00551	1.711517	3.56313
DMA-FSI	8.064451	-1.93874	-3.47887	-1.17467	-3.49676
DMA-IV	20.37837	1.538986	-5.8583	-1.81237	-1.4791
DMA-ALL	25.84689	10.12662	-3.36809	3.293883	6.420323

Table 3.5 The figures refer to percentages. When the value is positive, it means that the corresponding model generates higher trading returns compared to the HAR-RV model.

Annualized cumulative differences in trading returns (all vs HAR-OVX)

Days ahead	1	5	10	15	22
RW	-14.48038231	-17.6358	-12.5198	-11.9882	-8.21594
AR(1)	-12.4958594	-12.1586	-6.63311	-6.6676	-2.18967
HAR-RV	-3.268394224	4.105765	3.513359	2.54182	0.080991
HAR-OVX-GPR	5.06226805	2.996084	3.546176	3.893151	2.629676
HAR-OVX-EPU	-0.170648775	1.727819	3.583349	1.855768	0.972505
HAR-OVX-ADS	1.152948122	-2.26632	-1.81852	-3.32086	-3.50559
HAR-OVX-USFSI	8.675111846	0.12345	-1.36647	-2.00199	-1.649
HAR-OVX-OTHERFSI	2.940624693	6.080649	-3.63217	-5.96026	-2.94451
HAR-OVX-EMERGFSI	2.27381976	-1.77494	1.476811	1.087977	1.997972
HAR-OVX-VIX	11.10424621	4.952248	-0.43687	-0.45796	0.746899
HAR-OVX-VXD	8.552455086	1.174438	-0.81633	-1.58508	-1.69051
HAR-OVX-VXN	-0.605547928	4.380421	1.487984	0.270975	2.170355
DMA-UNC	11.154176	4.933118	1.507849	4.253337	3.644121
DMA-FSI	4.796057147	2.167021	0.034492	1.367149	-3.41577
DMA-IV	17.10997702	5.644751	-2.34495	0.729451	-1.39811
DMA-ALL	22.57849562	14.23239	0.145271	5.835702	6.501315

Table 3.6 The figures refer to percentages. When the value is positive, it means that the corresponding model generates higher trading returns compared to the HAR-OVX model.

3.8 Conclusion

Due to the fact that academics, investors and policy makers have shown great interest in oil price volatility, this study aims to fill the gap and answer the question "Which is the impact of the uncertainty environment on oil price volatility?". First of all, we started by categorizing the different indicators into uncertainty classes, and both individual models and combinations of those models are implemented through the DMA method. This chapter aims to extract information that no existing study has obtained by gathering all the uncertainty indicators in a modelling framework. The proposed models are based on the HAR-OVX model specification, since OVX is observed to be information-rich for future oil price volatility. One of the findings of this chapter is that the latter statement holds but only at short-run horizons. The HAR-OVX model does not provide additional information over the simple HAR structure specification at longer forecasting horizons.

At first, regarding short-run horizons, we conclude that DMA approaches including all indicators and especially the implied volatility indices that have been used in this study outperform the remaining models not only with respect to statistical evaluation techniques but also under a simple trading strategy that is performed in trading USO. The results of this trading strategy show that when combining all the uncertainty indicators under a DMA method, the cumulative returns are higher than 100% at the end of the out-of-sample forecasting period, which can be really useful for investors and professional forecasters.

Regarding mid- and long-run horizons, it is shown not only from the two loss functions and the MCS test, but also from the results of the trading strategy on USO, that the impact of geopolitical risk and economic policy uncertainty indices is high and the models including those factors, namely the individuals and the DMA-UNC, provide higher predictive accuracy compared to the remaining models.

Finally, it is recommended that forecasters should take into account the fact that implied volatility indices, namely the VIX, VXD and VXN, enhance the predictive accuracy of oil price volatility at short-run horizons, while the economic policy uncertainty and geopolitical risk indices are information-rich at mid- and long-run horizons. It is remarkable that the financial stress indices across regions do not significantly improve the forecasting ability especially for mid- and long-run horizons. Therefore, each of the uncertainty classes that are proposed plays a different role in forecasting oil price volatility. Finally, the proper combination of the aforementioned indicators is considered crucial for forecasting oil price volatility and can also generate high trading returns.

The results of this chapter indicate some interesting directions for future research. First, the implementation of additional combination techniques could be really interesting for both academics and investors. In addition to this, to investigate the impact of the uncertainty indicators in forecasting crude oil volatility index (OVX) would be a worthwhile research topic.

Chapter Four

Hedging opportunities for crude oil volatility

4.1 Introduction

In this chapter, the time-varying correlations between crude oil volatility and volatility measures of other three major asset classes are examined. There are published papers focusing on the relationship and the co-movements between crude oil and representatives of other asset classes, such as foreign exchange and stock markets. These interactions between crude oil, stock, foreign exchange markets and the market related to the macroeconomic conditions could be beneficial for investors that take positions on crude oil volatility and this is a primary target for investigation in this study. Nowadays, hedging is considered important for crude oil due to the high levels of volatility. Apart from the fact that most crude oil investors look for efficient hedging strategies in order to reduce the risk of their portfolios, they also focus on how to maximize their returns, which is not investigated in this study.

In this study, we examine whether hedging opportunities could be created by identifying the co-movements of crude oil and other assets and using them not only to compute the optimal portfolio weights as a hedging strategy, but also to measure the hedging effectiveness between the crude oil volatility and the volatility of the three major asset classes, namely the stock, the foreign exchange market and the market related to the macroeconomic conditions. It is important to mention that we focus on volatility and not on returns, since academics, investors and policy makers concentrate their attention on the global uncertainty, which is increased during the covid-19 crisis and, more specifically, due to the fact that crude oil has faced large deviations in recent years. During the Global Financial Crisis of 2007-2009, oil prices fluctuated from about \$60 to \$145 and then dropped again to \$30. Moreover, during 2014-2015, oil prices reduced by more than 75%, which is a signal that not only investors but also academics should give more attention to crude oil volatility, which has experienced large ups and downs.

It is important to mention the fact that due to the uncertainty of the global environment, a modeling framework that takes into account the time variation of the co-movements between the realized volatility of crude oil and the realized volatility measures of the other assets is implemented. The methodological framework, then, consists of the so called Dynamic Conditional Correlation model (DCC). The DCC model is used to capture the correlation clustering in financial time series and provides us with the conditional variance-covariance matrix, which will be used for the calculation of the optimal portfolio weights in different portfolios always including as an asset the WTI crude oil volatility. To the best of our knowledge, there is only one study focusing specifically on the correlation between crude oil volatility and volatility measures of several currencies by implementing the Diagonal BEKK model in order to estimate the time-varying correlations. However, the DCC model is considered to capture the problem of "curse of dimensionality" in multivariate modeling more efficiently. It also provides us with the ability to estimate the elements of the conditional variance-covariance matrix with several GARCH specifications by capturing the one best fitting to the characteristics of the time series, which is realized volatility in this case.

The main goal of this chapter is motivated by the work of Olstad et al., 2020, who investigate the dynamic correlations between the volatility measures of two oil benchmarks (both WTI and Brent) and six currencies, through a diagonal BEKK model. In this study, we enhance and extend their work by expanding the asset classes and not focusing exclusively on currencies, which represent the foreign exchange market. Moreover, high-frequency data is used in order to estimate a more accurate volatility estimator, which is the widely used realized volatility. To the best of our knowledge, there is no existing research focusing on the potential hedging opportunities of these asset classes (stock and foreign exchange markets and the market related to macroeconomic conditions) and conducting comparisons among them when referring to the interactions between crude oil volatility and the volatility of each of the other asset classes. For the assessment of whether these co-movements could be beneficial for crude oil volatility investors, a specific risk management strategy is applied, namely the optimal portfolio weights for two-asset portfolios that include WTI crude oil volatility by default and each one of the above mentioned volatility measures of the other three assets.

The main contribution of this chapter is the investigation of whether crude oil volatility could be hedged appropriately by using information from volatility measures of representative assets of stock market, foreign exchange market and the market related to macroeconomic conditions. Another contribution is the use of realized volatility as volatility measure instead of using conditional volatility or stochastic volatility, since a large number of studies have shown that realized volatility is a more accurate estimator for volatility and it is proved to produce better volatility forecasts (Engle & Sun, 2007; Hansen & Lunde, 2005; Tay et al., 2009). The attention of academics, investors and professional forecasters, in general, has been concentrated on realized volatility (Haugom et al., 2014; Phan et al., 2016; Prokopczuk et al., 2016; Sévi, 2014; Wen et al., 2016). More specifically, for the case of crude oil volatility, there are several studies focusing on the use of realized volatility in order to capture the intra-day information (Degiannakis & Filis, 2017; J. Liu et al., 2018; F. Ma et al., 2017; Prokopczuk et al., 2016). Finally, the findings that come to the conclusion of whether and which asset class can offer gains to crude oil volatility investors are also considered of major importance and fill the gap in the relevant literature.

The rest of the chapter is structured as follows. Section 4.2 reviews the existing relevant literature and gives more details on the theoretical background that the chapter is based on. Section 4.3 provides a detailed description of the data and the estimation of the realized volatility. Section 4.4 elaborates on the methodological part by providing details on the DCC-GARCH model specification, the calculation of the optimal portfolio weights and the hedging effectiveness, while Section 4.5 analyses the results of this study. Section 4.6 concludes the study and shares ideas for future research on this topic.

4.2 Theoretical background - Literature review

The theoretical background of this chapter consists of several parts but it mainly focuses on the relationship between crude oil volatility and volatility measures of other three major asset classes that comprise the financial markets, namely the stock market, the foreign exchange market and the market related to macroeconomic conditions. Thus, it is considered of vital importance to provide further details on the relationships between these assets and how they can be interconnected. Apart from these relationships, the most widely used methodological frameworks are presented in this section of the chapter and more details on the way that academics have implemented these econometric methods. Moreover, recent literature has come up with several ways to identify whether hedging crude oil can be beneficial for investors and stakeholders, in general. More specifically, in this study, we focus on opportunities related to risk reduction that could arise from using realized volatility of different asset classes in order to hedge crude oil realized volatility efficiently.

4.2.1 Relationship between crude oil and stock market

There are several papers focusing on the relationship between crude oil and stock markets at the level of returns. More specifically, Filis and Chatziantoniou, 2014 concentrate on aggregated stock market indices by also considering other macroeconomic variables, such as interest rates, industrial production and unemployment, in the structure of Vector Autoregressive (VAR) models. In contrast, there are studies that use a very common model for the identification of oil price effects on stock market, which is the well-known GARCH(1,1) model (Arouri & Nguyen, 2010; Broadstock et al., 2014). However, several authors investigate the time-varying relationship between oil and stock markets by using multivariate GARCH models, such as the BEKK model or the DCC model proposed by Engle, 2002. These studies include those of Filis et al., 2011 and Degiannakis et al., 2013. It is also important to mention that all the above papers use monthly data for the crude oil returns.

4.2.2 Relationship between crude oil and foreign exchange market

Apart from the relationship between crude oil and stock markets, the relationship of crude oil and foreign exchange markets is investigated at the volatility level. Regarding the literature that concentrates on returns level, there are several papers, such as Krugman, 1980, Aloui et al., 2013, Beckmann and Czudaj, 2013a, 2013b, which focus on the relationship of crude oil and foreign exchange markets at returns level using static frameworks. More specifically, Krugman, 1980 develops a simple model that takes into consideration some of the channels through which oil price changes affect exchange rates. Aloui et al., 2013 investigate the dependence between crude oil prices and U.S. dollar exchange rates using a copula-GARCH methodological approach and they provide evidence of significant dependence for almost all the oil-exchange rate pairs considered in their study. Beckmann and Czudaj, 2013a discriminate between long-and short-run dynamics of the relationship between crude oil prices and dollar exchange rates and conclude that this relationship is time-varying, suggesting that nonlinearities are an important issue when analyzing oil prices and exchange rates. In another paper, Beckmann and Czudaj, 2013b extend their previous study by showing that the results depend on which measure for exchange rate has been selected. Moreover, it plays a crucial role whether the time-varying relationship mainly runs from nominal exchange rates to nominal oil prices, or vice versa.

4.2.3 Relationship between crude oil and global macroeconomic conditions

In this chapter, we also focus on the relationship between crude oil realized volatility and volatility measure of the macroeconomic environment, which has not been examined extensively, in terms of volatility, so far. According to Yang and Zhou, 2017, the U.S. treasury bill yield represents the U.S. short-term interest rate. It is directly interconnected with the stock market volatility as written by Bailey and Stulz, 1989 and Bekaert et al., 2009. Furthermore, there is evidence that short-term interest rates forecast subsequent changes in the Fed's announced target rate (Fama, 2013; Hamilton & Oscar, 2002). This suggests that the U.S. short-term interest rate reflects the market's expectation of one dimension of the Fed's future monetary policy stance and, therefore, can be used as a representative of the asset class that is related to macroeconomic conditions.

4.2.4 Hedging strategies

The reason why academics and investors focus on hedging strategies comes from the fact that the need for reduction of their portfolios' risk is of major importance, since they need protection for their positions. A constant asset allocation of a portfolio over time would not allow for minimizing the portfolio variance and maximizing returns. However, this can be done by implementing a dynamic optimal portfolio strategy, which is applied in the current study, even if such a dynamic hedging strategy could result in high transaction costs. Apart from the optimal portfolio weights, there are optimal hedging ratios (conditional and unconditional) that are implemented for risk management purposes. The conditional optimal hedge ratios could be considered more precise but due to the frequent re-balancing, they require more transaction costs, and therefore lower profits (Cotter & Hanly, 2012; Fan et al., 2016). Recently, there are studies that capture time-variation in their optimal hedging strategies. For example, Basher and Sadorsky, 2015 examine the performance of the optimal hedging ratios, based on time-varying volatility models and conclude that these hedging ratios constructed by taking into account the time-variation perform better. Furthermore, Alizadeh et al., 2008 show that optimal hedging ratios, which are estimated by using dynamic conditional GARCH, MRS-GARCH and MRS-BEKK models yield better results compared to those that are estimated by using constant models. In this regard, this study is motivated by these findings and the use dynamic models for conditional volatility is preferred.

4.3 Data

4.3.1 Data description

This study focusing on realized volatility measures of different asset classes uses intra-day data for the WTI crude oil prices (WTI), the S&P500 index (SP500), the U.S. Dollar index (USDX) and the U.S. T-bills (TBILLS). Following Andersen, Bollerslev, Diebold, et al., 2003; Andersen et al., 2007 and several more recent studies (Degiannakis & Filis, 2017; Sévi, 2014; Yao et al., 2019), the time-series of the realized volatility measures are then constructed using 5-minute intra-day data of the front-month futures contracts for the WTI crude oil, the S&P500 index as a representative asset for the stock market asset class, the U.S. Dollar index as the representative of the foreign exchange market and the U.S. T-bills as a representative asset of the macroeconomic conditions. The sample period of this dataset starts from 4^{th} January 2010 to 30^{th} October 2017 and the daily realized volatility measures are constructed by using the common sample of the aforementioned representatives of the assets classes. Finally, all data are obtained from TickData.

4.3.2 Realized volatility estimation

In this chapter, we also work with annualized realized volatility series, which is defined as ARV_t.

Table 4.1 gives a summary of the descriptive statistics for all realized volatility measures of the different assets used in this study. We first observe that the mean of WTI crude oil realized volatility is higher than those of the other assets, with S&P500 the second highest. However, we see that S&P500 realized volatility is more volatile with higher coefficient of variation than WTI realized volatility. Moreover, the evolution of the volatility time series are portrayed in Figure 4.1.

Realized volatility measures	**WTI**	**S&P500**	**USDX**	**TBILLS**
Mean	28.31	12.96	6.86	5.02
Median	25.68	11.02	6.42	4.51
Minimum	4.66	1.60	1.44	1.71
Maximum	98.97	87.89	31.14	21.41
Std. Dev.	12.50	7.67	2.64	1.93
Skewness	1.58	2.66	1.45	2.44
Kurtosis	6.59	16.57	8.29	13.58
Coefficient of Variation	0.44	0.59	0.39	0.39

Table 4.1 Descriptive statistics of the realized volatility measures used in this study.

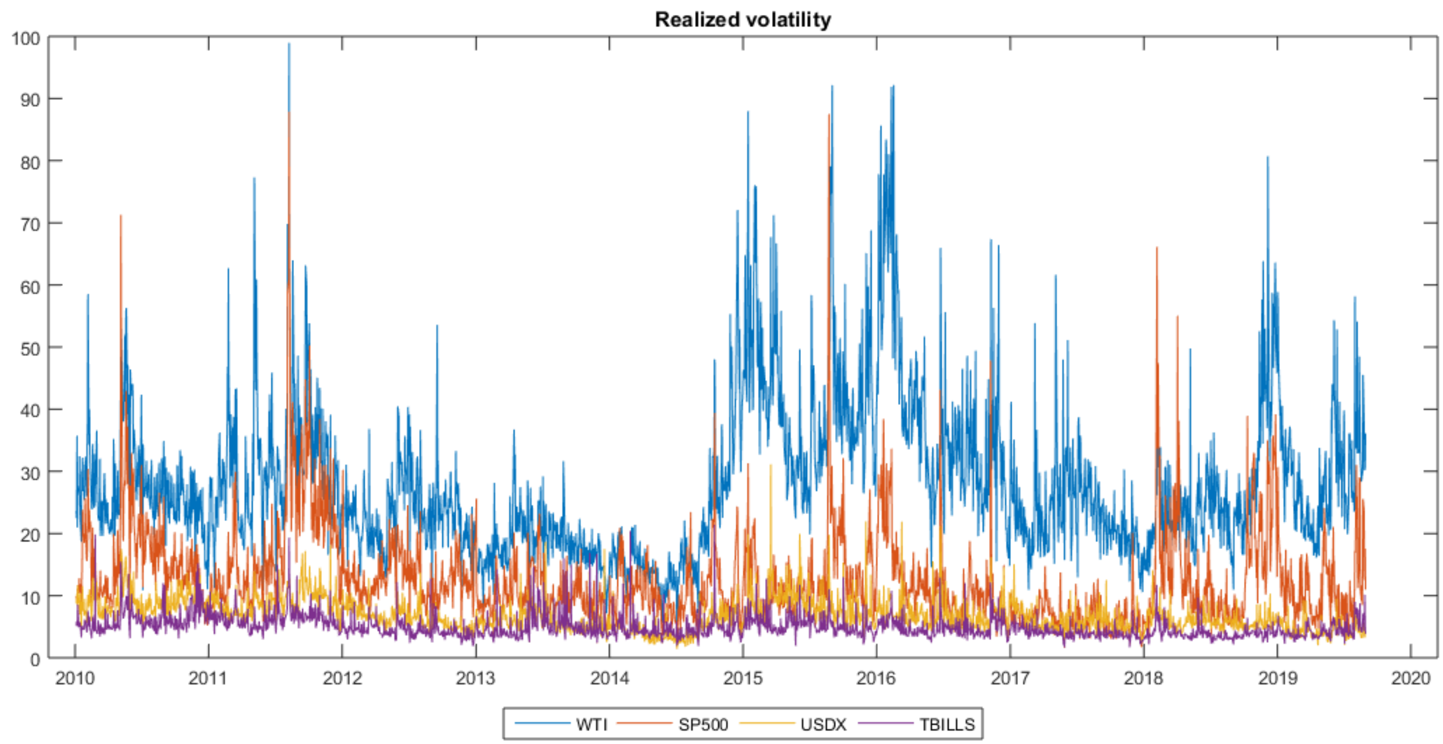

Figure 4.1 Evolution of the realized volatility measures for all assets over the entire sample period.

4.4 Methodology

4.4.1 DCC-GARCH model

In this study, the DCC-GARCH model of Engle, 2002 is used to estimate the conditional correlations between crude oil realized volatility and realized volatility of the other three asset classes. Let $\boldsymbol{y}_t$ be a $4x1$ vector of volatility measures meaning $\boldsymbol{y}_t = [ARV_t^{(WTI)}, ARV_t^{(SP500)}, ARV_t^{(USDX)}, ARV_t^{(TBILLS)}]'$.

The model is expressed as follows:

$$\begin{aligned} \boldsymbol{y}_t &= \boldsymbol{\mu} + \boldsymbol{\varepsilon}_t, \text{ for } t = 1, \ldots, T \\ \boldsymbol{\varepsilon}_t &= \boldsymbol{H}_t^{1/2} \boldsymbol{z}_t \end{aligned} \tag{4.1}$$

$\boldsymbol{H}_t$ is a $4x4$ positive definite conditional covariance matrix of $\boldsymbol{y}_t$ and $\boldsymbol{z}_t$ is an $4x1$ i.i.d random vector of errors. It is also important to note that the DCC-GARCH model of Engle, 2002 is estimated in two steps. First, the GARCH parameters are estimated. Second, the conditional correlations are estimated. The conditional covariance matrix $\boldsymbol{H}_t$ is defined as follows:

$$\boldsymbol{H}_t = \boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t. \tag{4.2}$$

$\boldsymbol{R}_t$ is the conditional correlation matrix and $\boldsymbol{D}_t$ is a diagonal matrix with time-varying standard deviations on the diagonal.

$$\boldsymbol{D}_t = \text{diag}(h_{1,t}^{1/2}, \ldots, h_{4,t}^{1/2}) \tag{4.3}$$

$$\boldsymbol{R}_t = (\text{diag}\boldsymbol{Q}_t)^{-1/2} \boldsymbol{Q}_t (\text{diag}\boldsymbol{Q}_t)^{-1/2} \tag{4.4}$$

The elements $h_{i,t}$ of the $\boldsymbol{H}_t$ covariance matrix are modeled using the GARCH(1,1) model[1] and can be written as:

$$h_{i,t} = \alpha_{i,0} + \alpha_{i,1}\varepsilon^2_{i,t-1} + \beta_i h_{i,t-1}, \text{ for } i = 1,\ldots,n. \tag{4.5}$$

Moreover, to ensure that $\boldsymbol{R}_t$ will be invertible and positive definite, $\boldsymbol{Q}_t$ is modeled as:

$$\boldsymbol{Q}_t = (1-a-b)\overline{\boldsymbol{Q}} + a\boldsymbol{z}_{t-1}\boldsymbol{z}'_{t-1} + b\boldsymbol{Q}_{t-1} \tag{4.6}$$

where the parameters a and b are non-negative such that a+b<1 and $\overline{\boldsymbol{Q}}$ is the $4x4$ unconditional correlation matrix of the standardized residuals $z_{i,t} = \frac{\varepsilon_{i,t}}{\sqrt{h_{i,t}}}$. The correlation estimator is then calculated as:

$$\rho_{i,j,t} = \frac{q_{i,j,t}}{\sqrt{q_{i,i,t}q_{j,j,t}}} \tag{4.7}$$

Figure 4.2, which depicts the evolution of the dynamic correlations over time, gives the sense that all realized volatility measures are positively correlated across almost the entire sample.

[1] It is noted that various specifications of the DCC model have been estimated, such as DCC-GARCH, DCC-GJR and DCC-EGARCH; DCC-GARCH(1,1) has been selected to produce the variance-covariance matrix, since it meets all the statistical assumptions required.

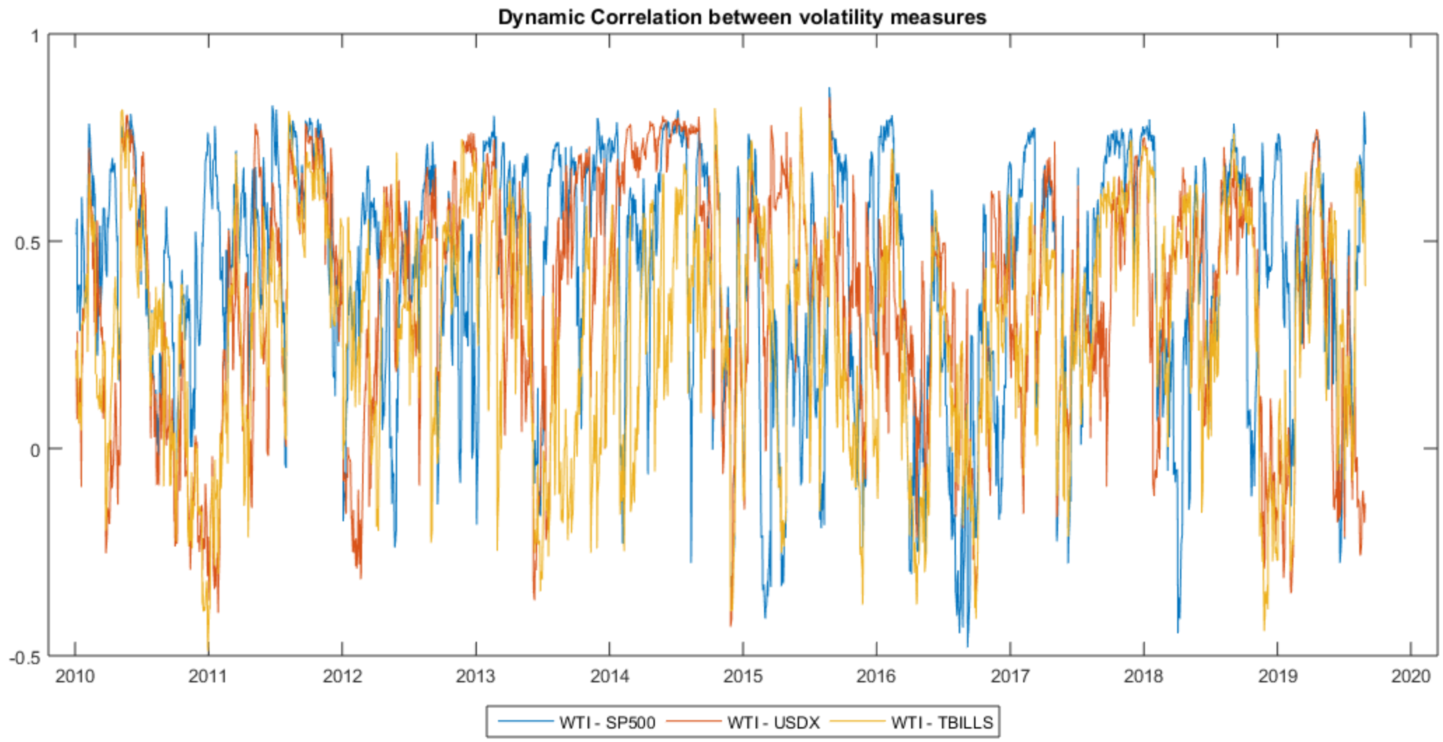

Figure 4.2 Dynamic correlations for WTI and the three asset classes used in this study.

4.4.2 Optimal portfolio weights

After having estimated the variance-covariance matrix of the realized volatility of crude oil and the other representatives of the main asset classes studied, it can be useful for volatility traders to estimate the optimal portfolio weights, which could help them protect their positions. The methodology implemented in this study has been used by several studies (Antonakakis et al., 2018; Hammoudeh et al., 2010; Syriopoulos et al., 2015) and assumes a \$1 USD portfolio, which consists of two assets. The first asset is the crude oil realized volatility and the second one is the realized volatility of the other asset classes. The calculation of the optimal portfolio weights is given by the following system of equations:

$$w_{AC,t} = \begin{cases} 0 & \text{if} \quad \frac{\sigma^2_{WT,t} - \sigma_{WT,AC,t}}{\sigma^2_{AC,t} + \sigma^2_{WT,t} - 2\sigma_{WT,AC,t}} < 0 \\ \frac{\sigma^2_{WT,t} - \sigma_{WT,AC,t}}{\sigma^2_{AC,t} + \sigma^2_{WT,t} - 2\sigma_{WT,AC,t}} & \text{if} \quad 0 < \frac{\sigma^2_{WT,t} - \sigma_{WT,AC,t}}{\sigma^2_{AC,t} + \sigma^2_{WT,t} - 2\sigma_{WT,AC,t}} < 1 \\ 1 & \text{if} \quad \frac{\sigma^2_{WT,t} - \sigma_{WT,AC,t}}{\sigma^2_{AC,t} + \sigma^2_{WT,t} - 2\sigma_{WT,AC,t}} > 1 \end{cases} \tag{4.8}$$

where $w_{AC,t}$ is the optimal weight in crude oil to be held by a two-asset portfolio. Moreover, in order to investigate and compare the performance between the optimal portfolio weights and the optimal hedging strategy, it is required to estimate the conditional variance of the portfolio constructed by using the optimal portfolio weights. This computation is given by the following equation:

$$\sigma^2_{OP,t} = w^2_{WT,t}\sigma^2_{WT,t} + w^2_{AC,t}\sigma^2_{AC,t} + 2w_{WT,t}w_{AC,t}\sigma_{WT,AC,t} \tag{4.9}$$

where $\sigma^2_{OP,t}$ denotes the conditional variance of the constructed portfolio.

Figures 4.4, 4.5 and 4.6 portray the conditional standard deviation obtained by the DCC-GARCH model for the single-asset portfolios and the two-asset portfolio constructed using the optimal portfolio weights. What is observed from these figures is that for all cases the standard deviation of the constructed portfolio is lower compared to the respective values for the single-asset portfolio.

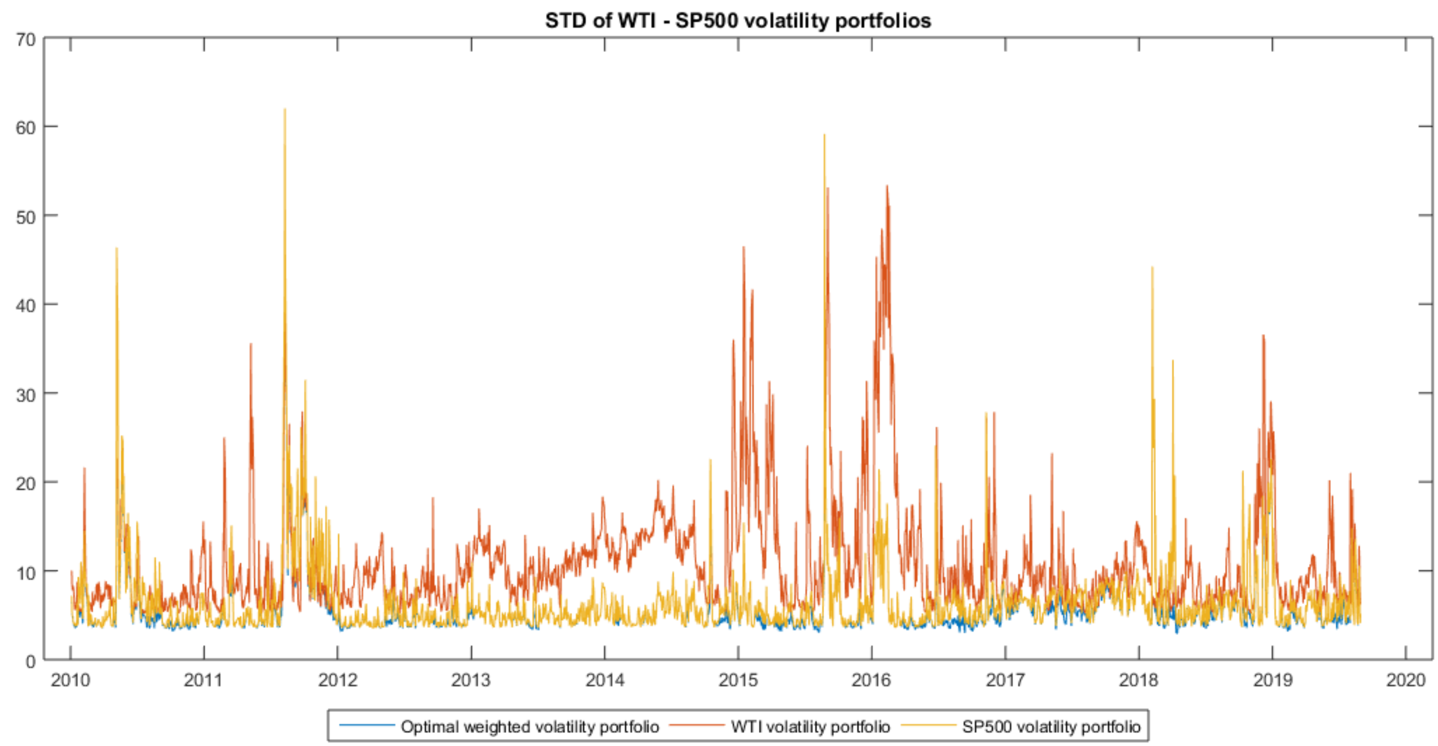

Figure 4.3 The conditional standard deviation of the two single-asset portfolios (WTI - SP500) and the two-asset volatility portfolio constructed by using the optimal portfolio weights.

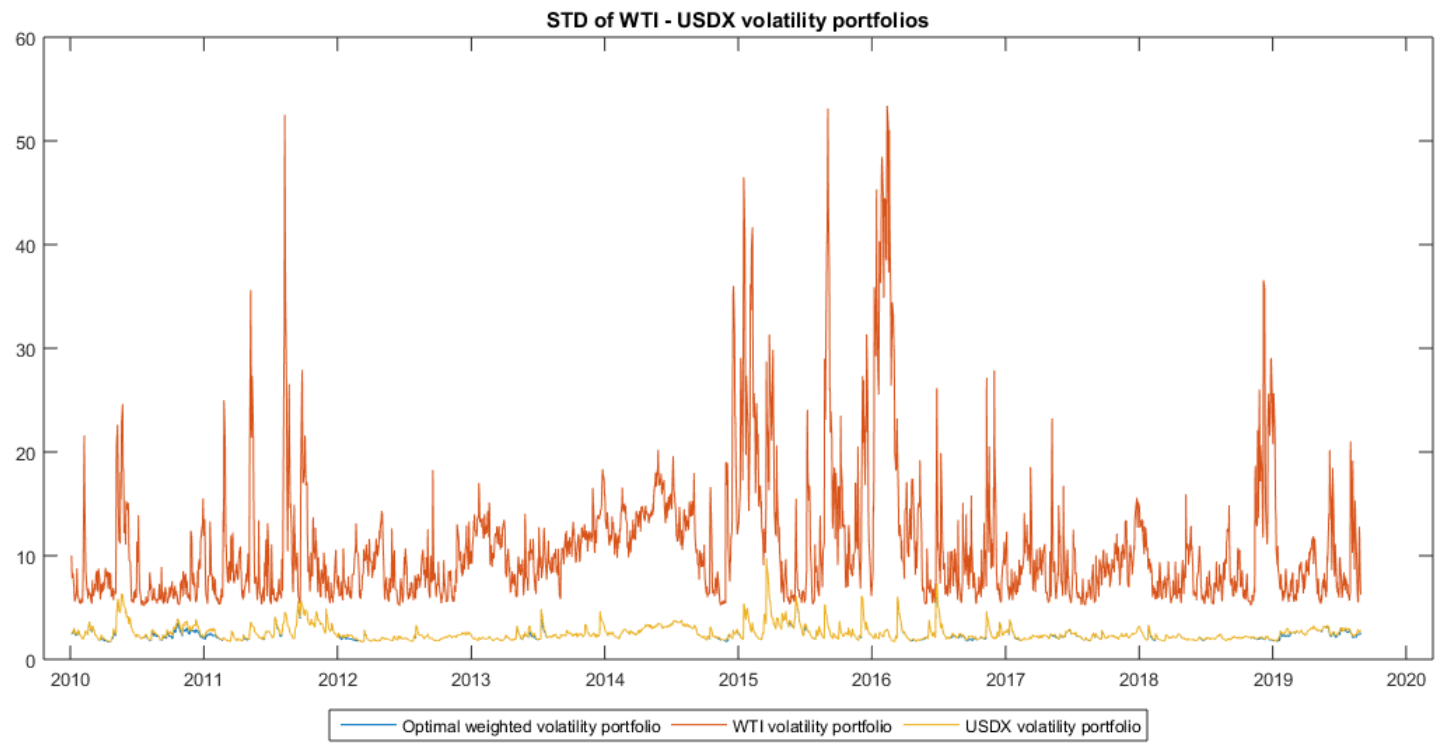

Figure 4.4 The conditional standard deviation of the two single-asset portfolios (WTI - USDX) and the two-asset volatility portfolio constructed by using the optimal portfolio weights.

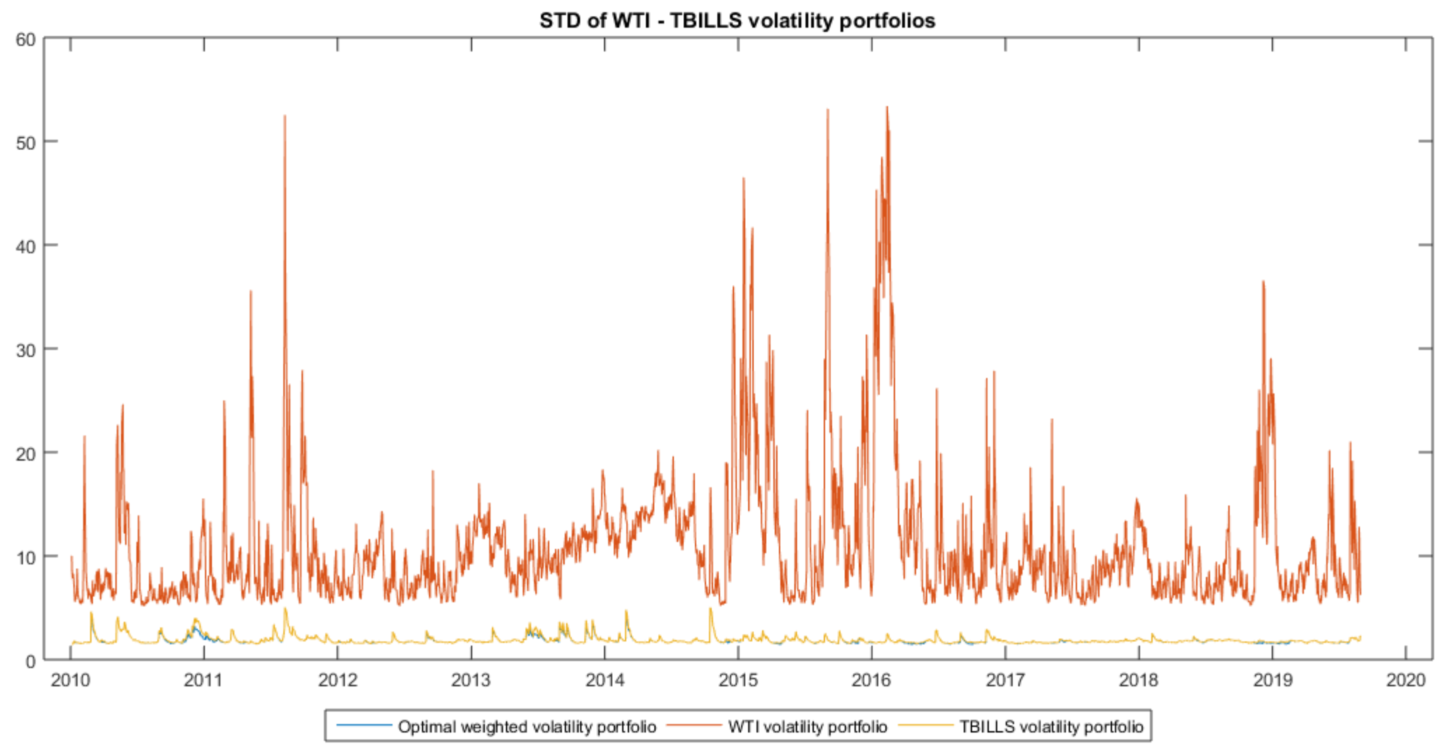

Figure 4.5 The conditional standard deviation of the two single-asset portfolios (WTI - TBILLS) and the two-asset volatility portfolio constructed by using the optimal portfolio weights.

4.4.3 Hedging effectiveness

An alternative risk management approach for an efficient hedging strategy is the hedging effectiveness ratio, which indicates the success of the hedging strategy in minimizing the risk of the hedged portfolio. The higher the ratio is, the higher the risk reduction. The estimate of the hedging effectiveness ratio is given by the following equation:

$$HE = \left[\frac{V_{UH} - V_H}{V_{UH}} \right] \tag{4.10}$$

where V_{UH} is the variance of a portfolio comprised of a single asset volatility, while V_H is the variance of an optimally weighted or optimal hedged portfolio which consists of crude oil volatility and the volatility of another asset class.

Finally, for robustness purposes, we also assess the results of the optimal portfolio weights strategy in a highly volatile period (from 01/12/2014 to 18/03/2016), which covers the oil collapse in 2014-2015, and a tranquil period (from 01/03/2017 to 31/10/2018).

4.5 Empirical findings

Regarding the time-varying correlation of WTI crude oil volatility and the volatility of the other assets, one first observation is that the correlation is almost always higher than 0 without much variation across time. It is also easily visible that the correlation of WTI crude oil volatility and the U.S. T-bills as well as the S&P500 appears to be highly volatile whereas the case of the correlation of WTI crude oil volatility and the volatility of U.S. dollar index seems to be more time-varying over time. Some of the variations appearing to the correlation time series of WTI and foreign exchange markets could be justified by geopolitical events that occurred (e.g. the political upheaval in Libya, Yemen and Bahrain during the period 2011-2012).

One of the main contributions of this study is the investigation of potential hedging opportunities between WTI and other assets, which represent the main asset classes, namely the stock market, the foreign exchange market and the market reflecting macroeconomic conditions. Thus, in order to assess these opportunities, we compute the optimal weights of a two asset portfolio, which consists of WTI crude oil volatility and the volatility of the other three asset classes under investigation. Figure 4.6 shows the evolution of the optimal portfolio weights for the three assets that are examined for their hedging power to WTI crude oil volatility over the entire sample. From the relevant graph, it is obvious that S&P500 takes values from 0% to 100% and its time series is time-varying with many variations over time. Moreover, from Table 4.2, it is evident that the highest proportion is given to U.S. T-bills (average daily value 98%) and the U.S. dollar index (average daily value 96.9%), which can be explained by the fact that WTI crude oil volatility is significantly higher relative to volatility of the U.S. dollar index and U.S. T-bills. It is observed that in the case of S&P500, the minimum value for the optimal weight is 0%, which is not the case for the other two assets, namely the U.S. dollar index and the U.S. T-bills, where the minimum values do not fall under 63%. As mentioned above, the optimal weight time series for S&P500 is highly volatile with higher standard deviation (approximately 0.25) relative to the other assets. From Table 4.2, we draw the conclusion that the portfolio standard deviation is reduced for all portfolios. However, the portfolio risk could be reduced at a higher level when optimal weights are allocated to the WTI - TBILLS portfolio, which seems to perform better in terms of minimized portfolio standard deviation.

Portfolio	Portfolio weights		Portfolio standard deviation		HE (%)
	WTI	Other asset	Optimal portf.	One-asset portf.	
Entire period					
WTI - S&P500	0.212	0.788	5.599	10.775 - 6.382	62.1 - 14.8
WTI - USDX	0.031	0.969	2.495	10.775 - 2.541	91.3 - 3.3
WTI - TBILLS	0.020	0.980	1.863	10.775 - 1.896	95.0 - 2.7
Volatile period					
WTI - S&P500	0.132	0.868	5.936	18.147 - 6.442	80.6 - 12.6
WTI - USDX	0.015	0.985	2.977	18.147 - 2.992	91.8 - 1.1
WTI - TBILLS	0.011	0.989	1.838	18.147 - 1.855	97.1 - 1.9
Tranquil period					
WTI - S&P500	0.302	0.698	5.688	8.407 - 7.048	47.7 - 20.7
WTI - USDX	0.014	0.986	2.168	8.407 - 2.181	92.0 - 1.1
WTI - TBILLS	0.009	0.991	1.773	8.407 - 1.779	94.6 - 0.7

Table 4.2 The overall performance, including the relevant portfolio weights and the portfolio standard deviation, of the optimal portfolio weights strategy in addition to the results for hedging effectiveness of the constructed portfolios.

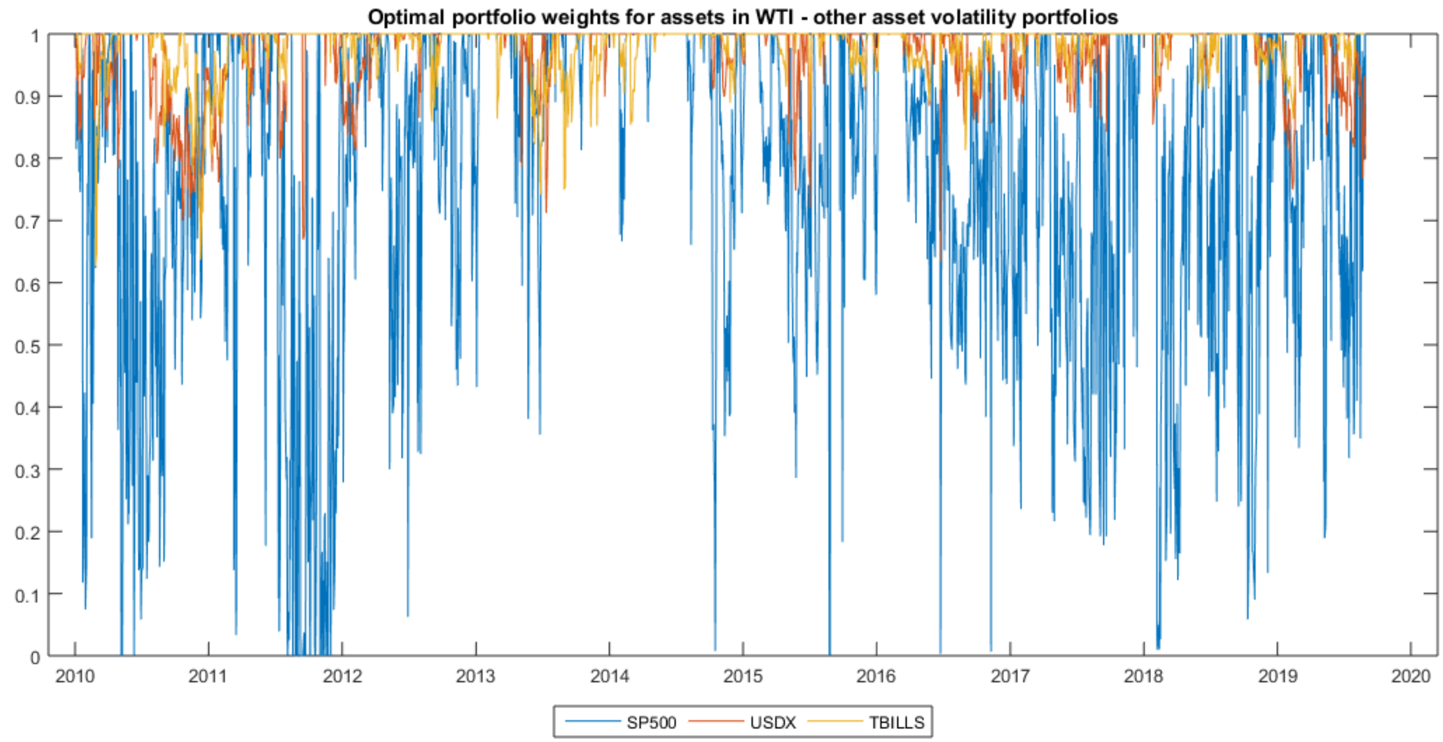

Figure 4.6 Optimal portfolio weights for the three investigated asset classes (stock market, foreign exchange market and market related to macroeconomic conditions).

In regard to the hedging effectiveness that is applied for risk management purposes, and more specifically for understanding the success level of the hedging strategy, it holds true that TBILLS seems to perform better for hedging WTI crude oil volatility. The hedging effectiveness for WTI crude oil volatility reaches 95% at the WTI-TBILLS portfolio, whereas the same measure is 91.3% for USDX, which is evidence for an efficient hedging of WTI crude oil volatility. In contrast, hedging effectiveness for the WTI - S&P500 portfolio is not considered adequate enough (approximately 62%), which gives the sense that S&P500 could not be an efficient hedger for WTI in terms of volatility.

Finally, it is interesting to mention that the reduction of the portfolio variance is particularly beneficial for WTI crude oil volatility investors given the fact that the hedging effectiveness against the unhedged asset portfolio ranges from 62% to 95%. Nevertheless, this statement does not hold true for the other asset classes volatility investors, since the hedging effectiveness is 14.8%, 3.3% and 2.7% for SP500, USDX and TBILLS, respectively. One key finding is that the hedging effectiveness increases during high volatile periods, which provide investors with information on how they can reduce their portfolio's risk and maximize their returns. In contrast, during more tranquil periods, the optimal portfolio weights strategy seems to create less hedging opportunities for WTI crude oil volatility, since hedging effectiveness is reduced.

4.6 Conclusion

This chapter examines whether there are hedging opportunities between WTI crude oil volatility and volatility measures of three other assets classes, namely the stock and foreign exchange markets and the market reflecting macroeconomic conditions, employing a dynamic conditional correlation model which captures time-variations in the correlation of the volatility time series. By employing this methodology, the variance-covariance matrix is used for implementing a hedging strategy based on the computation of the optimal portfolio weights. The empirical findings reveal that using the optimal portfolio weights in the WTI - TBILLS portfolio is beneficial for WTI crude oil volatility investors with hedging effectiveness reaching 95%. The U.S. dollar index is also considered an efficient asset for hedging WTI crude oil volatility, whereas S&P500 could be less appropriate for hedging purposes with hedging effectiveness ranging from 47.7% to 80% for volatile and tranquil periods, respectively. In this regard, from this chapter, we draw the conclusion that the optimal portfolio weights hedging strategy can be beneficiary and useful for WTI crude oil volatility investors and more specifically for the volatile periods. However, another finding of this chapter is that there is no evidence for hedging opportunities for USDX and TBILLS in a two-asset portfolio with allocated weight to WTI crude oil volatility. The hedging effectiveness for these two portfolios (WTI - USDX and WTI - TBILLS) ranges from 2.7% to 3.3%. The finding of this chapter can be considered really useful not only for investors but also for academics and policy makers, since crude oil is an asset which is considered of major importance for the global economy and therefore several stakeholders could be interested in exploring how WTI crude oil volatility could be hedged efficiently. One suggestion for future research is to focus not only on minimizing the portfolio's risk but also on maximizing returns or, from a regulatory perspective, on using these hedging strategies to estimate more efficiently Value-at-Risk. Finally, for robustness purposes, similar methodology could be employed for alternative assets as the representatives of these asset classes.

Chapter Five

Is oil price volatility important for the U.S. economy?

5.1 Introduction

Recent years, it is observed that the attention of researchers, investors and policy makers have been attracted by the evolution of crude oil volatility. Oil prices are subject to shocks driven by both supply and demand sides. Hamilton, 2009 studies the consequences of oil shocks for the economy and found that the oil shocks of 2007-2008 had a significant effect on consumption spending and purchases of automobiles. This can be justified by another study (Kilian & Vigfusson, 2017), which investigates the role of oil price shocks in causing U.S. recessions. In the latter study, the authors suggest that the recessionary effects of oil price shocks are modest and they suggest that unprecedented declines in the oil price should have strong effects on the economic outlook. These kind of episodes have been extensively investigated in several studies (Baumeister & Kilian, 2016; Edelstein & Kilian, 2007, 2009).

More specifically, the relationship between the oil price uncertainty and the macroeconomy has been studied in depth with the work of Ferderer, 1996 to be among the first published papers that concentrate on this relationship. In his study, he provides evidence that the standard deviation of oil prices can offer predictive information on the U.S. output growth. Moreover, he finds that the asymmetric relationship between oil prices and economic growth holds true in the case of oil price volatility. A crucial conclusion of the latter study is that oil price volatility has data-rich information for industrial production forecasts. In this chapter, we rely on the aforementioned statement and the purpose of this study is to investigate in depth the impact of not only the crude oil volatility, but also volatility measures of other markets, on the economic growth and more specifically on several disaggregated measures of industrial production. To the best of our knowledge, there is only one investigation (Elder, 2018), which focuses on the impact of oil price volatility on disaggregated measures of industrial production. The results of the aforementioned study indicate that the impact of oil price volatility is stronger in activities related to primary energy production and oil and gas drilling. However, these results come from an in-sample analysis without implementing a forecasting framework, which is the primary purpose of the present research.

There are several papers concentrating on the effect of oil prices on measures of the economic outlook. For example, Elder and Serletis, 2010 study the impact of the oil price uncertainty on several macroeconomic variables, such as investments, consumption and aggregate output, which is found to be negative and statistically significant. There are also studies focusing on the relationship between oil price volatility and macroeconomic indicators of specific countries. Rafiq et al., 2009 study the aforementioned relationship in the case of Thailand and find that there is causality running from oil price volatility to investment, unemployment rate, interest rate and trade balance. This impact seems to be short-lived and is mitigated after the financial crisis. Lardic and Mignon, 2006 study the relationship between oil prices and GDP in several European countries and provide evidence for an asymmetric co-integration. This asymmetric relationship is also investigated by Serletis and Mehmandosti, 2017, who find that the responses of economic growth to positive and negative oil price shocks does not offer information on whether there are asymmetries or not. Another work of van Eyden et al., 2019 studies the impact of oil price volatility on GDP growth for 17 member countries of the OECD.

The results of the latter study are similar to the other studies with the main finding being that oil price volatility has a negative and statistically significant impact on economic growth.

In this chapter, we primarily aim to find whether oil price volatility has predictive power on U.S. industrial production and more specifically on several disaggregated measures of industrial production in an out-of-sample analysis. As far as we are aware, there is no published work doing that. However, there are studies that investigate the relationship between oil price volatility and industrial production. For example, Jo, 2014 suggests that a shock of oil price uncertainty has negative impact on world industrial production. More specifically, high oil price uncertainty can significantly respond to decline in industrial production growth. Serletis and Istiak, 2013 is another study that aims to draw a conclusion on whether the relationship between oil prices and industrial production is symmetric for the G-7 countries and they provide different results for the countries under investigation. For the U.S., Romero-Meza et al., 2014 study the aforementioned relationship and conclude that there is a nonlinear and asymmetric relationship between oil prices and industrial production, and more interestingly there is evidence that oil prices lead U.S. industrial production. A more recent study of Alao and Payaslioglu, 2021 investigates the comovement of oil price volatility and industrial production in oil-exporting countries, such as Mexico and Brazil, and provide evidence for a dynamic linkage, which is temporary. Furthermore, Nonejad, 2021 in an out-of-sample investigation finds that oil prices do not provide forecast accuracy gains when developing models to forecast industrial production, which is explained by model misspecifications and not the fact that oil prices lack predictive information.

The index of industrial production, which is the most important macroeconomic indicator sampled at high frequency, is considered of vital importance for forecasting the short-term evolution of GDP globally. The frequency of the data used in studies that develop modeling frameworks for industrial production is mainly monthly, since this is the highest frequency at which industrial production is available. Moreover, all the potential drivers of the target variable, namely industrial production, are at the same frequency. Byers and Peel, 1995 implement non-linear autoregressive models to forecast industrial production for six countries and confirm that the time series appear to exhibit non-linearity. In terms of forecasting accuracy, Bodo et al., 2000 apply a wide range of models including ARIMA, VAR and conditional models, and find that a conditional error-correction model outperforms the competing models when generating industrial production forecasts. All these studies use monthly data for both the dependent and the explanatory variables. Moreover, Kawasaki and Franses, 2004 assume seasonal unit roots in the models used and find that this incorporation can contribute to more accurate industrial production forecasts. Another approach used for forecasting industrial production is that of Hassani et al., 2009, 2013, which is called singular spectrum analysis and seems to outperform the widely used ARIMA models at longer horizons.

However, in the present study, an alternative approach is implemented in order to generate out-of-sample industrial production forecasts. Over the last decades, research has focused on the role of the stock markets in measures of economic growth (King & Levine, 1993; Levine et al., 2000). In order to incorporate information coming from a higher frequency, a mixed frequency time series (MIDAS) methodological framework is implemented, initially proposed by Ghysels et al., 2006, which gives us the ability to use variables at daily frequency as drivers of the monthly industrial production. More specifically, a MIDAS model is developed in order to investigate the impact of oil price volatility measures sampled at daily frequency on the U.S. industrial production sampled at monthly frequency. Pan et al., 2018 use oil prices to forecast the U.S. real GDP by introducing a time-varying parameter MIDAS model and find that the proposed MIDAS model can outperform the competing models including a simple OLS regression. To the best of our knowledge, there is no other study investigating the impact of daily oil price volatility measures (constructed using intra-day data) on the monthly industrial production in the U.S. Therefore, this is considered to be the main contribution of our study to the literature. Apart from the methodological framework, the impact not only of oil price volatility but also of S&P500 realized volatility and other assets' realized volatility on industrial production growth is investigated in order to compare their predictive information.

As a key finding, we consider the fact that the crude oil realized positive semivariance has predictive power for industrial production and more specifically the energy related dissagregated measures of industrial production. Moreover, this study provides evidence for strong performance of the MIDAS modeling framework by incorporating daily volatility measures compared to a model that is limited to the inclusion of macroeconomic variables at the frequency of the target variable. Finally, it is observed that the exogenous variables that are added in the proposed model do not offer any additional predictive information.

The remainder of the chapter is structured as follows. In Section 5.2 we provide a brief description of the variables used in this chapter. In Section 5.3 the entire modeling framework is presented and also the way that forecasts are generated. The evaluation framework is presented in Section 5.4, while in Section 5.5 we discuss the out-of-sample results. Finally, Section 5.6 concludes the study.

5.2 Description of the variables

In this section, the variables that are used in this study are described in detail. More specifically, we start with the target variable, which is the U.S. industrial production and its dissagregated measures. We then continue with the higher frequency variables, which are the different volatility measures of crude oil market. For comparability reasons, the predictability of other markets' volatility measures, such as stock and foreign exchange markets, is tested. Finally, several exogenous variables at monthly frequency which are included in the implemented models as potential drivers of the U.S. industrial production, are described below in detail.

5.2.1 Industrial production and its disaggregated measures

In this chapter, we study the effect of oil price volatility measures on several components of industrial production. These components are special aggregate indices of U.S. industrial production and are compiled by the Federal Reserve Board[1]. These aggregates are grouped into energy-related and non energy-related industrial production indices. The largest component of the energy-related products is primary energy, which includes the extraction of crude oil, natural gas, coal, and nuclear and hydroelectric power generation. Another component is the drilling of oil and gas wells. Regarding the special aggregates for non energy-related production, the largest component is that of consumer goods excluding both technology and motor vehicles. Other important components of this group are motor vehicles and parts, and business equipment excluding technology and motor vehicles. These measures of industrial production are sampled at monthly frequency and the transformation applied in the logarithmic time series is the first difference.

Figure 5.1 displays the evolution of logarithmic returns of each industrial production index used in this chapter. It is observed that the special aggregate of drilling oil and gas wells is highly volatile compared to the remaining aggregates of industrial production. From Table 5.1, we observe that only the mean value of business equipment excluding motor vehicles is negative with an interestingly high level of coefficient of variation. All the remaining non energy-related indices seem to be at the same level in terms of variation.

[1]For further details see: https://www.federalreserve.gov/releases/g17/current/default.htm.

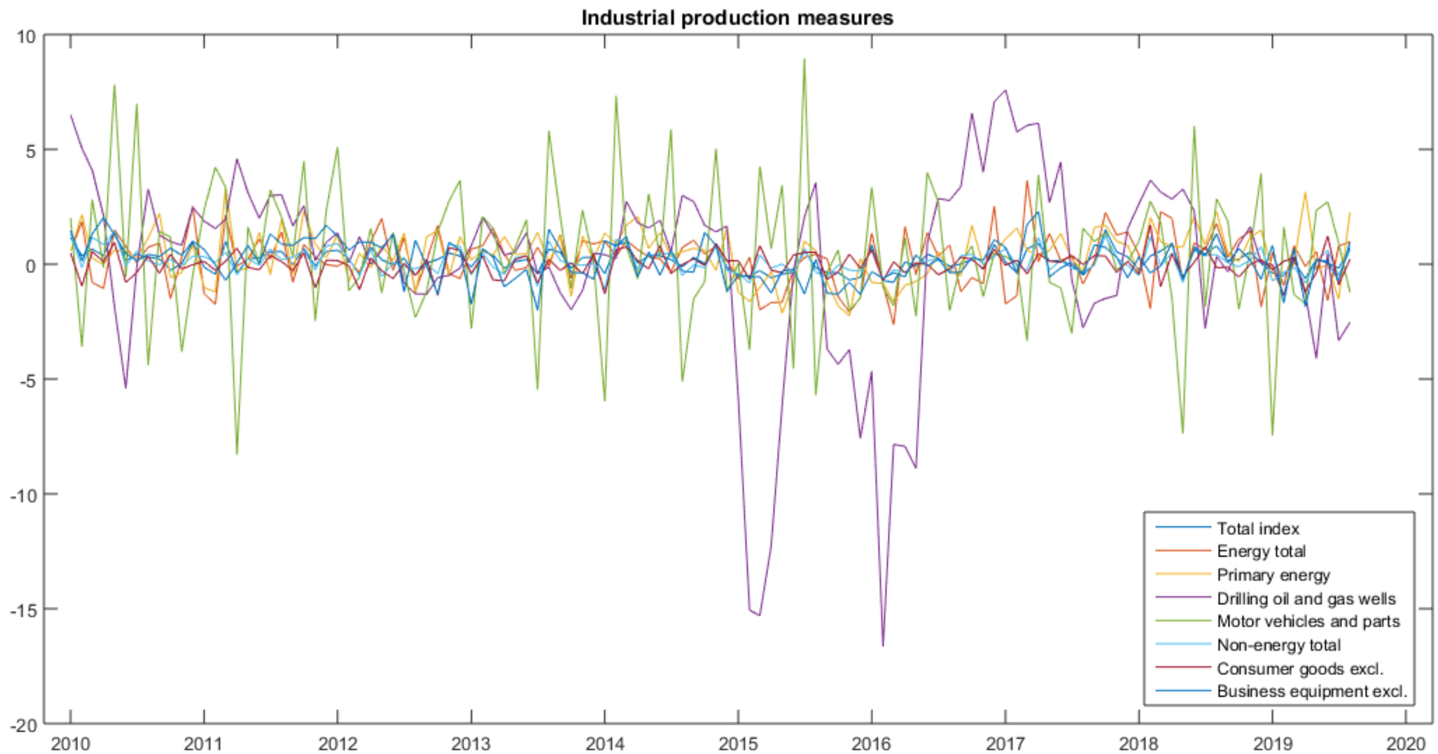

Figure 5.1 Time evolution of the special aggregates of industrial production.

Industrial production measures	**Total index**	**Energy total**	**Primary energy**	**Drilling oil and gas wells**
Mean	0.1660	0.2565	0.4189	0.1055
Median	0.1942	0.3183	0.4685	0.8402
Minimum	-0.7963	-2.6415	-2.2512	-16.6480
Maximum	1.5057	3.6460	3.3578	7.5801
Std. Dev.	0.4812	1.1547	1.1033	4.2090
Skewness	0.2010	-0.1045	-0.0486	-1.6645
Kurtosis	2.7299	2.9319	2.8585	7.0232
Coefficient of Variation	2.8986	4.5022	2.6341	39.8919
Jarque-Bera (p-value)	0.5000	0.5000	0.5000	0.0010
	Non-energy total	**Consumer goods**	**Business equipment**	**Motor vehicles and parts**
Mean	0.4613	0.1270	-0.0044	0.1654
Median	0.5184	0.0509	0.0726	0.1984
Minimum	-8.3066	-1.0989	-1.3589	-2.0013
Maximum	8.9628	1.4290	1.7121	2.2838
Std. Dev.	3.1551	0.5282	0.5362	0.8777
Skewness	-0.1309	0.2581	-0.0833	-0.2046
Kurtosis	3.5022	2.6820	3.3614	2.6218
Coefficient of Variation	6.8391	4.1599	-121.9147	5.3066
Jarque-Bera (p-value)	0.3825	0.3333	0.5000	0.3944

Table 5.1 Descriptive statistics for the dependent variables, namely the special aggregates of industrial production in U.S.

5.2.2 Realized volatility measures at daily frequency

In this study, the impact of crude oil realized volatility measures, constructed by intra-day data, on industrial production indices is investigated. We are based on the calculation of the realized volatility as noted in Chapter 2 and we use the annualized realized volatility, which is defined as RV_t. Moreover, in this study we use the daily positive and negative realized semi variance estimators, which are defined as $RSV_t^{(d+)}$ and $RSV_t^{(d-)}$, respectively.

All the aforementioned realized volatility measures are computed not only for WTI crude oil (WTI), but also for other assets. More specifically, these assets are the S&P500 (SP500) index, the U.S. dollar (DX) index and the U.S. T-bills (TBILL) and the respective realized volatility measures are computed as in Chapter 4. Additional details on their descriptive statistics and their evolution over time are presented in Table 4.1 and Figure 4.1, respectively.

5.2.3 Predictors at monthly frequency

We define a group of variables as potential drivers of industrial production and its special aggregates. More specifically, these variables are included on the right hand side of the MIDAS regression as explanatory variables sampled at monthly frequency, which is the same as that of each of the dependent variables, namely the several U.S. industrial production indices. This group consists of two subcategories.

The first one includes core variables reflecting macroeconomy and the second one contains some indicators that could effectively capture uncertainty. Regarding the first category of variables, some macroeconomic variables that are used by previous studies to explain GDP are included[2]. For example, one of the variables used to forecast macroeconomic variables is term spread, defined as the 10-year treasury bond minus the 3-month treasury bill rate. Another important variable used in this chapter as a potential predictor for industrial production indices is the default yield spread, defined as the BAA/AAA corporate bond yield spread. A core macroeconomic variable that can potentially hold predictive information on industrial production is the unemployment rate. The three above mentioned variables are extracted from the Federal Reserve Economic Data (FRED) online database[3]. Apart from these variables, some crude oil-related variables are incorporated in the methodology that are used for generating GDP forecasts (Nonejad, 2020a), since fundamentals of crude oil are potential drivers of industrial production and its aggregates. These crude oil-related variables are the U.S. crude oil production and the U.S. imports of crude oil, which are extracted from the Energy Information Administration (EIA)[4].

Moreover, indicators of uncertainty are investigated, which have been widely used in re-

[2] For further details see (Nonejad, 2020a, 2020b).

[3] Please find more details on the relevant link: https://fred.stlouisfed.org/.

[4] See https://www.eia.gov/petroleum/data.php for further details.

cent years. These are the U.S. economic policy uncertainty (USEPU) index, the geopolitical risk (GPR) index and the Partisan Conflict (PC) index. More specifically, the USEPU is based on newspaper archives and is constructed by using the following components: News Coverage about Policy-related Economic Uncertainty, Tax Code Expiration Data and Economic Forecaster Disagreement. With regard to the first component, it is constructed by performing monthly searches for terms, such as ’uncertainty’, ’economy’, ’congress’ and ’legislation’. The second component relies on reports by the Congressional Budget Office (CBO) that compile lists of temporary federal tax code provisions. The third and last one relies on data of the Federal Reserve Bank of Philadelphia’s Survey of Professional Forecasters. The second variable included in the group of explanatory variables, namely the geopolitical risk (GPR) index, captures adverse geopolitical events by using newspaper articles covering geopolitical tensions[5]. Finally, the PC index is added to the group of potential drivers. This indicator tracks the degree of political disagreement among U.S. politicians by measuring the frequency of newspaper articles reporting disagreement in a given month[6].

The frequency of all the variables presented in this section is monthly and the data is extracted from the corresponding website as referred to in their description. The transformations applied to individual variables are described below:

- *T10Y3M*: first difference.
- *BAA AAA*: first difference.
- *UNRATE*: first difference of logarithmic transformation.

[5] More details on the construction of the index can be found at the following link: https://www.matteoiacoviello.com/gpr.htm.

[6] For further details see https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/partisan-conflict-index.

- *OILPROD*: first difference of logarithmic transformation.

- *OILIMPORTS*: first difference.

Figure 5.2 displays the explanatory variables sampled at monthly frequency, which are considered drivers of the U.S. industrial production measures.

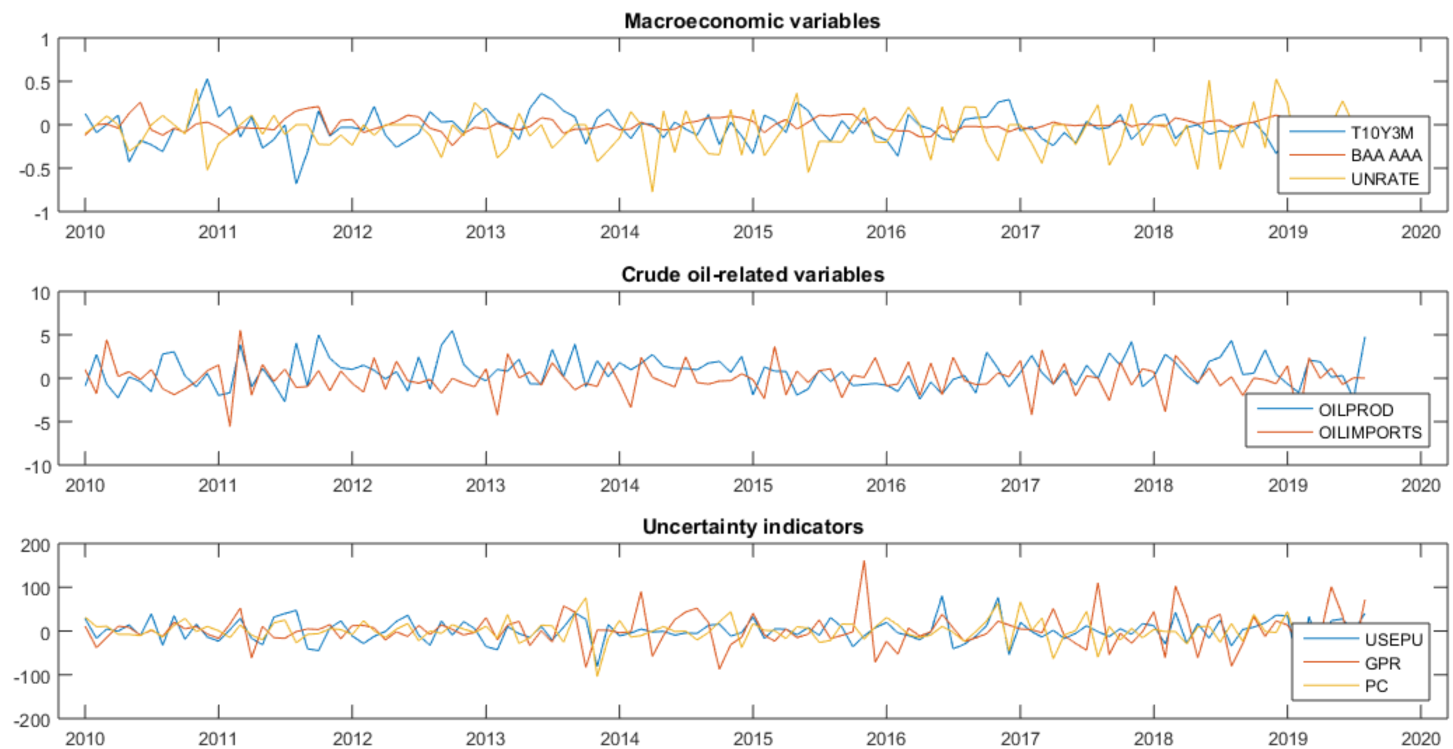

Figure 5.2 The time evolution of the group with the monthly explanatory variables consisting of three categories (three subplots), which are macroeconomic variables, oil-related variables and indicators of uncertainty.

Macroeconomic variables	T10Y3M	BAA AAA	UNRATE
Mean	-0.0336	-0.0019	-0.8484
Median	-0.0350	-0.0150	0.0000
Minimum	-0.6800	-0.2400	-7.7558
Maximum	0.5300	0.2600	5.2644
Std. Dev.	0.1727	0.0777	2.3584
Skewness	-0.1236	0.4255	-0.0819
Kurtosis	4.4460	3.9343	3.1122
Coefficient of Variation	-5.1356	-40.9719	-2.7797
Jarque-Bera (p-value)	0.0147	0.0267	0.5000

Crude oil-related variables	OILPROD	OILIMPORTS
Mean	0.7136	-327.4741
Median	0.6563	-1475.5000
Minimum	-2.7021	-55538.0000
Maximum	5.4857	55537.0000
Std. Dev.	1.7797	17838.7430
Skewness	0.4108	-0.1216
Kurtosis	2.6980	4.1232
Coefficient of Variation	2.4941	-54.4737
Jarque-Bera (p-value)	0.1029	0.0381

Uncertainty indicators	USEPU	GPR	PC
Mean	0.5969	1.4784	0.3374
Median	-0.5592	-1.5464	0.0084
Minimum	-94.1313	-87.2815	-104.0498
Maximum	80.5879	161.3418	76.3591
Std. Dev.	27.2313	38.6360	24.7834
Skewness	-0.2248	0.8127	-0.2640
Kurtosis	4.3211	5.6212	5.9216
Coefficient of Variation	45.6200	26.1342	73.4532
Jarque-Bera (p-value)	0.0181	0.0010	0.0010

Table 5.2 Descriptive statistics for the monthly explanatory variables used in this study, namely the macroeconomic variables, the oil-related variables, including the oil production and imports of crude oil, and the indicators of uncertainty.

5.3 Methodology

5.3.1 Simple model

For comparison reasons between the proposed MIDAS model and more naive models, a simple regression is used to generate 1-day ahead forecasts of industrial production. This model is written as:

$$y_t = \beta_0 + \sum_{i=1}^{p} \beta_i L^i y_t + \epsilon_t, \tag{5.1}$$

where y_t is the industrial production index at day t, L is the lag operator used for simplicity[7] and ε_t is a white noise. However, in this chapter a group of explanatory variables is added in the model's equation to explore whether the predictive ability of the models is enhanced. Therefore, this enhanced model including macroeconomic, oil-related and uncertainty-related variables is the following:

$$y_t = \beta_0 + \sum_{i=1}^{p} \beta_i L^i y_t + \sum_{j=1}^{N} \omega_j EV_t^j + \epsilon_t, \tag{5.2}$$

where N is the number of variables used as potential drivers of the industrial production measures. Each explanatory variable is expressed as EV_t^j at time t.

5.3.2 MIDAS modeling

Ghysels et al., 2004 built a new modeling framework that combines data from different frequencies, namely the so-called mixed data sampling (MIDAS) approach. This approach is used when the forecasters' objective is to estimate a number of hyperparameters relative to a

[7] The number of lags is denoted by p and in this study is empirically chosen to be 3 after several runs.

sampling rate of the higher frequency variable. MIDAS models have been widely used in order to forecast quarterly macroeconomic time series by using as predictors monthly or even daily data. In their study, Andreou et al., 2013 introduce regression-based methods in order to forecast quarterly economic activity using daily financial data. Their methodology relies on combinations of MIDAS regressions. Degiannakis, 2021 implements a MIDAS model to produce nowcasting values for real investment sampled at quarterly frequency by adding information from the stock market, which is on a daily sampling frequency. In the current study, we add information from daily realized volatility measures to the MIDAS model for predicting monthly aggregated measures of industrial production. The ADL-MIDAS model can therefore be written as:

$$y_t = \beta_0 + \sum_{i=1}^{p} \beta_i L^i y_t + \gamma \sum_{k=1}^{M} B(k;\boldsymbol{\theta}) L_{HF}^k x_t + \epsilon_t, \tag{5.3}$$

where the function $B(k;\boldsymbol{\theta})$ denotes the polynomial that determines the weights for the higher frequency variable. It is important to mention that we use the same notation across the whole modeling framework. Let y_t denote the lower frequency variable, which is the aggregated measure of industrial production. Since lags of the dependent variable are included in the right hand side of the equation, the lag operator L is used for simplicity reasons[8]. In practice, the first lag of the dependent variable y_t would be $Ly_t = y_{t-1}$, the second lag would be $L^2 y_t = y_{t-2}$ and so on. Moreover, apart from the lags of y_t, we are interested in the predictive information of a higher frequency variable, x_t, which is the realized volatility measure, sampled M times between samples of the dependent variable y_t. In this regard, lags of the higher-frequency variable x_t are included in the applied models by using L_{HF}, which denotes the lag operator

[8] After several runs, the number of lags included on the MIDAS model was chosen to be three, meaning that we incorporate information from the corresponding industrial production index 3 months before time t.

for the higher-frequency variable[9]. For example, if y_t is the dependent variable sampled at monthly frequency and x_t at daily frequency, $L_{HF}x_t$ denotes the day before the realization of the dependent variable at time t (i.e. the last day of the previous month).

Apart from the impact of the daily realized volatility measures for different assets, explanatory variables sampled at the same frequency as that of the dependent variable are also included. This is an enhanced version of the MIDAS model, as was done above in the case of the simple model. Therefore, the ADL-MIDAS-EX model is written as follows:

$$y_t = \beta_0 + \sum_{i=1}^{p} \beta_i L^i y_t + \gamma \sum_{k=1}^{M} B(k;\boldsymbol{\theta}) L_{HF}^k x_t + \sum_{j=1}^{N} \omega_j EV_t^j + \epsilon_t. \tag{5.4}$$

Regarding the weighting function, $B(k;\boldsymbol{\theta})$ can take different functional forms. According to Ghysels et al., 2004; Ghysels et al., 2006, a beta formulation is proposed, which is the following:

$$B(k;\theta_1,\theta_2) = \frac{f(\frac{k}{m},\theta_1,\theta_2)}{\sum_{j=1}^{m} f(\frac{j}{m},\theta_1,\theta_2)} \tag{5.5}$$

where

$$f(l,\theta_1,\theta_2) = \frac{l^{\theta_1-1}(1-l)^{\theta_2-1}\Gamma(\theta_1+\theta_2)}{\Gamma(\theta_1)\Gamma(\theta_2)}, \tag{5.6}$$

θ_1 and θ_2 are hyperparameters ruling the shape of the weighting function, and

$$\Gamma(\theta_p) = \int_0^{\infty} e^{-x} x^{\theta_p-1} dx \tag{5.7}$$

[9] We use 20 lags of the daily realized volatility measure in the MIDAS mode, since a trading month consists of 20 trading days. The entire modeling framework is run with 40 and 60 lags included in the MIDAS model but they do not offer further predictive information.

is the standard gamma function. Apart from this functional form, Ghysels et al., 2005; Ghysels et al., 2007 also proposed an exponential Almon specification, which is written as:

$$B(k;\theta_1,\theta_2)=\frac{e^{\theta_1 k+\theta_2 k^2}}{\sum_{j=1}^{m} e^{\theta_1 j+\theta_2 j^2}} \tag{5.8}$$

Under this modeling framework, we generate industrial production forecasts for all special aggregates and by using each of the realized volatility measures defined in Section 5.2. Therefore, let $\mathrm{IP}_t^{(i,RV_{asset})}$ denote the i measure of industrial production (i.e. primary energy). RV_{asset} declares the realized volatility measures of the asset added as the high frequency variable in the MIDAS model.

Regarding the prediction settings assumed in this chapter, we start with the sample used for constructing industrial production predictions. The initial sample period is T_0 = 53 months and the remaining 60 months are used for evaluating the industrial production forecasts. Moreover, a rolling window estimation is applied with fixed length of 53 months. More specifically, the parameters of the model are estimated by choosing data from the 1^{st} to the 53^{rd} and the industrial production forecasts are generated afterwards. Then, we re-estimate the parameters using data from the 2^{nd} to the 54^{th} and so on. The remaining period is the out-of-sample period and is denoted as T_1.

5.4 Evaluation framework

The evaluation framework employs two well-known loss functions, the Mean Squared Predicted Error (MSPE) and the Mean Absolute Error (MAE), which are defined as:

$$MSPE=\frac{1}{T_1}\sum_{t=1}^{T_1}(IP_{t+1|t}^{(i,RV_{asset})}-IP_{t+1}^{(i,RV_{asset})})^2, \tag{5.9}$$

and

$$MAE = \frac{1}{T_1} \sum_{t=1}^{T_1} \mid IP_{t+1|t}^{(i,RV_{asset})} - IP_{t+1}^{(i,RV_{asset})} \mid, \tag{5.10}$$

where $\text{IP}_{t+1|t}^{(i,RV_{asset})}$ is the 1-day-ahead forecast of the measure i of the industrial production generated by using the RV_{asset}, which is the realized volatility of the corresponding asset, $\text{IP}_{t+1}^{(i,RV_{asset})}$ is the IP at time $t+1$ and T_1 is the number of the out-of-sample data points.

5.5 Out-of-sample results

The results of the two statistical loss functions that are used, namely MSPE and MAE, can be found in Tables 5.3, 5.4, 5.5 and 5.6. From these results, we aim to investigate whether the inclusion of higher frequency variables can enhance the predictability of the models used for generating industrial production indices. Moreover, we study the predictive information of crude oil volatility and check whether another asset's realized volatility improves the forecasting performance of the models. Finally, it is important to find which realized volatility measure (e.g. realized positive semivariance) provides higher forecasting accuracy and which industrial production measure benefits from this.

Regarding the MSPE results, Table 5.3 shows the relevant values for the models that do not include explanatory variables at monthly frequency and they maintain only the realized volatility measures. On the other hand, Table 5.4 presents the MSPE results for models that do maintain information from explanatory variables sampled at monthly frequency.

Models - only RV	Total index	Energy total	Primary energy	Drill. oil & gas wells
AR(3)	0.3119	2.1865	1.6069	14.7489
RV WTI	0.2648	1.7009	1.2471	15.9834
RSVN WTI	0.2739	1.7699	1.4076	17.1185
RSVP WTI	0.2629	1.6729	1.2105	14.5529
RV SP500	0.3307	2.3214	1.6738	16.9372
RSVN SP500	0.3294	2.3581	1.6962	17.0998
RSVP SP500	0.3234	2.3124	1.6327	16.3909
RV DX	0.3272	2.3057	1.5881	16.0835
RSVN DX	0.3145	2.2846	1.6087	15.4928
RSVP DX	0.3366	2.2178	1.6028	16.2229
RV TBILL	0.3100	2.3191	1.5507	15.1160
RSVN TBILL	0.3031	2.3321	1.5558	15.0959
RSVP TBILL	0.3090	2.2940	1.4812	15.6413

Models - only RV	Non-energy total	Consumer goods	Business equipment	Motor veh. and parts
AR(3)	0.2674	0.2459	0.6906	7.7771
RV WTI	0.2876	0.2710	0.6751	8.6577
RSVN WTI	0.2849	0.2641	0.6671	8.8019
RSVP WTI	0.2980	0.2689	0.6827	8.0833
RV SP500	0.2920	0.2524	0.7301	9.0219
RSVN SP500	0.2702	0.2540	0.7521	8.9205
RSVP SP500	0.2971	0.2460	0.7033	9.0510
RV DX	0.2990	0.2802	0.7984	8.1429
RSVN DX	0.2896	0.2664	0.7749	7.9813
RSVP DX	0.3130	0.2694	0.8122	8.0959
RV TBILL	0.2762	0.2627	0.7206	7.9223
RSVN TBILL	0.2664	0.2701	0.7184	7.9252
RSVP TBILL	0.2748	0.2504	0.7208	7.7555

Table 5.3 The results of the MSPE loss function for all industrial production measures. In the first column, excluding the first model, which is a simple AR(3), the high frequency variable added in the MIDAS models is presented at each row. It is noted that no monthly explanatory variable is included as predictor in the models.

Models - all exog. var.	Total index	Energy total	Primary energy	Drill. oil and gas wells
ARX(3)	0.2988	2.5178	1.9186	18.2713
RV WTI	0.2757	1.8502	1.5788	18.3212
RSVN WTI	0.2748	1.9650	1.7745	19.0497
RSVP WTI	0.2809	1.9569	1.4537	16.8436
RV SP500	0.3208	2.8373	1.9456	19.9549
RSVN SP500	0.3064	2.9508	1.8995	21.0037
RSVP SP500	0.2888	2.6849	1.9466	19.4989
RV DX	0.3290	2.7798	1.9416	19.1481
RSVN DX	0.3279	2.7059	1.9027	18.7852
RSVP DX	0.3295	2.8004	2.0227	20.8795
RV TBILL	0.3099	2.6963	1.8972	19.1765
RSVN TBILL	0.2898	2.6345	1.8805	18.5070
RSVP TBILL	0.2847	2.6568	1.9664	18.8432

Models - all exog. var.	Non-energy total	Consumer goods	Business equipment	Motor veh. and parts
ARX(3)	0.2899	0.3299	0.7869	8.4717
RV WTI	0.3336	0.3681	0.8648	9.2466
RSVN WTI	0.3312	0.3712	0.9155	9.6983
RSVP WTI	0.3234	0.3428	0.9252	8.8911
RV SP500	0.3193	0.3197	0.8926	9.5129
RSVN SP500	0.3192	0.3347	0.8930	9.3904
RSVP SP500	0.3238	0.3160	0.8551	9.2596
RV DX	0.3029	0.3604	0.8447	9.3959
RSVN DX	0.3107	0.3405	0.8167	9.2895
RSVP DX	0.3114	0.3444	0.8884	9.2198
RV TBILL	0.3088	0.3398	0.8727	9.0689
RSVN TBILL	0.2932	0.3497	0.8659	9.1415
RSVP TBILL	0.3150	0.3565	0.7918	9.2094

Table 5.4 The results of the MSPE loss function for all industrial production measures. In the first column, excluding the first model, which is a simple AR(3), the high frequency variable added in the MIDAS models is presented at each row. It is noted that all monthly explanatory variables are also included as predictors in the models.

First of all, it is observed that MIDAS models that include crude oil realized positive semivariance outperform all the remaining models when referring to the energy-related industrial production measures. This means that volatility coming from positive crude oil returns has higher predictive information on industrial production. More specifically, the forecasting error of the models including crude oil realized positive semivariance can be reduced, compared to the simple AR(3) model, by more than 15% in total index and by almost 25% in case of primary energy. This holds true for the models that include other explanatory variables, such as uncertainty indicators. In that case, the forecasting error can be reduced by almost 20% compared to the ARX(3) model.

It is of major importance to mention that in order to generate forecasts of oil-related industrial production measures only WTI crude oil improves the forecasting performance compared to the other assets, namely the S&P500 index, the U.S. dollar index and the U.S. T-bills. From Tables 5.5 and 5.6, it is observed that the MAE results are qualitatively similar with the exception of the drilling oil and gas wells aggregated measure of industrial production, in which AR(3) beats all competing models, even those including explanatory variables, such as macroeconomic variables and oil fundamentals. In case of primary energy, the MIDAS model incorporating crude oil realized positive semivariance offers an almost 13% of reduction in forecasting error. Similar results are presented in the MIDAS models that incorporate not only realized volatility measures but also other potential determinants of industrial production. For example, the MIDAS model with crude oil realized positive semivariance and the entire set of explanatory variables sampled at monthly frequency outperforms the remaining models and reduces the forecasting error by almost 16%, in terms of MAE, compared to the ARX(3) model.

Models - only RV	Total index	Energy total	Primary energy	Drill. oil and gas wells
AR(3)	0.4526	1.1819	0.9740	2.5747
RV WTI	0.4182	1.0519	0.8273	2.8729
RSVN WTI	0.4226	1.0623	0.9086	3.1149
RSVP WTI	0.4177	1.0263	0.8461	2.6548
RV SP500	0.4611	1.2281	1.0364	2.8971
RSVN SP500	0.4634	1.2240	1.0748	2.9432
RSVP SP500	0.4552	1.2267	1.0405	2.8353
RV DX	0.4613	1.2565	0.9996	2.7461
RSVN DX	0.4547	1.2401	0.9970	2.6678
RSVP DX	0.4681	1.2207	0.9919	2.7872
RV TBILL	0.4473	1.2114	0.9758	2.5863
RSVN TBILL	0.4395	1.2110	0.9672	2.6455
RSVP TBILL	0.4478	1.2101	0.9759	2.7039

Models - only RV	Non-energy total	Consumer goods	Business equipment	Motor veh. and parts
AR(3)	0.4350	0.3889	0.6892	2.1753
RV WTI	0.4244	0.4110	0.6490	2.3016
RSVN WTI	0.4196	0.4098	0.6382	2.3092
RSVP WTI	0.4398	0.4130	0.6485	2.2128
RV SP500	0.4499	0.3800	0.6965	2.3969
RSVN SP500	0.4368	0.3925	0.7171	2.3749
RSVP SP500	0.4559	0.3742	0.6877	2.4213
RV DX	0.4686	0.4094	0.7406	2.2690
RSVN DX	0.4627	0.3959	0.7283	2.2280
RSVP DX	0.4731	0.4035	0.7461	2.2934
RV TBILL	0.4266	0.4056	0.7054	2.2308
RSVN TBILL	0.4169	0.4077	0.7078	2.2319
RSVP TBILL	0.4461	0.4025	0.6904	2.2140

Table 5.5 The results of the MAE loss function for all industrial production measures. In the first column, excluding the first model, which is a simple AR(3), the high frequency variable added in the MIDAS models is presented at each row. It is noted that no monthly explanatory variable is included as predictor in the models.

Models - all exog. var.	Total index	Energy total	Primary energy	Drill. oil and gas wells
ARX(3)	0.4521	1.2906	1.0963	3.0946
RV WTI	0.4148	1.0968	0.9406	3.1512
RSVN WTI	0.4200	1.1353	1.0159	3.2919
RSVP WTI	0.4129	1.1101	0.9182	2.9193
RV SP500	0.4670	1.3703	1.1465	3.3521
RSVN SP500	0.4494	1.3999	1.1360	3.4548
RSVP SP500	0.4400	1.3384	1.1442	3.2996
RV DX	0.4605	1.3462	1.1183	3.0821
RSVN DX	0.4742	1.3284	1.1102	3.0594
RSVP DX	0.4613	1.3335	1.1241	3.1672
RV TBILL	0.4740	1.3052	1.1165	3.2329
RSVN TBILL	0.4566	1.3065	1.0951	3.1121
RSVP TBILL	0.4378	1.3132	1.1285	3.0838

Models - all exog. var.	Non-energy total	Consumer goods	Business equipment	Motor veh. and parts
ARX(3)	0.4444	0.4470	0.6834	2.2179
RV WTI	0.4442	0.4708	0.6851	2.3277
RSVN WTI	0.4444	0.4711	0.7125	2.4441
RSVP WTI	0.4316	0.4587	0.7096	2.2698
RV SP500	0.4623	0.4300	0.7292	2.3664
RSVN SP500	0.4503	0.4461	0.7321	2.3784
RSVP SP500	0.4578	0.4277	0.7231	2.3683
RV DX	0.4525	0.4603	0.7187	2.3726
RSVN DX	0.4519	0.4480	0.7200	2.4022
RSVP DX	0.4552	0.4558	0.7273	2.3970
RV TBILL	0.4498	0.4616	0.7265	2.3892
RSVN TBILL	0.4281	0.4622	0.7279	2.3893
RSVP TBILL	0.4562	0.4782	0.6982	2.3649

Table 5.6 The results of the MAE loss function for all industrial production measures. In the first column, excluding the first model, which is a simple AR(3), the high frequency variable added in the MIDAS models is presented at each row. It is noted that all monthly explanatory variables are also included as predictors in the models.

With regard to the non energy-related industrial production measures, the results for both MSPE and MAE are mixed. In general, the MIDAS models including both realized volatility measures cannot beat the simple AR(3) and ARX(3) models. This happens only in case of business equipment, where crude oil realized negative semivariance can reduce forecasting error by 2% in comparison with the results of a simple AR(3) model. Concerning the predictive ability of the MIDAS models including realized volatility of the other assets, the results show that the difference is almost zero, which means that the simple models do not benefit from the inclusion of higher frequency variables representing uncertainty in financial markets.

Finally, we compare the results of the models including the monthly explanatory variables and those that have only realized volatility measures as potential determinants of the industrial production measures. It is obvious that MIDAS models including only realized volatility measures and particularly those of WTI crude oil, outperform the models including additional explanatory variables, namely the macroeconomic, oil-related and uncertainty-related variables. For example, in the case of primary energy industrial production, the MIDAS models without monthly explanatory variables can reduce the forecasting error by more than 20%, which means that daily realized volatility measures can adequately improve the forecasting performance of a simple model. Qualitatively similar are the results for the non energy-related industrial production measures, even if, in this case, a simple AR(3) seems to perform better than more sophisticated models.

5.6 Conclusion

In this chapter, we focus on the impact of daily crude oil realized volatility measures on special aggregates of industrial production in an out-of-sample investigation. More specifically, models that incorporate variables at both monthly and daily frequencies are constructed with the main target of generating industrial production forecasts. To the best of our knowledge, there is no existing work doing that. The contribution of this study is manifold. First of all, we propose models with daily crude oil realized volatility measures, constructed by using intraday data, for forecasting special aggregates of industrial production in order to find whether the forecasting performance of state of the art models is enhanced. Moreover, we investigate whether realized volatility of other assets can offer forecasting gains and if so which aggregates of industrial production can benefit from the inclusion of these measures. Finally, we select a group of potential determinants of industrial production, including macroeconomic variables, indicators of uncertainty and other oil-related variables and their predictive information is investigated in an out-of-sample analysis.

The values of the statistical loss functions show that, in case of energy-related industrial production measures, the forecasting performance of the MIDAS models including WTI crude oil realized positive semivariance is significantly improved compared to simpler models that do not include daily realized volatility measures. This holds true for both MSPE and MAE results. With regard to the non energy-related industrial production measures, it is observed that neither crude oil realized volatility nor realized volatility of other assets can help to improve the forecasting accuracy. However, there is only one case, that of business equipment excluding motor vehicles, where the MIDAS model with crude oil realized negative semivariance as predictor beats the remaining models, even those that incorporate other exogenous variables, such as factors of uncertainty. The main conclusion is that daily WTI crude oil price volatility is a significant driver for energy-related industrial production aggregates and has higher predictive information compared to several macroeconomic variables, oil fundamentals and indicators of uncertainty. This provides professional forecasters and policy makers with evidence that they should use the MIDAS modeling framework to forecast monthly industrial production. For the non energy-related industrial production aggregates, we draw the conclusion that more sophisticated models including exogenous information cannot beat a simple AR(3) model.

The results of this chapter might be inspiring for academics and policy makers and open new avenues for future research on this area. In this regard, the same methodology could be applied for generating other macroeconomic variables that reflect economic outlook in the U.S. or in other countries. Moreover, the evaluation framework could be enhanced by using more statistical tests that can further justify the importance of oil price volatility for industrial production forecasts. Finally, researchers can change the MIDAS model into a time-varying MIDAS model by taking into account potential structural changes in the macroeconomic environment.

Chapter Six

Thesis Conclusion

First of all, it would be important to mention that through this dissertation we tried to create a story that starts from what matters when developing crude oil volatility forecasting frameworks and which exogenous factors have high predictive information on crude oil realized volatility afterwards. However, since crude oil investors seek ways that will help them minimize the risk of their portfolios, we investigated whether there are hedging opportunities for portfolios comprised of crude oil volatility and other assets' volatility. Finally, the importance of the daily crude oil realized volatility on the U.S. economic outlook is studied, which suggests that policy makers should take crude oil volatility into account through the decision making process. Therefore, we hope that the overview of this thesis presented above provides evidence for an integrated analysis on the significance and uniqueness of the crude oil realized volatility and we believe that this dissertation is going to have practical usefulness and be material for future research on this topic.

More specifically, in this PhD thesis we bring together all the different elements in this line of research and provide an answer as to what matters most for multi-step forecasts of crude oil RV. Previous studies have shown that in the case of WTI crude oil the HAR-RV model outperforms all other competing forecasting models. Therefore, the first study (Chapter 2) aims to improve the forecasting performance of the HAR-RV model in order to capture the changes of the coefficients by allowing time-variation in HAR-type models' parameters and by implementing not only the direct approach but also the iterated methodology for obtaining multi-period forecasts for RV and OVX. The goals of this study are considered of major importance for the rest of the thesis, since we rely on these RV measures in order to find potential hedging opportunities and to study the effect of these RV measures on the U.S. economic outlook. Therefore, evaluating the aforementioned models, it is obvious for the case of RV that the impact of realized semi variance components is highly significant, and TVP models which include these components outperform the remaining models at all forecasting horizons. Regarding the OVX forecasts, it is interesting to note that iterated forecasts including continuous components from the decomposition of quadratic variation provide much better performance in longer horizons, relative to other models. It is important to mention here that developing MATLAB and R scripts for the entire methodological framework presented above was one of the main challenges of this part of the thesis.

Apart from the methodological procedures, it is important to identify the drivers of crude oil price volatility in an out-of-sample analysis. According to the second study (Chapter 3), it is suggested that forecasters should take into account the fact that IV indices, namely VIX, VXD and VXN, enhance the predictive accuracy of oil price volatility at short-run horizons, while the economic policy uncertainty and geopolitical risk indices are information-rich at mid- and long-run horizons. Finally, the proper combination of the aforementioned indicators, which is done using the DMA approach, is considered crucial for forecasting oil price volatility and can also generate high trading returns. The findings of the Chapters 2 and 3 could be considered useful for investors, who search for accurate realized volatility forecasts in order to maximize their profits. With regard to the latter implication, we would like to provide some suggestions for future research.

In the third study of the thesis (Chapter 4), in which we examine whether there are hedging opportunities between WTI crude oil volatility and volatility measures of three other assets classes, the variance-covariance matrix is used for implementing a hedging strategy based on the computation of the optimal portfolio weights. The empirical findings reveal that using the optimal portfolio weights in the WTI - TBILLS portfolio is beneficial for WTI crude oil volatility investors with hedging effectiveness reaching 95%. The U.S. dollar index is also considered an efficient asset for hedging WTI crude oil volatility, whereas the S&P500 could be less appropriate for hedging purposes with hedging effectiveness ranging from 47.7% to 80% for volatile and tranquil periods, respectively.

According to the final study of the thesis (Chapter 5), in which we study whether the crude oil volatility measures are important for the U.S. economy, the forecasting performance of the MIDAS models including WTI crude oil realized positive semivariance is significantly improved compared to the remaining models in the case of energy-related industrial production. This provides professional forecasters and policy makers evidence that they should use the MIDAS modeling framework to forecast monthly industrial production. With regard to the non energy-related industrial production measures, it is observed that neither crude oil RV nor RV of other assets can help to improve the forecasting accuracy in terms of either statistical loss function.

One of the main limitations of the present thesis is that the covid-19 period is not included in the analysis due to the fact that our intra-day dataset is limited to 7 years (from 2010 to 2017). The crude oil market was subject to severe shocks during the covid-19 pandemic, which could be interesting to be investigated in depth in terms of out-of-sample forecasting. We would therefore motivate future studies to focus on this specific period (02/2019-now) and check whether our findings hold true for the latter period as well. Our 7-years dataset (intra-day data) creates another limitation when referring to Chapter 5, which focuses on forecasting monthly industrial production. More specifically, we could have developed forecasting frameworks for the U.S. GDP and its main components but the small sample period does not allow that because of the quarterly frequency of the macroeconomic variables. Finally, in the present thesis we develop forecasting frameworks for the crude oil market, which could be enhanced by developing modelling frameworks for other markets, as well, in order to compare the findings.

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APPENDIX

Appendix One

Chapter 2 - TVP equations

The one-step ahead forecast and posterior distributions are developed with the significant contribution of kalman filtering, which helps us update the equations of the system. Kalman filtering begins with the result:

$$(\boldsymbol{\alpha}_{t-1} \mid y^{t-1}) \sim N(\hat{\boldsymbol{\alpha}}_{t-1}, \boldsymbol{\Sigma}_{t-1|t-1}), \tag{A.1}$$

Kalman filtering proceeds using:

$$(\boldsymbol{\alpha}_{t} \mid y^{t-1}) \sim N(\hat{\boldsymbol{\alpha}}_{t-1}, \boldsymbol{\Sigma}_{t|t-1}), \tag{A.2}$$

where $\boldsymbol{\Sigma}_{t|t-1} = \boldsymbol{\Sigma}_{t-1|t-1} + \boldsymbol{\Sigma}_{u_t}$.

Raftery et al. (2010) note that things simplify substantially if this latter equation is replaced by:

$$\boldsymbol{\Sigma}_{t|t-1} = \frac{1}{\lambda}\boldsymbol{\Sigma}_{t-1|t-1}. \tag{A.3}$$

Involving the forgetting factor λ ($0 \leqslant \lambda \leqslant 1$), that refers to a gradual evolution of coefficients[1]. It is common to choose a value of λ near one, suggesting a gradual evolution of coefficients. Take into consideration Raftery et al. (2010), we also set it $\lambda = 0.99$. Moreover, we have $\boldsymbol{\Sigma}_{u_t} = (1-\lambda^{-1})\boldsymbol{\Sigma}_{t-1|t-1}$.

[1]In econometrics the forgetting factor approach has long been implemented in the state space literature going back to Fagin (1964) and Jazwinsky (1970). The name "forgetting factor" is proposed by the fact that this specification implies that observations j periods in the past are weighted by λ^j.

However, according to Grassi et al. (2017), the updating equation of $\boldsymbol{\Sigma_{t|t-1}}$ in (A.2) is perturbed by a function of the squared prediction errors. Thus, the equation (A.3) is replaced by

$$\boldsymbol{\Sigma_{t|t-1}} = \boldsymbol{\Sigma_{t-1|t-1}}. \tag{A.4}$$

We should also note at this point that under the contribution of those two specifications, we no longer have to estimate $\boldsymbol{\Sigma_{u_t}}$. Kalman filtering is then completed by the updating equation:

$$(\boldsymbol{\alpha_t} \mid y^t) \sim N(\boldsymbol{\hat{\alpha}_t}, \boldsymbol{\Sigma_{t|t}}), \tag{A.5}$$

where

$$\boldsymbol{\hat{\alpha}_{t|t}} = \boldsymbol{\hat{\alpha}_{t|t-1}} + \boldsymbol{\Sigma_{t|t-1}} \boldsymbol{x_t'} (\hat{H}_t + \boldsymbol{x_t} \boldsymbol{\Sigma_{t|t-1}} \boldsymbol{x_t'})^{-1} (y_t - \boldsymbol{x_t} \boldsymbol{\hat{\alpha}_{t-1}}), \tag{A.6}$$

and

$$\boldsymbol{\Sigma_{t|t}} = \boldsymbol{\Sigma_{t|t-1}} - \boldsymbol{\Sigma_{t|t-1}} \boldsymbol{x_t'} (\hat{H}_t + \boldsymbol{x_t} \boldsymbol{\Sigma_{t|t-1}} \boldsymbol{x_t'})^{-1} \boldsymbol{x_t} \boldsymbol{\Sigma_{t|t-1}}, \tag{A.7}$$

for the case of the forgetting factor's approximation. Nevertheless, the updating step in (A.7) is replaced by

$$\boldsymbol{\Sigma_{t|t}} = \boldsymbol{\Sigma_{t|t-1}} - \boldsymbol{\Sigma_{t|t-1}} \boldsymbol{x_t'} (\hat{H}_t + \boldsymbol{x_t} \boldsymbol{\Sigma_{t|t-1}} \boldsymbol{x_t'})^{-1} \boldsymbol{x_t} \boldsymbol{\Sigma_{t|t-1}} + \beta \cdot max\left[0, FL\left(\frac{\varepsilon_t^2}{\hat{H}_t} - 1\right)\right] \cdot I, \tag{A.8}$$

where $\varepsilon_t = y_t - \boldsymbol{x_t} \boldsymbol{\hat{\alpha}_{t-1}}$ and the estimated error variance is calculated by the following[2]:

$$\hat{H}_t = \kappa \hat{H}_{t-1} + (1-\kappa)\varepsilon_t^2. \tag{A.9}$$

[2]The design parameters β and κ are set as 1e-10 and 0.94, respectively.

Recursive forecasting is implemented by using the predictive distribution,

$$(y_t \mid y^{t-1}) \sim N(\boldsymbol{x_t}\hat{\boldsymbol{\alpha}}_{\boldsymbol{t-1}}, \hat{H}_t + \boldsymbol{x_t}\boldsymbol{\Sigma}_{\boldsymbol{t|t-1}}\boldsymbol{x_t'}). \tag{A.10}$$

According to Koop and Korobilis (2012), these results are analytical and, thus, Markov chain Monte Carlo (MCMC) algorithm is not required.

Appendix Two

Chapter 2 - Results based on the MAE

Realized volatility - MAE ratios of HAR-type models to the RW model

Days ahead	1	5	10	15	22	44	66
RW	6.63	8.37	9.44	9.85	11.01	12.52	14.11
OLS - DIRECT							
HAR-RV	0.86	0.86	0.85	0.89	0.86	0.88	0.84
HAR-C	0.86	0.86	0.86	0.89	0.87	0.89	0.84
HAR-RSV	0.86	0.86	0.85	0.88	0.85	0.89	0.86
OLS - ITERATED							
HAR-RV	0.86	0.86	0.85	0.89	0.86	0.88	0.86
HAR-C	0.86	0.86	0.86	0.90	0.87	0.89	0.86
HAR-RSV	0.86	0.85	0.84	0.88	0.85	0.87	0.86
TVP - DIRECT							
HAR-RV	0.86	0.85	0.85	0.88	0.86	0.89	0.87
HAR-C	0.86	0.85	0.85	0.88	0.86	0.89	0.87
HAR-RSV	0.86	0.85	0.83	0.85	0.84	0.87	0.87
TVP - ITERATED							
HAR-RV	0.86	0.85	0.84	0.87	0.83	0.83	0.81
HAR-C	0.86	0.85	0.84	0.87	0.84	0.83	0.81
HAR-RSV	0.86	0.84	0.83	0.86	0.82	0.83	0.82

Table B.1 The results of the MAE loss function for different forecasting horizons regarding RV forecasting errors. Values represent ratios of HAR-type models to the RW model. A ratio below 1 suggests that MAE of the corresponding HAR-type model outperforms that of the RW model. We present the actual MAE values only for the RW model.

Oil price implied volatility index - MAE ratios of HAR-type models to the RW model

Days ahead	1	5	10	15	22	44	66
RW	1.37	2.94	3.97	4.90	5.84	8.39	10.37
OLS - DIRECT							
HAR-OVX	1.00	1.01	1.02	1.02	1.02	0.97	0.93
HAR-OVX-C	2.75	1.52	1.32	1.22	1.19	1.05	0.94
HAR-OVX-RSV	2.61	1.48	1.29	1.20	1.17	1.05	0.95
OLS - ITERATED							
HAR-OVX	1.00	1.02	1.03	1.01	1.01	1.00	0.97
HAR-OVX-C	2.75	1.52	1.29	1.20	1.13	1.01	0.93
HAR-OVX-RSV	2.61	1.46	1.23	1.15	1.09	0.99	0.93
TVP - DIRECT							
HAR-OVX	1.00	0.99	1.00	0.99	1.00	0.99	0.99
HAR-OVX-C	2.74	1.54	1.37	1.25	1.21	1.12	1.03
HAR-OVX-RSV	2.56	1.53	1.34	1.26	1.20	1.11	1.05
TVP - ITERATED							
HAR-OVX	1.00	1.01	1.02	1.00	0.99	0.95	0.91
HAR-OVX-C	2.74	1.49	1.27	1.17	1.10	0.96	0.87
HAR-OVX-RSV	2.56	1.46	1.23	1.14	1.07	0.95	0.88

Table B.2 The results of the MAE loss function for different forecasting horizons regarding OVX forecasting errors. Values represent ratios of HAR-type models to the RW model. A ratio below 1 suggests that MAE of the corresponding HAR-type model outperforms that of the RW model. We present the actual MAE values only for the RW model.

Appendix Three

Chapter 3 - DMA approach

In this part, we concentrate on the one-step ahead forecasting procedure in order to show the updating steps of the DMA approach in detail.

The main methodological approach of the updating equations of the TVP model is based on the Kalman filter, which begins with the result:

$$(\boldsymbol{\alpha_{t-1}} \mid y^{t-1}) \sim N(\boldsymbol{\hat{\alpha}_{t-1}}, \boldsymbol{\Sigma_{t-1|t-1}}), \tag{C.1}$$

The Kalman filtering process proceeds as follows:

$$(\boldsymbol{\alpha_{t}} \mid y^{t-1}) \sim N(\boldsymbol{\hat{\alpha}_{t-1}}, \boldsymbol{\Sigma_{t|t-1}}), \tag{C.2}$$

where $\boldsymbol{\Sigma_{t|t-1}} = \boldsymbol{\Sigma_{t-1|t-1}} + \boldsymbol{\Sigma_{u_t}}$.

Since we are motivated by the approach that Grassi et al. (2017) propose, the updating equation of $\boldsymbol{\Sigma_{t|t-1}}$ is perturbed by a function of the squared prediction errors, which is shown in the updating steps. At this step, we assume the following:

$$\boldsymbol{\Sigma_{t|t-1}} = \boldsymbol{\Sigma_{t-1|t-1}}. \tag{C.3}$$

At this point, we have to mention that due to the fact that we use the aforementioned approach, we no longer have to estimate $\boldsymbol{\Sigma_{u_t}}$. Kalman filter procedure is completed by the updating equation:

$$(\boldsymbol{\alpha_{t}} \mid y^{t}) \sim N(\boldsymbol{\hat{\alpha}_{t}}, \boldsymbol{\Sigma_{t|t}}), \tag{C.4}$$

where

$$\boldsymbol{\hat{\alpha}_{t|t}} = \boldsymbol{\hat{\alpha}_{t|t-1}} + \boldsymbol{\Sigma_{t|t-1}} \boldsymbol{x_t'} (\hat{H}_t + \boldsymbol{x_t} \boldsymbol{\Sigma_{t|t-1}} \boldsymbol{x_t'})^{-1} (y_t - \boldsymbol{x_t} \boldsymbol{\hat{\alpha}_{t-1}}), \tag{C.5}$$

and

$$\boldsymbol{\Sigma_{t|t}} = \boldsymbol{\Sigma_{t|t-1}} - \boldsymbol{\Sigma_{t|t-1}} \boldsymbol{x_t'} (\hat{H}_t + \boldsymbol{x_t} \boldsymbol{\Sigma_{t|t-1}} \boldsymbol{x_t'})^{-1} \boldsymbol{x_t} \boldsymbol{\Sigma_{t|t-1}} + \beta \cdot max\Big[0, FL\Big(\frac{\varepsilon_t^2}{\hat{H}_t} - 1\Big)\Big] \cdot I, \tag{C.6}$$

where $\varepsilon_t = y_t - \boldsymbol{x_t} \boldsymbol{\hat{\alpha}_{t-1}}$ and the estimated error variance is calculated by the following[1]:

$$\hat{H}_t = \kappa \hat{H}_{t-1} + (1-\kappa)\varepsilon_t^2. \tag{C.7}$$

Recursive forecasting is implemented by using the predictive distribution,

$$(y_t \mid y^{t-1}) \sim N(\boldsymbol{x_t} \boldsymbol{\hat{\alpha}_{t-1}}, \hat{H}_t + \boldsymbol{x_t} \boldsymbol{\Sigma_{t|t-1}} \boldsymbol{x_t'}). \tag{C.8}$$

After having estimated each individual model of the K combinations under the TVP approach, which is explained analytically in the previous part, the DMA averages the forecasts obtained by the individual models using $\pi_{t|t-1,k}$ as weights for $k = 1, \dots, K$ over the out-of-sample period. Those DMA forecasts can be expressed as:

$$E(y_t \mid y^{t-1}) = \sum_{k=1}^{K} \pi_{t|t-1,k} \boldsymbol{x_{t-1}^{(k)}} \boldsymbol{\hat{\alpha}_{t-1}^{(k)}} \tag{C.9}$$

where $\boldsymbol{\hat{\alpha}_{t-1}^{(k)}}$ are the Kalman filter estimates of the state-space model at time $t-1$.

At this point, probability in the forecasting model has to be determined. As proposed by Raftery et al. (2010), the relation between $\pi_{t|t-1,k}$ and $\pi_{t-1|t-1,k}$ is described as:

[1] The design parameters β and κ are set as 1e-10 and 0.94, respectively.

$$\pi_{t|t-1,k} = \frac{\pi^{\alpha}_{t-1|t-1,k}}{\sum_{l=1}^{K} \pi^{\alpha}_{t-1|t-1,l}} \tag{C.10}$$

where $0 < \alpha \leqslant 1$ is a forgetting factor[2], which is constant and smaller than 1.

The updating equation is defined as follows:

$$\pi_{t|t,k} = \frac{\pi_{t|t-1,k} f_k(y_t \mid y^{t-1})}{\sum_{l=1}^{K} \pi_{t|t-1,l} f_l(y_t \mid y^{t-1})} \tag{C.11}$$

where $f_k(y_t \mid y^{t-1})$ is the predictive density of model k. The main idea of this updating equation is that a model, which had a better forecasting performance in the past, will receive higher weight at time t.

[2] In this study, we follow Koop and Korobilis (2012) in setting $\alpha = 0.99$.